

Impedance boundary condition for vector potentials on thin layers and its application to integral equations*

H. Igarashi^{1,a,b}, A. Kost¹, and T. Honma²

¹ Institut für Elektrische Energietechnik, TU Berlin, Einsteinufer 11, 10587 Berlin, Germany

² Faculty of Engineering, Hokkaido University Kita 13, Nishi 8, Kita-ku, Sapporo, 060, Japan

Received: 20 March 1997 / Accepted: 27 August 1997

Abstract. Thin layers of magnetic substance are often used in magnetic shieldings. Since the scale of the spatial change in electromagnetic fields in the direction of the thickness of such a thin layer is considerably different from that in the transverse directions, the numerical treatment of the interior electromagnetic fields is formidable. In this paper, it is shown that the impedance boundary conditions on the surfaces of the thin magnetic layer, which have been written in former papers by the scalar potentials, can be expressed in terms of the vector potentials. Moreover, a new numerical method for analysis of eddy currents on thin magnetic layers is introduced, in which the quasi-static magnetic field in an air region ambient the thin layer is analyzed by solving the boundary integral equations under the impedance boundary conditions without any numerical treatment of the interior field. This formulation has no difficulties even when the skin depth is very short compared with the thickness of the layer. It is shown that the numerical results obtained by the present method in two-dimensional and axisymmetric systems agree well with the results analytically obtained or computed by a conventional numerical method.

PACS. 02.70.Pt Boundary-integral methods – 07.55.Nk Magnetic shielding in instruments – 41.20.-q Electric, magnetic and electromagnetic fields

1 Introduction

Magnetic conductive materials with thickness considerably smaller compared with their overall size are often used in magnetic shielding. Those thin layers have high permeability, so that magnetic fields generated by a noise source are prevented from penetration due to the magnetostatic effect at low frequencies. The layers also have high conductivity which allows eddy currents to flow to block electromagnetic noises at high frequencies. Under these circumstances the scale of the spatial change in electromagnetic fields in the direction of the thickness of the layers can become more than a few thousand times shorter compared with that in their transverse directions. When those fields are numerically analyzed by conventional numerical methods such as the Finite Difference Method (FDM) and Finite Element Method (FEM), the distance between nodes would also have considerably different spatial scales. This kind of ill structure leads to a numerical instability in the solution of a system of matrix equations. If the spatial scales in the domain discretization are made

comparable in all the directions to avoid the above difficulty, the resultant number of unknowns becomes too huge for practical treatments.

The Impedance Boundary Conditions (IBCs) have been introduced to model magnetic fields effectively around a thin conductive magnetic layer without any numerical treatments of the interior electromagnetic fields [1,2]. In this method the equations governing quasi-static magnetic fields around the thin layer are numerically solved under the IBCs. The interior electromagnetic field in the direction of thickness is analytically evaluated to obtain the IBCs. Hence this method is expected to be valid for arbitrary ratio of the skin depth to the thickness of the layer, until the wavelength of applied electromagnetic fields becomes comparable to the overall size of the layer.

The IBCs have been so far written in terms of the magnetic scalar potential. This formulation results in IBCs that take the form of a pair of second-order partial differential equations with a differential operator defined in the local coordinates on the tangential plane of the layer. These equations must be numerically solved in conjunction with the equations for ambient magnetic fields.

Since the ambient magnetic fields usually extend to infinite, the Boundary Element Method (BEM) is thought to be suitable for their analysis. It is, however, difficult to find Green's functions, which plays a crucial role in the BEM

* This paper was presented at NUMELEC'97.

