

AXISYMMETRIC COLLISION PROBLEM FOR TWO IDENTICAL ELASTIC SOLIDS OF REVOLUTION

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A direct central collision of two identical bodies of revolution is studied. A nonstationary mixed boundary-value problem with an unknown moving boundary is formulated. Its solution is represented by a series in term of Bessel functions. An infinite system of Volterra equations of the second kind for the unknown expansion coefficients is derived by satisfying the boundary conditions. The basic characteristics of the collision process are determined depending on the curvature of the frontal surface of the bodies.

Keywords: elastic systems, direct central collision, identical bodies, nonstationary axisymmetric problem, ultrasonic stage

The present paper further develops the technique for determining the time-dependent collision characteristics of blunted elastic bodies. On the assumption that the bodies deform elastically during the collision (i.e., irreversible strains are absent or negligible) we have developed a method for solving the direct central collision problem for two identical elastic solids of revolution within the framework of the nonstationary theory of elasticity. The method is based on the approach proposed in [6, 7] to describe an impact of a body on the surface of a compressible fluid and in [8, 9] to describe an impact of a rigid body on the free surface of an elastic half-space. The recent publication [5] set forth a method of attacking the elastic collision problem for two long cylindrical bodies that contact along and translate transversely to a common generatrix during the (direct central) collision. We will apply this method to the axisymmetric collision problem for two identical elastic solids of revolution. Assuming low frontal curvature of the colliding bodies, small depths of their mutual penetration, and small elastic strains, we reduce the boundary conditions on the front to a flat surface. Let the time interval of interest be commensurable with the collision duration and such that the elastic waves reflected from the rear and side surfaces have no time to come back to the contact area and, hence, do not affect the interaction process. This makes it possible to formulate the boundary conditions on those surfaces somewhat arbitrarily, which, in turn, facilitates solving the problem. Since the contact area is varying during the collision and unknown beforehand, the following mixed boundary conditions are specified on the fronts of the bodies: (i) within the contact region, the elastic deformation rates and normal stresses are equal, the latter remaining compressive during the collision; (ii) the tangential stresses remain zero over the entire surface (friction between the bodies is neglected); and (iii) outside the contact region, the surfaces of the bodies are free from normal stresses. If the bodies have finite masses, then the relative velocity varies during the collision, and, therefore, the governing system of dynamic elastic equations should be supplemented with the equations of rigid-body translation with the corresponding initial conditions. Thus, we have an initial-boundary-value problem where the boundary moves with a variable velocity and is not known beforehand. The solution of this problem is represented as Fourier-Bessel series. Satisfying the mixed boundary conditions results in an infinite system of integral Volterra equations of the second kind for the expansion coefficients. This system is reduced, using quadrature formulas, to a system of algebraic equations, which can be solved numerically. Solving the latter system simultaneously with the equation of motion (which is also discretized) yields the stresses, forces of reaction, contact area, velocity, etc. as functions of the initial velocity, the masses of the

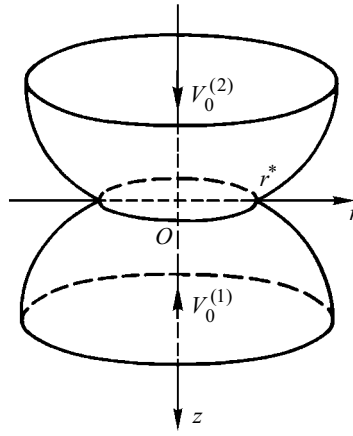


Fig. 1

bodies, and the elastic characteristics of their materials. The recent publications [2, 4, 14, 15] are devoted to nonstationary mixed problems with an unknown boundary, and the paper [13] to recent experiments on colliding elastic bodies.

1. Formulation of the Problem. Consider two elastic identical solids of revolution moving toward each other and colliding at $t = 0$. The relative velocity of their centers of mass is directed along the common axis of revolution and is equal to $V_0 = V_0^{(2)} - V_0^{(1)}$ at the initial time (direct central collision).

Let the bodies be referred to a cylindrical coordinate system $Orz\theta$ such that the point of initial contact is at its origin and the Oz -axis is parallel to the axis of revolution (Fig. 1). Since all the sections of the system of bodies by planes containing the Oz -axis are equivalent, we can restrict the consideration to one of the sections—the plane zOr . The contact area, varying during the collision, is determined, at every instant, by a boundary value of the radial coordinate r^* , which is moving and unknown beforehand. The elastic properties of the bodies are defined by the bulk modulus K and the shear modulus μ . The velocities of compression and shear waves are defined by

$$c_p = \left[\left(K + \frac{4}{3} \mu \right) / \rho \right]^{1/2}, \quad c_s = (\mu / \rho)^{1/2},$$

where ρ is the density of the material the bodies are made of.

