

PERIODIC SYSTEM OF CLOSELY LOCATED HOLES IN A QUASIORTHOTROPIC PLANE

M. P. Savruk,^{1,2} A. Kazberuk,³ and A. B. Chornen'kyi¹

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We study a plane periodic problem of the theory of elasticity for a quasiorthotropic plane with an infinite series of closely located curvilinear holes by the method of singular integral equations. By using a unified approach to the solution of the problems of stress concentration factors near holes with sharp and rounded vertices, we obtain the stress intensity factors at sharp vertices of the holes. The stress concentration factors at rounded and sharp tips of two-sided parabolic and U-shaped notches are found as a result of the limit transition.

Keywords: quasiorthotropic material, periodic system of holes, method of singular integral equations, stress intensity factor, two-sided notch.

Periodic problems of the theory of elasticity for a plane with infinite series of smooth curvilinear holes were studied by numerous researchers. The numerical values of stress concentration factors (SCF) were, for the most part, obtained for elliptic and circular holes in isotropic [1–4] and anisotropic [5–8] planes for large distances between these holes. The stress intensity factors (SIF) and SCF at sharp and rounded vertices of closely located holes in an isotropic plane were found by the method of singular integral equations (SIE) [9, 10] with the use of a unified approach to the solution of the problems of stress concentration near the holes with the same vertices.

In what follows, by using this approach, we solve a plane periodic problem for the quasiorthotropic plane weakened by an infinite series of closely located curvilinear holes. In the case where the holes approach each other, from this solution, we obtain the SCF at the rounded tips of two-sided notches in the quasiorthotropic plane as a result of the limit transition.

Integral Equation of the Problem

Consider a quasiorthotropic plane (degenerate orthotropic material [10–15]) weakened by a periodic series of curvilinear holes. The origin of a Cartesian coordinate system Oxy is located inside a hole with smooth contour L in the main strip of periods with width d . The Ox - and Oy -axes are directed along the principal axes of orthotropy. The holes are uniformly distributed along the Ox -axis. As a positive direction, we choose the direction of traversing the contour L for which the domain of the body S remains to the left (Fig. 1).

At infinity, the quasiorthotropic plane is stretched by stresses

$$\sigma_x^\infty = q \quad \text{and} \quad \sigma_y^\infty = p$$

¹ Karpenko Physicomechanical Institute, Ukrainian National Academy of Sciences, Lviv, Ukraine.

² Corresponding author; e-mail: savruk@ipm.lviv.ua.

³ Politechnika Białostocka, Białystok, Poland.

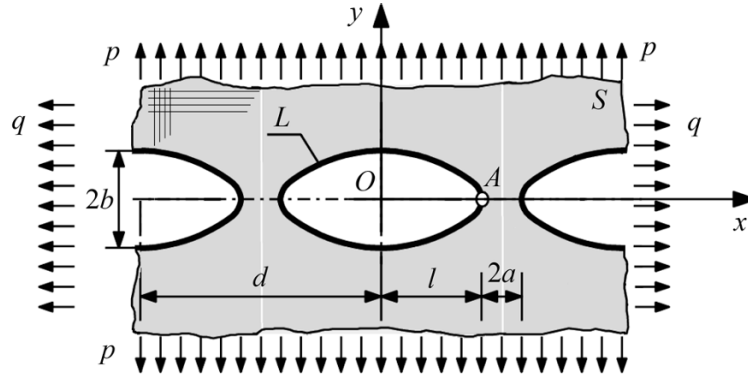


Fig. 1. Periodic system of curvilinear holes in a quasiorthotropic plane.

and, on the edges of the holes, we specify a self-balanced load

$$N^*(t) + iT^*(t) = p_*(t), \quad t \in L,$$

where $N^*(t)$ and $T^*(t)$ are, respectively, the normal and tangential components of the vector of stresses.