^a e-mail: iga@em.huee.hokudai.ac.jp

^b Present address: Dept. Electrical Eng., Faculty of Engineering, Hokkaido University Kita 13, Nishi8, Kita-ku, Sapporo, 060, Japan

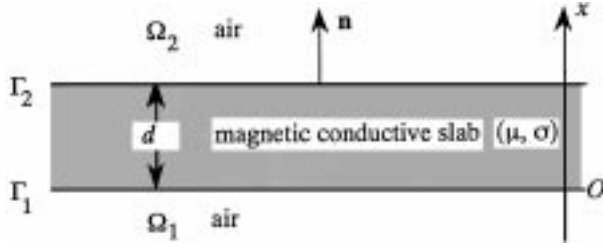


Fig. 1. Cross section of a thin slab.

formulation, corresponding to the locally-defined differential operator in the equations of IBCs. In contrast to BEM, the equations of IBCs are suitably solved by either FDM or FEM. These methods, however, have drawbacks in the treatment of infinite regions. Nevertheless, the magnetic fields around a thin magnetic layer have been analyzed by FEM on the basis of IBCs [2–4]. An FE-BE hybridization has also been proposed to compensate the disadvantages in FEM and BEM [1].

It is also noticed that the magnetic scalar potential can be defined only in a simply connected region. Hence, in this scalar formulation, a cut must be introduced such that a multiply connected region is subdivided into simply connected regions. This topological limitation is one of the drawbacks of the magnetic scalar potential method.

The above mentioned problems are thought to stem from the use of the magnetic scalar potential in IBCs. In this paper, the vector potential is employed to express IBCs. This representation results in a pair of first order differential equations. The IBCs in this formulation can be imposed directly on the equations for the ambient magnetic fields, without a numerical solution of themselves. This formulation allows us to analyze the fields only using BEM, and to relax the topological restrictions.

The remainder of this paper will be organized as follows. The next section introduces IBCs for the thin layer, and they are written in terms of the vector potential. The third section describes how to impose IBCs on the boundary integral equations. The fifth section gives some numerical examples for the test of the present method, while the last section includes concluding remarks.

2 Impedance boundary conditions

2.1 Basic formulation

Let us consider an infinitely wide slab, with thickness d , permeability μ and conductivity σ , immersed in a time-harmonic magnetic field, whose cross section is shown in Figure 1. The electromagnetic property of this slab is assumed to be linear. The electromagnetic fields in a curved finite-width layer which is sufficiently thin compared with its overall size are expected, at least locally, to be suitably approximated by this slab model.

Ampère's law under the quasi-static approximation is given by

$$\text{curl } \mathbf{H} = \sigma \mathbf{E}, \quad (1)$$

where $\mathbf{H}$ and $\mathbf{E}$ are magnetic and electric fields, respectively. By integrating both sides of equation (1) over the cross section of the slab, we have

$$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \int_0^d \sigma \mathbf{E}_t dx, \quad (2)$$

where $\mathbf{n}$ is the unit normal vector, the suffices 1 and 2 denote the quantities on the surfaces Γ_i , $i = 1, 2$, $\mathbf{E}_t$ denotes the tangential component of $\mathbf{E}$, that is $\mathbf{E}_t = \mathbf{n} \times (\mathbf{E} \times \mathbf{n})$. Equation (1) means that the current tangential to a layer gives rise to the jump across the thin layer in the component of $\mathbf{H}$ perpendicular to the current.

Similarly, Faraday's law

$$\text{curl } \mathbf{E} = -j\omega\mu\mathbf{H}, \quad (3)$$

gives

$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = -j\omega \int_0^d \mu \mathbf{H}_t dx, \quad (4)$$

where ω is the angular frequency.

We will evaluate the integrals in the right hand side of equations (2, 4) below.

From equations (1, 3), we obtain equations in the following form

$$\text{curl curl } \mathbf{u} + \gamma^2 \mathbf{u} = 0, \quad (5)$$

where $\mathbf{u}$ corresponds to either $\mathbf{H}$ or $\mathbf{E}$, and $\gamma = \sqrt{j\omega\sigma\mu}$. When the slab is sufficiently thin, the spatial change in electromagnetic fields in the direction of the thickness is expected to be much larger than that in the transverse directions. Hence equation (5) can be reduced to

$$\frac{d^2 \mathbf{u}}{dx^2} - \gamma^2 \mathbf{u} = 0. \quad (6)$$

The solution to equation (6) can be written in the form

$$\mathbf{u} = \mathbf{a}e^{\gamma x} + \mathbf{b}e^{-\gamma x}, \quad (7)$$

where $\mathbf{a}$ and $\mathbf{b}$ are constants, which are expressed in terms of the boundary values $\mathbf{u}_i$ on Γ_i as follows:

$$\begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = \frac{1}{2 \sinh(\gamma d)} \begin{bmatrix} -e^{-\gamma d} & 1 \\ e^{\gamma d} & -1 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}. \quad (8)$$