Let us introduce the following dimensionless variables:

$$\bar{r} = \frac{r}{R}, \quad \bar{z} = \frac{z}{R}, \quad \bar{u}_j^{(i)} = \frac{u_j^{(i)}}{R}, \quad \bar{t} = \frac{c_0 t}{R}, \quad \bar{M} = \frac{M}{\rho R^3}, \quad \bar{\sigma}_{jk}^{(i)} = \frac{\sigma_{jk}^{(i)}}{K},$$

$$\bar{F} = \frac{F}{KR^2}, \quad c_0 = \left(\frac{K}{\rho} \right)^{1/2}, \quad \alpha = \frac{c_p}{c_0}, \quad \beta = \frac{c_s}{c_0}, \quad b = \frac{\beta}{\alpha}, \quad j, k = r, z, \quad i = 1, 2,$$

where R is a characteristic linear dimension (such as the distance from the focus to the vertex of the parabola that, revolving, generates the surface of the body); $u_j^{(i)}$ are the components of the displacement vector; $\sigma_{jk}^{(i)}$ are the components of the stress tensor; M is the mass of the body; and c_0 is the velocity of sound waves in the material with zero shear modulus. We will hereafter use only dimensionless variables; therefore, the overbars can be omitted. During collision, the bodies are in an asymmetric stress-strain state. The deformation of the bodies is described by two scalar wave potentials $\varphi^{(i)}$ and $\psi^{(i)}$ that satisfy the equations [10]

$$\frac{\partial^2 \varphi^{(i)}}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi^{(i)}}{\partial r} + \frac{\partial^2 \varphi^{(i)}}{\partial z^2} = \frac{1}{\alpha^2} \frac{\partial^2 \varphi^{(i)}}{\partial t^2}, \quad \frac{\partial^2 \psi^{(i)}}{\partial r^2} + \frac{1}{r} \frac{\partial \psi^{(i)}}{\partial r} + \frac{\partial^2 \psi^{(i)}}{\partial z^2} = \frac{1}{\beta^2} \frac{\partial^2 \psi^{(i)}}{\partial t^2}. \quad (1.1)$$

The displacements and stresses are expressed in terms of $\varphi^{(i)}$ and $\psi^{(i)}$ as follows:

$$\begin{aligned} u_r^{(i)} &= \frac{\partial \varphi^{(i)}}{\partial r} + \frac{\partial^2 \psi^{(i)}}{\partial r \partial z}, \quad u_z^{(i)} = \frac{\partial \varphi^{(i)}}{\partial z} - \frac{\partial^2 \psi^{(i)}}{\partial r^2} - \frac{1}{r} \frac{\partial \psi^{(i)}}{\partial r}, \\ \sigma_{zz}^{(i)} &= (1-2b^2) \frac{\partial^2 \varphi^{(i)}}{\partial t^2} + 2\beta^2 \left(\frac{\partial^2 \varphi^{(i)}}{\partial z^2} - \frac{\partial^3 \psi^{(i)}}{\partial r^2 \partial z} - \frac{1}{r} \frac{\partial^2 \psi^{(i)}}{\partial r \partial z} \right), \\ \sigma_{rz}^{(i)} &= 2\beta^2 \left(\frac{\partial^2 \varphi^{(i)}}{\partial r \partial z} - \frac{1}{2\beta^2} \frac{\partial}{\partial r} \left(\frac{\partial^2 \psi^{(i)}}{\partial t^2} \right) + \frac{\partial^3 \psi^{(i)}}{\partial r \partial z^2} \right), \quad i=1,2. \end{aligned} \quad (1.2)$$