The problem is solved by the method of SIE [9, 10]. We use the method of superposition and seek complex stress potentials for an orthotropic body in the form [16]

$$\Phi_1^*(z_1) = \Phi_1^0(z_1) + \Phi_1(z_1), \quad \Psi_1^*(z_1) = \Psi_1^0(z_1) + \Psi_1(z_1),$$

where

$$\Phi_1^0(z_1) = \frac{p + \gamma^{-2}q}{4} \quad \text{and} \quad \Psi_1^0(z_1) = \frac{p - \gamma^{-2}q}{2} + i\gamma^{-1}\tau$$

are given functions that describe the homogeneous stress-strain state in a solid quasiorthotropic plane without holes and the potentials $\Phi_1(z_1)$ and $\Psi_1(z_1)$ determine the perturbed state caused by the holes. Here, $z_1 = x + i\gamma y$ is a complex variable and γ is the parameter of orthotropy. In the plane stressed state, this parameter has the form $\sqrt[4]{E_x/E_y}$ [12, 13].

By using the analogy between the plane problems of the theory of elasticity for isotropic and quasiorthotropic bodies [12], we choose the potentials $\Phi_1(z_1)$ and $\Psi_1(z_1)$ for the quasiorthotropic plane with periodic system of holes in the form

$$\begin{aligned} \Phi_1(z_1) &= \frac{1}{2d} \int_{L_1} \cot \frac{\pi}{d} (t_1 - z_1) g'_1(t_1) dt_1, \\ \Psi_1(z_1) &= \frac{1}{2d} \int_{L_1} \left\{ \cot \frac{\pi}{d} (t_1 - z_1) \overline{g'_1(t_1)} d\bar{t}_1 - \left[\cot \frac{\pi}{d} (t_1 - z_1) + \frac{\pi}{d} (\bar{t}_1 - t_1 + z_1) \csc^2 \frac{\pi}{d} (t_1 - z_1) \right] g'_1(t_1) dt_1 \right\}. \end{aligned} \quad (1)$$

Here, $g'_1(t_1)$ is an unknown continuous complex-valued function given on a contour L_1 in the plane z_1 that corresponds to the contour L in the plane z . Subscript “1” in all complex quantities indicates that they are

related to the plane $z_1 = x + i\gamma y$. Setting the parameter $\gamma = 1$, we arrive at the corresponding relations (without subscript 1) for the isotropic material [16]. The parameters corresponding to the planes z and z_1 satisfy certain relationships, namely,

$$\begin{aligned} t_1 &= \frac{1}{2}[(1+\gamma)t + (1-\gamma)\bar{t}], \\ \frac{dt_1}{dt} &= \frac{1}{2}\left[(1+\gamma) + (1-\gamma)\frac{d\bar{t}}{dt}\right]. \end{aligned} \quad (2)$$

For the determination of the perturbed state, we get the following boundary condition on the contour L :

$$N(t) + iT(t) = p(t) = p_*(t) - \left\{ \Phi_0(t) + \overline{\Phi_0(t)} + \frac{d\bar{t}}{dt} [t\overline{\Phi'_0(t)} + \overline{\Psi_0(t)}] \right\}, \quad t \in L, \quad (3)$$

and, in addition, the stresses vanish at infinity.

Let X_n and Y_n be the Cartesian components of the vector of stresses acting on the curvilinear contour L from the outer normal n . They are related to the normal and tangential components of stresses N and T by the formula [17]

$$X_n + iY_n = -i \frac{dt}{ds} (N + iT) = \frac{d}{ds} \left(\frac{\partial F}{\partial y} - i \frac{\partial F}{\partial x} \right), \quad (4)$$

where s is the angular abscissa on the contour L corresponding to the point $t = x + iy \in L$.

By using representations (3) and (4), we find

$$\left(\left(\frac{i}{\gamma} \right) X_n - Y_n \right) \frac{ds}{dt_1} = \Phi_1(t_1) + \overline{\Phi_1(t_1)} + \frac{d\bar{t}_1}{dt_1} (t_1 \overline{\Phi'_1(t_1)} + \overline{\Psi_1(t_1)}), \quad t_1 \in L_1.$$