The integration of $\mathbf{u}$ along x is then performed to give

$$\int_0^d \mathbf{u} dx = \frac{1}{\gamma} \tanh\left(\frac{\gamma d}{2}\right) (\mathbf{u}_1 + \mathbf{u}_2). \quad (9)$$

Inserting equation (9) to equations (2, 4), and considering the fact that the tangential components of $\mathbf{H}$ and $\mathbf{E}$ are continuous across Γ_i , we consequently have

$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = -\frac{1}{Z_a} (\mathbf{E}_{t1} + \mathbf{E}_{t2}), \quad (10)$$

$$\mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = Z_b (\mathbf{H}_{t1} + \mathbf{H}_{t2}), \quad (11) \quad \text{provided that}$$

where $Z_a \equiv \gamma/[\sigma \tanh(\gamma d/2)]$ and $Z_b \equiv \gamma \tanh(\gamma d/2)/\sigma$. Equation (10) can be rewritten by operating $\mathbf{n} \times$ to both sides as

$$\mathbf{n} \times (\mathbf{E}_1 + \mathbf{E}_2) = Z_a (\mathbf{H}_{t1} - \mathbf{H}_{t2}). \quad (10')$$

Equations (10', 11) represent IBCs for a thin conductive magnetic layer, in which Z_a and Z_b have the dimension of $[\Omega]$.

We observe in equations (10', 11) an analogy to the equation of a plane electromagnetic wave, that is

$$\mathbf{n} \times \mathbf{E} = Z_0 \mathbf{H}, \quad (12)$$

where $Z_0 \equiv \sqrt{\mu/\epsilon}$ is the wave impedance, ϵ is the permittivity, and here $\mathbf{n}$ corresponds to the direction of the propagation. The basis of the above derivation has been given in [1, 2], and extended in [4].

2.2 Vector potential

We will here express IBCs in terms of the vector potential $\mathbf{A}$.

By substituting the relation $\mathbf{B} = \text{curl } \mathbf{A}$ in equation (3), the electric field $\mathbf{E}$ can be written in terms of $\mathbf{A}$ and the scalar potential φ as

$$\mathbf{E} = -j\omega \mathbf{A} - \text{grad} \varphi. \quad (13)$$

In the layer with a constant σ , free of true charge, $\mathbf{E}$ satisfies

$$\text{div } \mathbf{E} = 0. \quad (14)$$

In addition, we impose the Coulomb gauge on $\mathbf{A}$, that is

$$\text{div } \mathbf{A} = 0. \quad (15)$$

Taking divergence of equation (13), and substituting equations (14, 15) into it, we have

$$\nabla^2 \varphi = 0, \quad \text{in the layer.} \quad (16)$$

On the inner sides of Γ_i , representing it by Γ_i^+ , the eddy currents are perpendicular to the normal vector $\mathbf{n}$, so that $\mathbf{E} \cdot \mathbf{n} = 0$ holds there. Thus if $\mathbf{A} \cdot \mathbf{n} = 0$ on Γ_i^+ , then

$$\mathbf{n} \cdot \text{grad } \varphi = 0, \quad \text{on } \Gamma_i^+, \quad (17)$$

must hold. We conclude from equations (16, 17) that φ must be constant in the layer, which can be set to zero without loss of generality. Consequently, we have

$$\mathbf{E} = -j\omega \mathbf{A}, \quad \text{in the layer,} \quad (18)$$

$$\mathbf{A} \cdot \mathbf{n} = 0 \quad \text{on } \Gamma_i^+. \quad (19)$$

The similar discussion has been given in [5] for FEM using both vector and scalar potentials for thick materials.