In formulating the boundary conditions, we assume that the frontal curvature of the bodies and the depths of their mutual penetration are quite small and that the time interval of interest is not greater than the collision duration (the elastic waves reflected from the side surfaces have no time to come back to the contact area and, hence, do not affect the interaction process). These assumptions enable us to linearize the boundary conditions and specify them at the plane $z=0$. Then, in the absence of friction between the contacting surfaces, the boundary conditions include equality of the normal stresses and displacements within the contact region, absence of the normal stresses outside the contact region, and absence of the tangential stresses everywhere on the surfaces of the bodies. Note that the contact stresses must be compressive;

$$u_z^{(1)} \Big|_{z=0} + s^{(1)} = u_z^{(2)} \Big|_{z=0} + s^{(2)}, \quad |r| \in (0..|r^*|), \quad s^{(i)}(0) = 0, \quad (1.3)$$

$$\sigma_{zz}^{(1)} \Big|_{z=0} = \sigma_{zz}^{(2)} \Big|_{z=0}, \quad |r| \in (0..|r^*|), \quad (1.4)$$

$$\sigma_{zz}^{(1)} \Big|_{z=0} > 0, \quad \sigma_{zz}^{(2)} \Big|_{z=0} > 0, \quad |r| \notin (0..|r^*|),$$

$$\sigma_{zz}^{(i)} \Big|_{z=0} = 0, \quad |r| \notin (0..|r^*|), \quad (1.5)$$

$$\sigma_{rz}^{(i)} \Big|_{z=0} = 0, \quad |r| \geq 0, \quad i=1,2. \quad (1.6)$$

In (1.3), $s^{(i)}$ ($i=1,2$) is the rigid displacement of the i th body. For convenience, let us rearrange relation (1.3) in terms of the deformation rates of the contacting surfaces:

$$V_0^{(1)} + \frac{\partial u_z^{(1)}}{\partial t} \Big|_{z=0} = V_0^{(2)} + \frac{\partial u_z^{(2)}}{\partial t} \Big|_{z=0}, \quad |r| \in (0..|r^*|), \quad (1.7)$$

where $V_0^{(i)}$ ($i=1,2$) is the velocity of the i th body.

The initial conditions are as follows:

$$\varphi^{(i)} = \psi^{(i)} = \dot{\varphi}^{(i)} = \dot{\psi}^{(i)} = 0, \quad t=0. \quad (1.8)$$

Owing to the choice of the time interval of interest, we can find a finite (in r) domain such that the waves reflected from the side boundaries would not have time to come back to the contact region. Thus, the interacting bodies can be replaced by elastic semiinfinite cylinders ($|r| \leq l$). The above boundary conditions are realized at their end surfaces, and the boundary conditions at the side surfaces of the half-strip may be dealt with as we see fit. In particular, to simplify the separation of variables in the general solution, we assume that the following conditions are satisfied at the side surfaces:

$$u_z^{(i)} \Big|_{|r|=l} = 0, \quad \sigma_{rz}^{(i)} \Big|_{|r|=l} = 0. \quad (1.9)$$

Perturbations are absent at infinity:

$$\begin{aligned}\varphi^{(1)} &\rightarrow 0, & \psi^{(1)} &\rightarrow 0, & z &\rightarrow \infty, \\ \varphi^{(2)} &\rightarrow 0, & \psi^{(2)} &\rightarrow 0, & z &\rightarrow -\infty.\end{aligned}\quad (1.10)$$

The boundary-value problem posed should be supplemented with the equation of motion of the center of mass of each body:

$$\mu_1 \frac{dV_c^1}{dt} = F(t), \quad \mu_2 \frac{dV_c^2}{dt} = -F(t), \quad (1.11)$$

$$F(t) = 2\pi \int_0^{r^*} r \sigma_{zz}(t, r, 0) dr, \quad \otimes \quad V_c^{(1)}(0) = V_0^{(1)}, \quad V_c^{(2)}(0) = V_0^{(2)}. \quad (1.12)$$

In the general case, the contact boundary r^* is determined from the nonpenetration condition

$$w(t) - f(r) - u_z(t, r, 0) = \begin{cases} 0, & 0 \leq r \leq r^*(t), \\ < 0, & r > r^*(t), \end{cases} \quad (1.13)$$

where $w(t)$ is the displacement of the center of mass of one of the bodies; and $f(r)$ is the curve that, when revolved about the Oz -axis, describes the surface of the body.

In a simpler formulation, the point of change in the boundary conditions, r^* , is determined as the radial coordinate of the intersection point of the undeformed boundaries if the depth of mutual penetration is z^* .

Relations (1.1)–(1.13) constitute a mixed initial–boundary-value problem with an unknown moving boundary.