Satisfying the boundary condition (3) with the help of potentials (1), we reduce the formulated boundary-value problem to the SIE [10]

$$\frac{1}{\pi} \int_{L_1} [K_1(\tau_1, t_1) g'_1(\tau_1) d\tau_1 + L_1(\tau_1, t_1) \overline{g'_1(\tau_1)} d\bar{\tau}_1] - \frac{1}{2i} \frac{M_0}{(t_1)^2} \frac{d\bar{t}_1}{dt_1} + \frac{a_0}{l} \frac{ds_1}{dt_1} = \tilde{P}_1(t_1), \quad t_1 \in L_1 \quad (5)$$

for the determination of the unknown function $g'_1(t_1)$. To the left-hand side of Eq. (5), we add the following functionals that are equal to zero (under the condition of self-balanced loading on the edge of the hole):

$$M_0 = i \int_{L_1} [\bar{\tau}_1 g'_1(\tau_1) d\tau_1 - \tau_1 \overline{g'_1(\tau_1)} d\bar{\tau}_1], \quad a_0 = \int_{L_1} g'_1(\tau_1) d\tau_1.$$

These functionals guarantee the uniqueness of solution for an arbitrary right-hand side $\tilde{P}_1(t_1)$ [16]. In this case, it is also taken into account that the origin of the coordinates $z = 0 \notin S$ is located inside the hole, s_1 is the angular abscissa of the point t_1 on the contour L_1 , and l is an arbitrary parameter with the dimension of length.

The kernels and the right-hand side of the integral equation (5) are given by the formulas

$$K_1(\tau_1, t_1) = \frac{\pi}{2d} \left[\cot \frac{\pi}{d} (\tau_1 - t_1) + \frac{d\bar{t}_1}{dt_1} \cot \frac{\pi}{d} (\bar{\tau}_1 - \bar{t}_1) \right],$$

$$L_1(\tau_1, t_1) = \frac{\pi}{2d} \left[\left(1 - \frac{d\bar{t}_1}{dt_1} \right) \cot \frac{\pi}{d} (\bar{\tau}_1 - \bar{t}_1) - \frac{\pi}{d} \frac{d\bar{t}}{dt_1} (\tau_1 - t_1 + \bar{t}_1 - \bar{\tau}_1) \csc^2 \frac{\pi}{d} (\bar{\tau}_1 - \bar{t}_1) \right],$$

$$\tilde{P}_1(t_1) = \frac{1}{2\gamma} \left[(1 + \gamma)p(t) - (1 - \gamma)\overline{p(\bar{t})} \frac{d\bar{t}}{dt} \right] \frac{dt}{dt_1}.$$

We represent the parametric equation of the contour L in the form

$$\tau = l\omega(\xi), \quad \tau \in L, \quad 0 \leq \xi \leq 2\pi.$$

After the corresponding change of variables ($t = l\omega(\eta)$), we reduce the integral equation (5) to the dimensionless canonical form

$$\frac{1}{\pi} \int_0^{2\pi} [M_1(\xi, \eta)u_1(\xi) + N_1(\xi, \eta)\overline{u_1(\xi)}] d\xi = \tilde{P}_1(\eta), \quad 0 \leq \eta \leq 2\pi, \quad (6)$$

where

$$M_1(\xi, \eta) = lK_1(l\omega_1(\xi), l\omega_1(\eta)) - \frac{\overline{\omega_1(\xi)}}{2\omega_1^2(\eta)} \frac{\overline{\omega_1'(\eta)}}{\omega_1'(\eta)} + \frac{|\omega_1'(\eta)|}{\omega_1'(\eta)},$$

$$N_1(\xi, \eta) = lL_1(l\omega_1(\xi), l\omega_1(\eta)) + \frac{\omega_1(\xi)}{2\omega_1^2(\eta)} \frac{\overline{\omega_1'(\eta)}}{\omega_1'(\eta)},$$

$$u_1(\xi) = g_1'(\omega_1(\xi)) \frac{\omega_1'(\xi)}{p}, \quad \tilde{P}_1(\eta) = \frac{\tilde{P}_1(\omega_1(\eta))}{p}, \quad l\omega_1(\eta) = t_1.$$