By inserting equation (18) into equations (10', 11), and noting that the tangential components of $\mathbf{A}$ are continuous across Γ_i since $\mathbf{B} \cdot \mathbf{n}$ is continuous there, we have

$$\alpha \mathbf{n} \times (\mathbf{A}_1 + \mathbf{A}_2) = (\mathbf{B}_{t1} - \mathbf{B}_{t2}), \quad (20)$$

$$\mathbf{n} \times (\mathbf{A}_1 - \mathbf{A}_2) = \beta (\mathbf{B}_{t1} + \mathbf{B}_{t2}), \quad (21)$$

where $\alpha \equiv \omega \mu_0 / (j Z_a)$, $\beta \equiv j Z_b / (\omega \mu_0)$ and $\mathbf{B}_i$ denotes the magnetic induction on the outer sides of Γ in air. The constant α has the dimension $[1/\text{m}]$ while β has $[\text{m}]$.

When the continuity of $\mathbf{A} \cdot \mathbf{n}$ across Γ_i , which is, however, by no means physically required, is further assumed, the condition consistent with equation (19)

$$\mathbf{A}_1 \cdot \mathbf{n} = \mathbf{A}_2 \cdot \mathbf{n} = 0 \quad (22)$$

can be additionally imposed on Γ .

2.3 Magnetic scalar potential

The IBCs have originally been derived using the magnetic scalar potential ϕ that satisfies $\mathbf{H} = -\text{grad } \phi$. We briefly review here the formulation using ϕ .

By taking divergence of IBCs (Eqs. (10', 11)), we have

$$\text{div}_t (\mathbf{H}_{t1} - \mathbf{H}_{t2}) = -\alpha (H_{n1} + H_{n2}), \quad (23)$$

$$\text{div}_t (\mathbf{H}_{t1} + \mathbf{H}_{t2}) = -\frac{1}{\beta} (H_{n1} - H_{n2}), \quad (24)$$

where $H_{ni} = \mathbf{n} \cdot \mathbf{H}_i$, $i = 1, 2$, and div_t represents the tangential divergence which is defined in a local orthogonal coordinates (t_1, t_2) on the plane tangential to Γ as

$$\text{div}_t \mathbf{P} \equiv \frac{1}{h_1 h_2} \left(\frac{\partial}{\partial t_1} h_2 P_1 + \frac{\partial}{\partial t_2} h_1 P_2 \right), \quad (25)$$

where h_i are the metric coefficients and $\mathbf{P}$ is an arbitrary differentiable vector.

Equations (23, 24) are then written in terms of ϕ as

$$\text{div}_t \text{grad}_t (\phi_1 - \phi_2) = -\alpha \frac{\partial}{\partial n} (\phi_1 + \phi_2), \quad (26)$$

$$\text{div}_t \text{grad}_t (\phi_1 + \phi_2) = -\frac{1}{\beta} \frac{\partial}{\partial n} (\phi_1 - \phi_2), \quad (27)$$

where $\text{grad}_t \equiv \mathbf{e}_i \partial / (h_i \partial t_i)$, $\mathbf{e}_i$ are the basis vectors of the system (t_1, t_2) . Equations (26, 27) are IBCs written in terms of the magnetic scalar potential ϕ [1, 2].

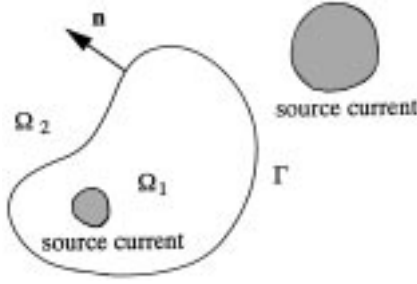


Fig. 2. Closed boundary Γ and air regions Ω_1 and Ω_2 .

2.4 Discussion

We here make a comparison between the vector-potential and scalar-potential formulations. Since equations (26, 27) are a pair of second-order differential equations, they must be numerically solved in conjunction with the equations for the magnetic fields in the air region ambient Γ . In contrast to this scalar-potential formulation, the vector-potential one, (Eqs. (20, 21)), comprises a pair of first-order differential equations which, as well as equation (22), can be imposed directly on the equations for the air region without solving them, as shown in the next section. Moreover, the scalar-potential formulation with ϕ is valid only when the domain is simply connected unless a cut is introduced. The present vectorial formulation using $\mathbf{A}$ does not suffer from this topological restriction.