2. Solution Technique. Applying the integral Laplace transform in t with a parameter s to system (1.1) and taking the initial conditions (1.8) into account, we obtain the system of equations

$$\begin{aligned}\frac{\partial^2 \varphi^{(i)L}}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi^{(i)L}}{\partial r} + \frac{\partial^2 \varphi^{(i)L}}{\partial z^2} &= \frac{s^2}{\alpha^2} \varphi^{(i)L}, \\ \frac{\partial^2 \psi^{(i)L}}{\partial r^2} + \frac{1}{r} \frac{\partial \psi^{(i)L}}{\partial r} + \frac{\partial^2 \psi^{(i)L}}{\partial z^2} &= \frac{s^2}{\beta^2} \psi^{(i)L},\end{aligned} \quad (2.1)$$

where the superscript L denotes a Laplace-transformed variable.

Let the solution of system (2.1) have the form of series in terms of Bessel functions on the interval $[0, 1]$ that contains the perturbed region.

As indicated above, the colliding bodies are identical in shape and material. In this case, the general solution of system (2.1) with (1.9) and (1.10) has the form

$$\begin{aligned}\varphi^{(1)L} &= \sum_{n=0}^{\infty} A_n^{(1)}(s) \cdot e^{-zP} J_0(\lambda_n r), & \psi^{(1)L} &= \sum_{n=0}^{\infty} B_n^{(1)}(s) \cdot e^{-zS} J_0(\lambda_n r), \\ \varphi^{(2)L} &= \sum_{n=0}^{\infty} A_n^{(2)}(s) \cdot e^{zP} J_0(\lambda_n r), & \psi^{(2)L} &= \sum_{n=0}^{\infty} B_n^{(2)}(s) \cdot e^{zS} J_0(\lambda_n r), \\ P &= \sqrt{s^2 / \alpha^2 + \lambda_n^2}, & S &= \sqrt{s^2 / \beta^2 + \lambda_n^2},\end{aligned} \quad (2.2)$$

where λ_n are the roots of the equation $J_1(\lambda_n l) = 0$, $n = \overline{0, \infty}$, and $A_n^{(i)}$ and $B_n^{(i)}$ are coefficients to be determined from the boundary conditions (1.6).

Let us expand the desired functions on $z = 0$ into a series in terms of Bessel functions:

$$\sigma_{zz}^{(i)}(t, r, 0) = \sum_{n=0}^{\infty} \sigma_{zzn}^{(i)}(t) J_0(\lambda_n r), \quad V^{(i)} = \sum_{n=0}^{\infty} V_n^{(i)}(t) J_0(\lambda_n r), \quad (2.3)$$

$$\sigma_{rz}^{(i)}(t, r, 0) = \sum_{n=0}^{\infty} \sigma_{zzn}^{(i)}(t) J_0(\lambda_n r), \quad i = 1, 2. \quad (2.4)$$

Consider an auxiliary nonmixed problem. Let the rate of deformation of the frontal surfaces be given and the tangential stresses be absent:

$$V^{(i)}(t, r, 0) = \left. \frac{\partial u_z^{(i)}}{\partial t} \right|_{z=0}, \quad (2.5)$$

$$\sigma_{rz}^{(i)}(t, r, 0) = 0, \quad i = 1, 2. \quad (2.6)$$

Applying the integral Laplace transform to (2.6), we find the relationship between $A_n^{(i)}$ and $B_n^{(i)}$:

$$A_n^{(i)} = (-1)^{i+1} \frac{s^2 + 2\beta^2 \lambda_n^2}{2\beta^2 P} B_n^{(i)}, \quad i = 1, 2, \quad n = \overline{0, \infty}. \quad (2.7)$$

Using relations (1.2), (2.3), (2.5), and (2.7), we obtain the following relationship among the coefficients of the n th harmonics:

$$\sigma_{zzn}^{(i)L} = (-1)^i \alpha V_n^{(i)L} \left(1 - \frac{\sqrt{s^2 + \alpha^2 \lambda_n^2} - s}{\sqrt{s^2 + \alpha^2 \lambda_n^2}} + 4\beta^2 \lambda_n^2 \left(\frac{1}{\sqrt{s^2 + \alpha^2 \lambda_n^2}} - \frac{\beta}{\alpha} \frac{1}{\sqrt{s^2 + \beta^2 \lambda_n^2}} \right) \left(\frac{1}{s} + \frac{\beta^2 \lambda_n^2}{s^3} \right) \right), \quad i = 1, 2. \quad (2.8)$$