For the smooth curvilinear holes, the unknown 2π -periodic function $u_1(\xi)$ is continuous but, at the vertices of the contour with large linear curvature, it has quasi-singularities, which is explained by high levels of stress concentration. Thus, in order to improve the convergence of the numerical solution of the integral equation (6) in the case where the contour L has k cyclically located vertices, we use a nonlinear transformation of variables [18]

$$\xi = G(\tau), \quad 0 \leq \tau \leq 2\pi, \quad \eta = G(\theta), \quad 0 \leq \theta \leq 2\pi, \quad (7)$$

where

$$G(\tau) = \tau - \frac{1}{k} \sin k\tau, \quad 0 \leq \tau \leq 2\pi.$$

After the change of variables (7) in the integral equation (6), we get

$$\frac{1}{\pi} \int_0^{2\pi} \left[M_1(\xi, \eta) u_1(\tau) + N_1(\xi, \eta) \overline{u_1(\tau)} \right] G'(\tau) d\tau = \tilde{P}_1(\theta), \quad 0 \leq \theta \leq 2\pi, \quad (8)$$

where

$$u_1(\tau) = u_1(G(\tau)), \quad \tilde{P}_1(\theta) = \tilde{P}_1(G(\theta)).$$

Solving the SIE (8) by the method of quadrature, we arrive at a system of linear algebraic equations [9, 10, 14, 16]

$$\frac{1}{2n} \sum_{k=1}^{4n} \left[M_1(\xi_k, \eta_m) u_1(\tau_k) + N_1(\xi_k, \eta_m) \overline{u_1(\tau_k)} \right] G'_1(\tau_k) = \tilde{P}_1(\theta_m), \quad m = 1, \dots, 4n, \quad (9)$$

where the density of the quadrature nodes and collocation points

$$\xi_k = G(\tau_k), \quad \tau_k = \frac{\pi(2k-1)}{4n}, \quad k = 1, \dots, 4n,$$

$$\eta_m = G(\theta_m), \quad \theta_m = \frac{\pi(m-1)}{2n}, \quad m = 1, \dots, 4n$$

increases near the vertices of the hole as a result of transformation (7).

Assume that the contour L has two axes of geometric and force symmetry, which coincide with the Ox - and Oy -axes. Then the order of the system of algebraic equations (9) can be made four times lower. By using the corresponding conditions of double symmetry [10]

$$u_1(2\pi - \xi) = \overline{u_1(\xi)} \quad \text{and} \quad u_1(\pi - \xi) = -\overline{u_1(\xi)}$$

for the unknown function $u_1(\xi)$, we arrive at the following system of n complex linear algebraic equations:

$$\left\{ \begin{array}{l} \frac{1}{n} \operatorname{Re} \sum_{k=1}^n \left[M_1^*(\xi_k, \eta_m) u_1(\tau_k) \right] G'_1(\tau_k) = \tilde{P}_1(\theta_m), \quad m = 1, n+1, \\ \frac{1}{2n} \sum_{k=1}^n \left[M_1^*(\xi_k, \eta_m) u_1(\tau_k) + N_1^*(\xi_k, \eta_m) \overline{u_1(\tau_k)} \right] G'_1(\tau_k) = \tilde{P}_1(\theta_m), \quad m = 2, \dots, n, \end{array} \right. \quad (10)$$

for the evaluation of n unknown values of the complex functions $u_1(\tau_k)$, $k = 1, \dots, n$. Here, it is denoted

$$M_1^*(\xi_k, \eta_m) = M_1(\xi_k, \eta_m) - N_1(\pi - \xi_k, \eta_m) - M_1(\pi + \xi_k, \eta_m) + N_1(2\pi - \xi_k, \eta_m),$$

$$N_1^*(\xi_k, \eta_m) = N_1(\xi_k, \eta_m) - M_1(\pi - \xi_k, \eta_m) - N_1(\pi + \xi_k, \eta_m) + M_1(2\pi - \xi_k, \eta_m).$$