Moreover, the eddy current near the edge of a layer behaves in different ways depending on its direction, as pointed out in [4]. Namely the component of the eddy current perpendicular to the edge must vanish on the edge while the parallel component can exist there. The constants α and β in IBCs should be determined at the edges to satisfy these conditions. Inevitably they must have different values for different directions. The scalar formulation (Eqs. (26, 27)), however, cannot handle this directional dependence, in contrast to the vectorial formulation (Eqs. (20 to 22)). A similar problem occurs at corners of a layer in scalar formulation.

These are the advantages of the present formulation over the one with the scalar magnetic potential.

3 Integral equations with IBCs

This section will describe how IBCs obtained in the previous section are applied to the integral equations governing the magnetic field ambient a thin layer. Though the present form of IBCs can be, in principle, applied to three-dimensional problems, two-dimensional and axisymmetric problems are considered here for simplicity. Figure 2 shows a thin closed layer Γ surrounded by finite and infinite air regions Ω_1 and Ω_2 . The thickness d of Γ is so small that it is neglected in a geometrical sense to identify the outer surface Γ_2 with the inner one Γ_1 , but has influence on electromagnetic fields through IBCs. The layer Γ would be composed of magnetic conductive material, and possibly of air in part.

3.1 Two-dimensional system

We introduce the Cartesian coordinates (x, y, z) to represent a two-dimensional system independent from z . The magnetic field in this system can be expressed in the form

$$\mathbf{B} = \text{grad } A \times \text{grad } z, \quad (28)$$

$$\mathbf{A} = A \text{grad } z, \quad (29)$$

where A denotes the z -component of $\mathbf{A}$. The substitution of equations (28, 29) into equations (20, 21) gives

$$\frac{\partial A_1}{\partial n} - \frac{\partial A_2}{\partial n} = \alpha (A_1 + A_2), \quad (30)$$

$$(A_1 - A_2) = \beta \left(\frac{\partial A_1}{\partial n} + \frac{\partial A_2}{\partial n} \right). \quad (31)$$

Note that the boundary condition (Eq. (22)) automatically holds in this case. Equations (30, 31) can be written in the matrix form as

$$\begin{bmatrix} A_1 \\ \frac{\partial A_1}{\partial n} \end{bmatrix} = \frac{1}{1 - \alpha\beta} \begin{bmatrix} 1 + \alpha\beta & 2\beta \\ 2\alpha & 1 + \alpha\beta \end{bmatrix} \begin{bmatrix} A_2 \\ \frac{\partial A_2}{\partial n} \end{bmatrix}, \quad (32)$$

where the non-dimensional constant $\alpha\beta$ is expressed as $\alpha\beta = [\tanh(\gamma d/2)]^2$. We can show that $\alpha = 0$ and $\beta = -\mu d/(2\mu_0)$ in the static limit ($\omega \rightarrow 0$), and $\alpha = 0$ and $\beta = -d/2$ in air ($\sigma = 0$, $\mu = \mu_0$) at arbitrary frequencies.

The integral equations governing the quasi-static magnetic field in the air regions Ω_1 and Ω_2 are derived from the Laplace equation for A . They are written in the form

$$CA_1(\mathbf{x}) = \int_{\Gamma} \left[\frac{\partial A_1(\mathbf{x}')}{\partial n'} G(\mathbf{x}, \mathbf{x}') - A_1(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right] ds' + A_{s1}(\mathbf{x}), \quad (33)$$

$$(1 - C)A_2(\mathbf{x}) = - \int_{\Gamma} \left[\frac{\partial A_2(\mathbf{x}')}{\partial n'} G(\mathbf{x}, \mathbf{x}') - A_2(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right] ds' + A_{s2}(\mathbf{x}), \quad (34)$$

where A_{si} , $i = 1, 2$ represent the source fields in Ω_i , and C is a constant coming from the Cauchy singularity in the integrals in equations (33, 34) which is, for example, $1/2$ if Γ is smooth. G is the free space Green function, *i.e.*, $G = -\log(R)/(2\pi)$, where R is the distance between the source and field points.

Equations (33, 34) are discretized by BEM to solve them numerically, while IBC (Eq. (32)) is used to connect the unknowns in Ω_1 to those in Ω_2 .