Recovering the original functions and using the convolution theorem [3], we get

$$\sigma_{zzn}^{(i)} = (-1)^i \alpha \left(V_n^{(i)}(t) + \int_0^t V_n^{(i)}(\tau) F_n(t - \tau) d\tau \right), \quad i = 1, 2, \quad (2.9)$$

$$F_n(t) = -\lambda_n \alpha J_1(\lambda_n \alpha t) + 4\beta^2 \lambda_n^2 \int_0^t \left(J_0(\lambda_n \alpha \tau) - \frac{\beta}{\alpha} J_0(\lambda_n \beta \tau) \right) \left(1 + \beta^2 \lambda_n^2 \frac{(t - \tau)^2}{2} \right) d\tau, \quad (2.10)$$

where $J_0(t)$ and $J_1(t)$ are the Bessel functions of the first kind and of the zero and first orders, respectively [1].

Expression (2.9) relates the expansion coefficients for the normal stress and deformation rate. Using this expression, we will satisfy the boundary conditions of the mixed problem. Then conditions (1.4), (1.5), and (1.7) can be written as

$$\begin{aligned} -2V^{(1)} - \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(1)}(\tau) F_n(t - \tau) d\tau + \int_0^t V_n^{(2)}(\tau) F_n(t - \tau) d\tau \right) J_0(\lambda_n r) + V_0(t) &= 0, \quad |l| \leq r^*, \\ V^{(1)} + \sum_{n=0}^{\infty} \int_0^t V_n^{(1)}(\tau) F_n(t - \tau) d\tau J_0(\lambda_n r) &= 0, \quad r^* \leq |l| \leq l, \end{aligned} \quad (2.11)$$

$$2V^{(2)} + \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(2)}(\tau) F_n(t - \tau) d\tau + \int_0^t V_n^{(1)}(\tau) F_n(t - \tau) d\tau \right) J_0(\lambda_n r) + V_0(t) = 0, \quad |l| \leq r^*,$$

$$V^{(2)} + \sum_{n=0}^{\infty} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau J_0(\lambda_n r) = 0, \quad r^* \leq l \leq l, \quad (2.12)$$

where $V_0 = V_0^{(2)} - V_0^{(1)}$.

The equations in (2.11) and (2.12) can be united using the Heaviside function $H(x)$:

$$\begin{aligned} & -2V^{(1)} - \sum_{n=0}^{\infty} \int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau J_0(\lambda_n r) \\ & = \left(-V_0(t) + \sum_{n=0}^{\infty} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau J_0(\lambda_n r) \right) H(r^* - r) + \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau \right) J_0(\lambda_n r) H(r - r^*), \end{aligned} \quad (2.13)$$

$$\begin{aligned} & 2V^{(2)} + \sum_{n=0}^{\infty} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau J_0(\lambda_n r) \\ & = \left(-V_0(t) - \sum_{n=0}^{\infty} \int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau J_0(\lambda_n r) \right) H(r^* - r) - \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau \right) J_0(\lambda_n r) H(r - r^*). \end{aligned} \quad (2.14)$$

Performing series expansions and taking the orthogonality of Bessel functions into account in (2.13) and (2.14), we arrive at two infinite systems of integral Volterra equations of the second kind for $V_n^{(1)}$ and $V_n^{(2)}$:

$$\begin{aligned} & V_n^{(1)}(t) + \frac{1}{2} \int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau + \frac{1}{2} \sum_{m=0}^{\infty} \alpha_{mn} \int_0^t V_m^{(2)}(\tau) F_m(t-\tau) d\tau \\ & + \frac{1}{2} \sum_{m=0}^{\infty} \alpha'_{mn} \int_0^t V_m^{(1)}(\tau) F_m(t-\tau) d\tau = \frac{1}{2} V_{0n}(t), \end{aligned} \quad (2.15)$$

$$\begin{aligned} & V_n^{(2)}(t) + \frac{1}{2} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau + \frac{1}{2} \sum_{m=0}^{\infty} \alpha_{mn} \int_0^t V_m^{(1)}(\tau) F_m(t-\tau) d\tau \\ & + \frac{1}{2} \sum_{m=0}^{\infty} \alpha'_{mn} \int_0^t V_m^{(2)}(\tau) F_m(t-\tau) d\tau = -\frac{1}{2} V_{0n}(t). \end{aligned} \quad (2.16)$$