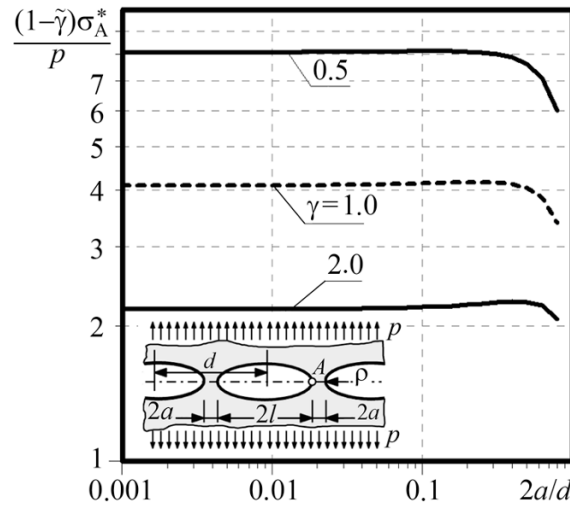


Fig. 2. Dependences of the function $(1-\tilde{\gamma})\sigma_A^*/p$ at the vertices of elliptic holes in a stretched elastic plane on the parameter $2a/d$ ($\chi = 0.1$).

After the solution of Eqs. (10), we determine the complex stress potentials (1) and (2) with the help of which we can find the stress-strain state in the entire elastic domain. The normal stresses σ^s along the edge of the hole are expressed directly via the obtained function $u_1(t)$ [9, 10].

Periodic System of Elliptic Holes

Consider a system of elliptic holes in an elastic quasiorthotropic plane stretched at infinity by stresses $\sigma_y^\infty = p$ in the direction perpendicular to the line of centers of the holes whose contours are free of loads ($\sigma_x^\infty = q = 0$, $p_*(t) = 0$). The semiaxes of the ellipse are denoted by l and b . We introduce the following dimensionless parameters: $\tilde{\gamma} = 2l/d$ is the ratio of the major semiaxis of the ellipse to the width of the band of periods d ; $(1-\tilde{\gamma}) = 2a/d$ is the relative distance between the holes, where a is the distance between the vertices of the neighboring contours; $\chi = \rho/a = \varepsilon\tilde{\gamma}/(1-\tilde{\gamma})$ is the ratio of the radius of curvature ρ to the distance between the contours, where $\varepsilon = \rho/l$ is the relative radius of curvature at the vertex of the hole (Fig. 1).

The numerical results required to find the SCF near the vertices of elliptic holes were obtained by the method of SIE [10] for the geometric sizes $0.001 \leq \chi \leq 1$ and $0.001 \leq 2a/d < 1$ and various values of the orthotropy parameter γ (Fig. 2).

On finding the SCF for a periodic system of elliptic holes in the elastic quasiorthotropic plane for small relative distances between the neighboring holes ($\tilde{\gamma} \rightarrow 1$) (Fig. 2), we can perform the limit transition to a two-sided symmetric parabolic notch in the quasiorthotropic plane stretched at infinity by the forces $P = pd$ (Fig. 3):

$$k_t = \frac{\sigma_S^*}{P/2a} = \lim_{\tilde{\gamma} \rightarrow 1} (1-\tilde{\gamma}) \frac{\sigma_A^*}{p}, \quad (11)$$

where σ_S^* is the normal load at the tip of this notch.