It would be possible to model multi-layers by multiplying the transform matrix in equation (32) successively for each layer.

3.2 Axisymmetric system

The cylindrical coordinates (r, θ, z) are here introduced to consider an axisymmetric system. It is assumed that the system is independent from θ , and $\mathbf{B}$ has no θ -component. The magnetic field in this case is expressed as

$$\mathbf{B} = \text{grad } \psi \times \text{grad } \theta, \quad (35)$$

$$\mathbf{A} = \psi \text{grad } \theta, \quad (36)$$

where ψ is the stream function defined by $\psi \equiv rA_\theta$. The IBCs for this case can be obtained after the same process as in the two-dimensional case. The result is

$$\begin{bmatrix} \psi_1 \\ \frac{\partial \psi_1}{\partial n} \end{bmatrix} = \frac{1}{1 - \alpha\beta} \begin{bmatrix} 1 + \alpha\beta & 2\beta \\ 2\alpha & 1 + \alpha\beta \end{bmatrix} \begin{bmatrix} \psi_2 \\ \frac{\partial \psi_2}{\partial n} \end{bmatrix}. \quad (37)$$

The integral equations for this case are written in the form

$$C\psi_1(\mathbf{x}) = \int_{\Gamma} \left[\frac{\partial \psi_1(\mathbf{x}')}{\partial n'} G(\mathbf{x}, \mathbf{x}') - \psi_1(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right] \frac{ds'}{r'} + \psi_{s1}(\mathbf{x}), \quad (38)$$

$$(1 - C)\psi_2(\mathbf{x}) = - \int_{\Gamma} \left[\frac{\partial \psi_2(\mathbf{x}')}{\partial n'} G(\mathbf{x}, \mathbf{x}') - \psi_2(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right] \frac{ds'}{r'} + \psi_{s2}(\mathbf{x}), \quad (39)$$

where $\psi_{si}, i = 1, 2$ represent the source fields in Ω_i , and here G is given by

$$G = \frac{\sqrt{rr'}}{\pi k} \left[\left(1 - \frac{k^2}{2} \right) K(k) - E(k) \right], \quad (40)$$

where $k^2 \equiv 4rr'/[(r' + r)^2 + (z' - z)^2]$, (r', z') and (r, z) are the field and source points, respectively, and K and E represent the complete elliptic integrals of the first and second kind.

4 Numerical results

In this section, the present method will be applied to two-dimensional and axisymmetric problems to test its validity. The unknowns are assumed to be constant in each boundary element in all the BEM computations.

4.1 Two-dimensional problems

The first problem is evaluation of the shielding factor S of an infinitely-long cylindrical shell. A uniform time-harmonic magnetic field is applied in the transverse direction on the x - y plane. The radius a of the cylinder is

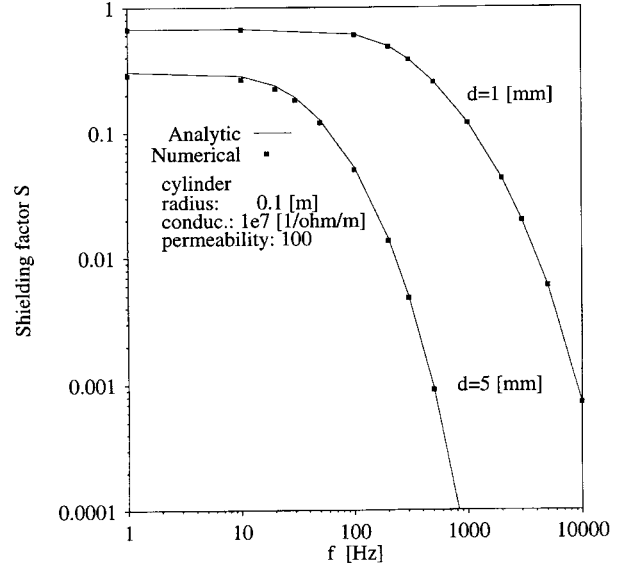


Fig. 3. Frequency characteristic of shielding factor S for a cylindrical shell.