The coefficients of systems (2.15) and (2.16) are defined by

$$\begin{aligned} & V_{00} = \frac{r^{*2}}{l^2} V_0(t), \quad V_{0n} = \frac{2r^* J_1(\lambda_n r^*)}{l^2 \lambda_n J_0^2(\lambda_n l)} V_0(t), \\ & \alpha_{00} = \frac{r^{*2}}{l^2}, \quad \alpha'_{00} = \frac{1}{l^2} (l^2 - r^{*2}), \quad \alpha_{m0} = -\alpha'_{m0} = \frac{2r^* J_1(\lambda_m r^*)}{l^2 \lambda_m}, \quad m \neq 0, \\ & \alpha_{mn} = -\alpha'_{mn} = \frac{2r^*}{l^2 J_0^2(\lambda_n l) (\lambda_m^2 - \lambda_n^2)} (\lambda_m J_0(\lambda_n r^*) J_1(\lambda_m r^*) - \lambda_n J_0(\lambda_m r^*) J_1(\lambda_n r^*)), \end{aligned}$$

$$m \neq n, \quad \alpha_{nn} = 1 - \alpha'_{nn} = \frac{r^{*2}}{l^2 J_0^2(\lambda_n l)} \left(J_1^2(\lambda_n r^*) + J_0^2(\lambda_n r^*) \right). \quad (2.17)$$

The structure of the infinite systems (2.15) and (2.16) (they differ only by the sign of the right-hand side) suggests that it is sufficient to solve just one of them because $V_n^{(2)}(t) = -V_n^{(1)}(t)$. Thus, when the colliding bodies are identical, the mixed boundary-value problem reduces to an infinite system of integral Volterra equations of the second kind (2.15) (or (2.16)).

System (2.15) should be supplemented with the equation of motion (1.11), where the force $F(t)$ is defined by

$$F(t) = 2\pi \sum_{n=0}^{\infty} \sigma_{zzn}(t) J_1(\lambda_n r^*) r^*. \quad (2.18)$$

After determining the coefficients $V_n(t)$ from (2.15) (or (2.16)), we can calculate the stresses $\sigma_{zz}(t)$ from (2.3), (2.9), and (2.10).

3. Supersonic Stage. Consider the earliest, so-called “supersonic” stage of collision. At this stage, a point r^* of the contact boundary moves with a velocity greater than that of waves; and the perturbations in the bodies do not reach the frontal surface. Therefore, the boundary conditions at this surface may be dealt with as we see fit. Let a velocity normal to the frontal surface of the body be specified on it (equal to zero outside the contact region). Then the boundary conditions at $z = 0$ ($0 \leq |r| \leq l$) are the following:

$$\left. \frac{\partial u_z^{(1)}}{\partial t} \right|_{z=0} = V_0 + \left. \frac{\partial u_z^{(2)}}{\partial t} \right|_{z=0}, \quad \sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}, \quad \sigma_{rz}^{(i)} = 0, \quad i = 1, 2, \quad (3.1)$$

where $V_0 = H(r^* - |r|) V_0(t) = \sum_{n=0}^{\infty} V_{0n}(t) J_0(\lambda_n r)$ and $V_0(t) = V_0^{(2)}(t) - V_0^{(1)}(t)$.

The coefficients $V_{0n}(t)$ are defined by (2.14), and the normal stresses are compressive.

Satisfying the boundary conditions (3.1) in the Laplace-transform space, we determine the coefficients $A_n^{(i)}$ and $B_n^{(i)}$:

$$A_n^{(i)} = -1 \frac{s^2 + 2\beta^2 \lambda_n^2}{2s^3 P} V_{0n}^L, \quad B_n^{(i)} = (-1)^i \frac{\beta^2}{s^3} V_{0n}^L, \quad i = 1, 2, \quad n = \overline{0, \infty}. \quad (3.2)$$

Using (1.2), (2.2), and (3.2), we obtain the following expression for the Laplace-transformed normal stress on the surface of the bodies:

$$\sigma_{zz}^L = \sum_{n=0}^{\infty} \sigma_{zzn}^L J_0(\lambda_n r),$$

$$\sigma_{zzn}^L = -\frac{\alpha}{2} V_{0n}^L \left(1 - \frac{\sqrt{s^2 + \alpha^2 \bar{n}^2} - s}{\sqrt{s^2 + \alpha^2 \bar{n}^2}} + 4\beta^2 \bar{n}^2 \left(\frac{1}{\sqrt{s^2 + \alpha^2 \bar{n}^2}} - \frac{\beta}{\alpha} \frac{1}{\sqrt{s^2 + \beta^2 \bar{n}^2}} \right) \left(\frac{1}{s} + \frac{\beta^2 \bar{n}^2}{s^3} \right) \right). \quad (3.3)$$