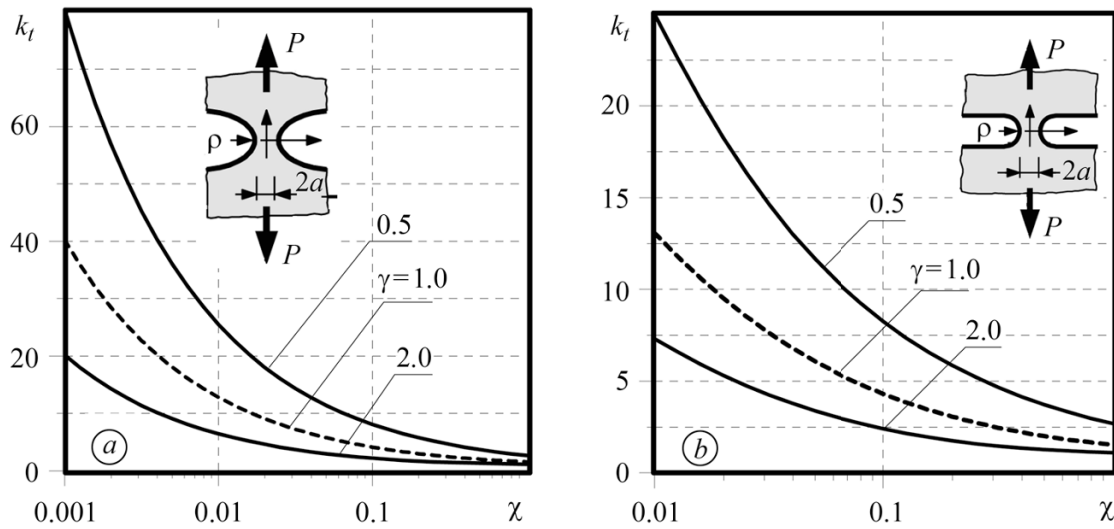


Fig. 3. Dependences of the SCF k_t at the tips of two-sided parabolic (a) and U-shaped (b) notches on the parameter $\chi = \rho/a$.

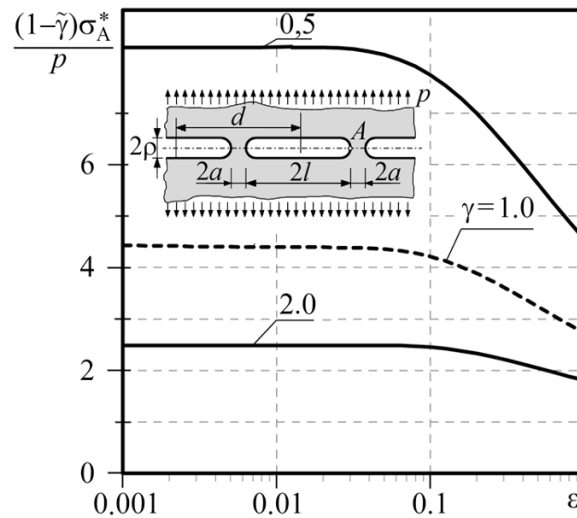


Fig. 4. Dependences of the function $(1 - \tilde{\gamma})\sigma_A^*/p$ at the tips of narrow slots on the relative radius of curvature $\varepsilon = \rho/l$ ($\chi = 0.1$).

If the direction of tension of the quasiorthotropic plane coincides with the direction of the larger modulus of elasticity ($\gamma < 1$), then the SCF k_t at the tip of the parabolic notch is larger than in the isotropic plane ($\gamma = 1$), and vice versa (Fig. 3).

System of Physical Slots

Similarly, we obtain the solution of the problem of stress concentration in a stretched quasiorthotropic plane with periodic system of physical slots of length $2l$ (whose contours form two parallel segments connected by semicircles of radius ρ) (Fig. 4).

We represent the parametric equation of the contour L doubly symmetric about the Ox - and Oy -axes in the following form:

$$t = l\omega(\xi) = l \begin{cases} \omega_0(\xi), & 0 \leq \xi < \frac{\pi}{2}, \\ -\overline{\omega_0(\pi - \xi)}, & \frac{\pi}{2} \leq \xi < \pi, \\ -\omega_0(\xi - \pi), & \pi \leq \xi < \frac{3\pi}{2}, \\ \overline{\omega_0(2\pi - \xi)}, & \frac{3\pi}{2} \leq \xi < 2\pi, \end{cases} \quad (12)$$

where the function $\omega_0(\xi)$ describes a part of the contour L lying in the fourth quarter of the coordinate system

$$\omega_0(\xi) = \begin{cases} 1 - \varepsilon + \varepsilon(\cos c\xi - i \sin c\xi), & 0 \leq \xi < \frac{\pi}{2c}, \\ \varepsilon c \left(\frac{\pi}{2} - \xi \right) - i\varepsilon, & \frac{\pi}{2c} \leq \xi \leq \frac{\pi}{2}. \end{cases}$$

The SCF at the tips of a two-sided U-shaped notch in the case where the relative distances between the slots tend to zero are obtained with the help of the limit transition (Fig. 3b).