Recovering the original functions and using the convolution theorem, we obtain

$$\sigma_{zzn} = -\frac{\alpha}{2} \left(V_{0n}(t) + \int_0^t V_{0n}(\tau) F_n(t - \tau) d\tau \right), \quad (3.4)$$

where $F_n(t)$ is defined by (2.10).

Thus, at the supersonic stage, the solution is much simpler and reduced to an infinite sequence of linear integral equations (3.3) for the coefficients $V_{0n}(t)$ defined by (2.17). It is well to bear in mind that relation (3.4) must be used simultaneously with (1.11) and (2.15).

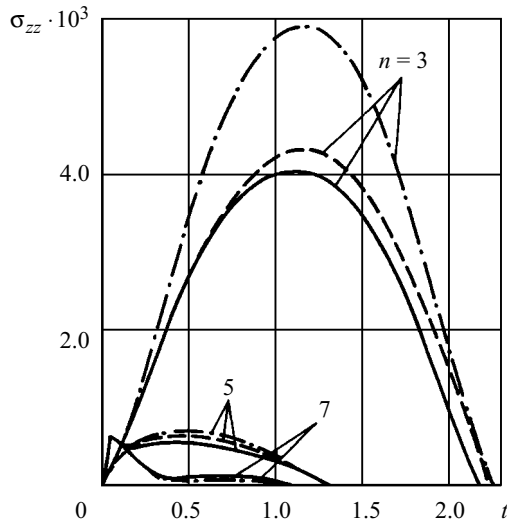


Fig. 2

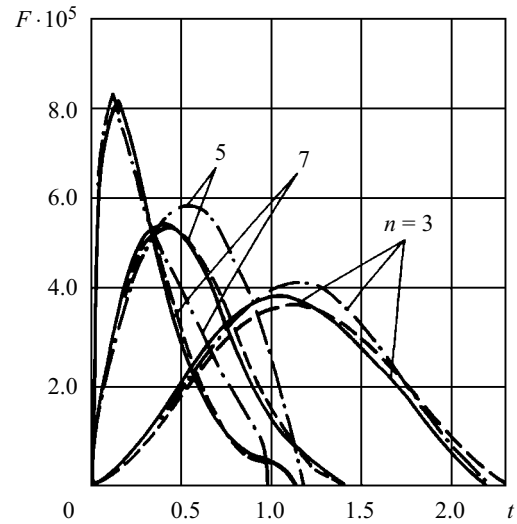


Fig. 3

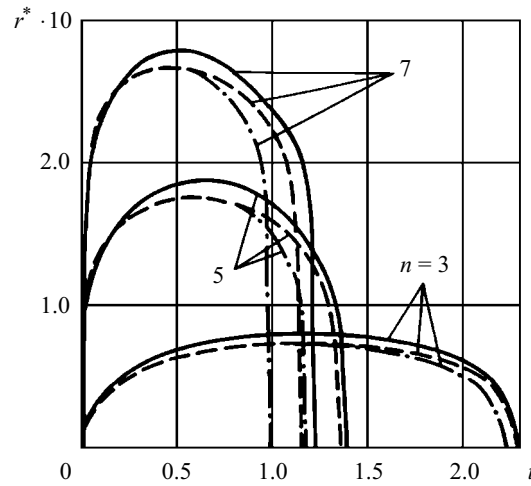


Fig. 4

4. Solution Algorithm and Results. The integrals in (2.9), (2.15), and (3.4) were evaluated by the Gregory formula [11]. Equation (1.11) with a truncated right-hand side was solved by the Adams method [3]. Differences up to the fourth order were retained in both Gregory and Adams formulas. Infinite systems of integral equations were subjected to reduction. The order of reduction was selected from the considerations of practical convergence. In particular, reliable numerical results were obtained with 80 equations in the infinite system (2.12) and $\Delta t = 0.025$ as a step of integration. The Gibbs phenomena were smoothed by using Gibbs factors [11].

The task was to determine the normal stress on the surface of the bodies, the interaction forces, the variable contact area, and the relative velocity depending on the shapes of the colliding bodies.

The collision problem was solved for two identical bodies whose surfaces are described by the parabola $z = r^n$ revolved about the z -axis ($n = 3, 5, 7$). As n increases, so does the bluntness of the frontal surface of the bodies. Three formulations were considered: the supersonic problem (3.4), (1.10), the mixed problem (2.15), (1.10), and the mixed problem that accounts for the change in the contact area because of the deformation of the contacting bodies. In the first two formulations, the contact boundary is determined as the radial coordinate of the intersection point of the undeformed contours of the bodies, z^* being the depth of mutual penetration. In the third formulation, the contact boundary is determined from relation (1.13). In the figures, the dash-dot line corresponds to the supersonic problem, the dashed line to the mixed problem, and the solid line to the case of accounting for the deformation of the surface of the interacting bodies on the contact boundary (the third problem).

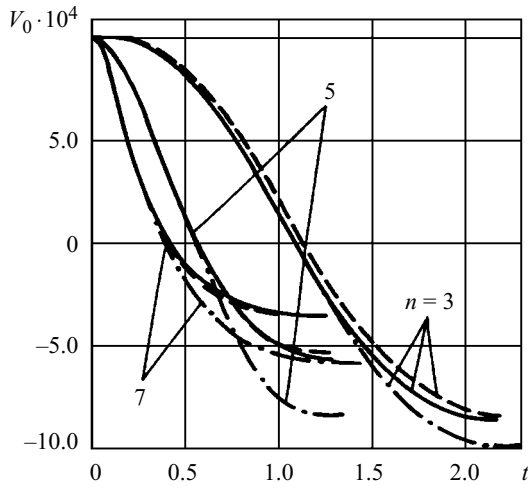


Fig. 5

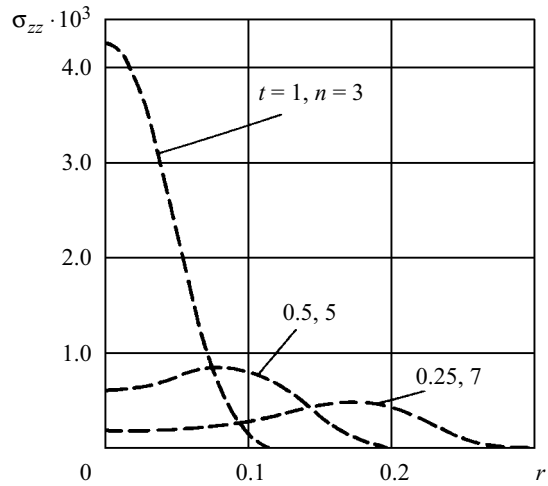


Fig. 6

Figures 2–6 present the results obtained in all the three formulations for $V_0 = 1.0 \cdot 10^{-3}$, $\mu = 0.45K$, and $M = 0.05$.

Figure 2 shows how the normal stress at the frontal point varies with time. For $n = 3$ (more pointed front of the paraboloid), the supersonic solution is reliable only at the earliest stage of interaction and the maximum value of σ_{zz} is much different from that obtained in the exacter formulations. For $n = 7$, the curves of the mixed and supersonic problems practically coincide with that of the third problem. As n (degree of bluntness) increases, the stress reaches the maximum level earlier, this level decreasing.

Figure 3 shows how the force of interaction between the bodies varies during collision. For $n = 5$ and $n = 7$, all the curves are practically indistinguishable. On the time interval considered, the force of interaction peaks, its maximum level being reached earlier and increasing with n .

Figure 4 demonstrates how the contact area varies with time. As n increases, the contact boundary extends and the collision duration drops.

Figure 5 shows the relative velocity of the bodies versus time. For $n = 5$ and $n = 7$, all the curves practically coincide. As n increases, the relative velocity decreases faster and the collision duration decreases.

An analysis of Figs. 2–5 reveals that the supersonic solution can be used as an upper estimate for the maximum contact stresses and interaction force, and the curves of the mixed problem can be used to approximate those of the third problem.

Figure 6 shows the distribution of normal stress over the surface of the interacting bodies. As n increases, the maximum stress shifts from the center to the edge of the contact region and is reached earlier. The stress distribution shown in the figure is in qualitative agreement with that (for various values of n) in two parabolic bodies under static compression [12].

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