The values of the SCF k_t at the tip of the two-sided U-notch in the quasiorthotropic plane are quite close to the values obtained for the two-sided parabolic notch in the case where $0.01 \leq \chi \leq 1$ (see Fig. 3).

System of Rhombic Holes

Consider the problem of the stress concentration at rounded (by arcs of radius ρ) vertices of rhombic holes periodically distributed in a stretched quasiorthotropic plane. Assume that the internal corners of a rhomb with vertices on the Ox -axis are equal to 2β ($0 \leq 2\beta \leq \pi$), $2l$ is the projection of the contour of the hole L onto the Ox -axis, and $2a$ is the distance between the vertices of the neighboring holes.

According to the values of SCF for the rounded vertices, we can find the SIF at the corresponding sharp vertices of the holes [10]

$$\tilde{K}_I^V = \frac{1}{R_I} \lim_{\rho \rightarrow 0} [(2\pi\rho)^{\lambda_I} \sigma_A^*], \quad (13)$$

where R_I is the coefficient of influence of rounding of the notch on the stresses acting at the tip of a sharp angular notch [10, 19] and λ_I is the singularity exponent of the stresses. For the quasiorthotropic material, we find [10]

$$\lambda_I = 1.247 \cos \tilde{\beta} - 1.312 \cos^2 \tilde{\beta} + 0.8532 \cos^3 \tilde{\beta} - 0.2882 \cos^4 \tilde{\beta},$$

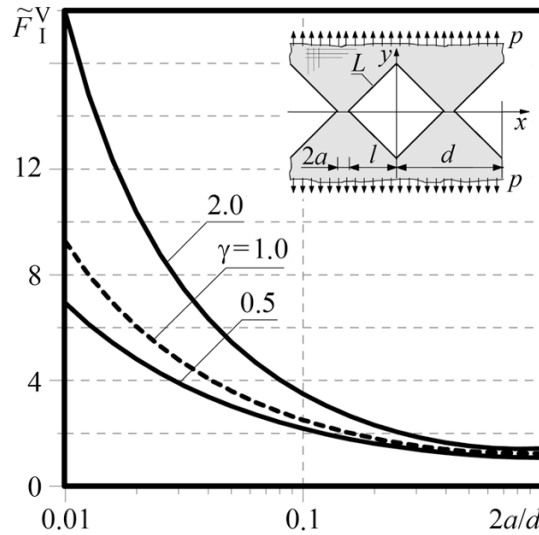


Fig. 5. Dependence of the dimensionless SIF $\tilde{F}_I^V$ at sharp vertices of square holes on the relative distance between them $2a/d$.

where

$$\tilde{\beta} = \pi - \beta_1(\alpha), \quad \beta_1(\alpha) = \pi + \arctan(\gamma \tan \alpha), \quad \alpha = \pi - \beta.$$

By using this method, we establish dependences of the dimensionless SIF $\tilde{F}_I^V = \tilde{K}_I^V / (p(\pi l)^{\lambda_1})$ at the sharp vertices of the square holes ($2\beta = \pi/2$) on the relative distance between these holes (Fig. 5).

Unlike the SCF at the rounded tips of the notches (see Fig. 3), the values of the SIF for the sharp vertices of the square holes (Fig. 5) in the quasiorthotropic plane are higher in the case where this plane is stretched along the axis of orthotropy for which $\gamma > 1$. This conclusion follows from the definition of the SIF (13) if the dependence of the coefficient R_I on the orthotropy parameter γ is taken into account [10].

CONCLUSIONS

We study the plane periodic problem for a quasiorthotropic plane with infinite series of closely located curvilinear holes (physical slots, elliptic and rhombic holes). We determine the SIF and SCF at the sharp and rounded vertices of the holes in the quasiorthotropic plane in uniaxial tension. By the limit transition, in the case where the holes approach each other, we obtain the SIF and SCF near the sharp and rounded vertices of two-sided notches in the quasiorthotropic plane. The influence of the main orthotropy parameter on the stressed state of the quasiorthotropic plane with symmetric holes and notches is investigated.

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