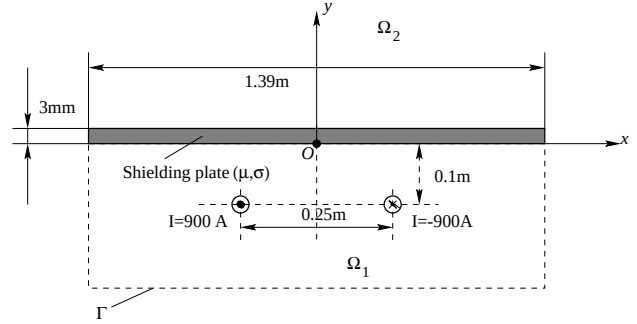


Fig. 4. A thin shielding plate above anti-parallel currents.

taken to be 0.1 [m], and the conductivity σ and permeability μ of the thin layer are chosen as $1 \times 10^7 [\Omega^{-1}\text{m}^{-1}]$ and $100\mu_0$. The shielding factor S is defined as

$$S = \frac{|\mathbf{H}|_{\text{shield}}}{|\mathbf{H}|_{\text{no shield}}}, \quad (41)$$

where $\mathbf{H}$ is evaluated at the center of the cylinder.

The frequency characteristics of S are shown in Figure 3. The solid lines represent the analytical values [6] while the dots the numerical results computed by the present method. The shielding factor S begins to decrease when the skin depth δ becomes comparable to the thickness d of the shell, *i.e.*, at $f \simeq 250$ [Hz] and $f \simeq 10$ [Hz] for $d = 1$ [mm] and $d = 5$ [mm], respectively. The analytical and computed results agree well with each other over the computed range.

The second problem, shown in Figure 4, is evaluation of the magnetic field near a thin metallic plate, under which a pair of anti-parallel currents is located. This is a simplified model of a magnetic shield by which magnetic fields generated from underground power cables are

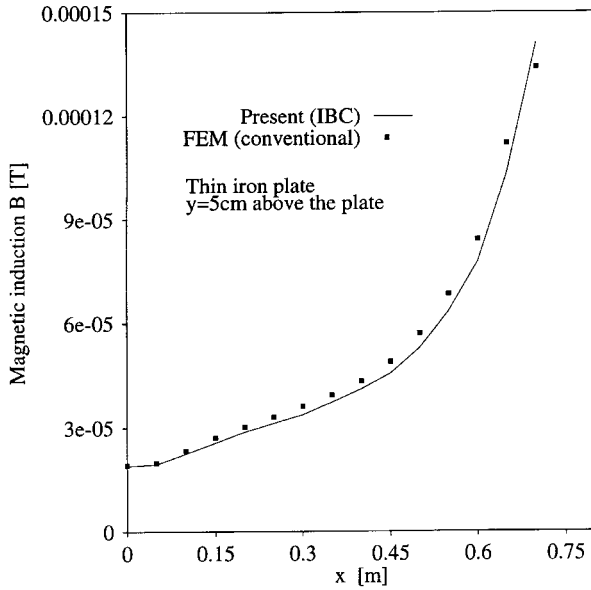


Fig. 5. Spatial distribution of magnetic induction above the plate.

intended to be blocked. The plate is assumed to be sufficiently long in the direction of the currents in comparison with its width in x -direction, so that a two-dimensional analysis in the x - y plane can be performed. The geometrical parameters are shown in Figure 4. The currents are sinusoidal ACs with amplitude 900 [A] and frequency 50 [Hz]. The permeability and conductivity of the plate are: $\mu = 1000\mu_0$, $\sigma = 6.41 \times 10^6$ [$\Omega^{-1}\text{m}^{-1}$]. The skin depth δ for this problem is 0.89 [mm], which is less than one third of the thickness d of the plate. The inner domain Ω_1 with the currents and the outer domain Ω_2 are separated by the plate and fictitious boundary in air.

The magnetic induction $|\mathbf{B}|$ is evaluated at the points $x \geq 0$, $y = 0.05$ [m]. This model is analyzed also by a conventional method based on FEM which makes no use of IBCs but an adaptive mesh generation technique [7]. The agreement between both numerical results, shown in Figure 5, is satisfactory. It should be noticed that the conventional FEM would require an enormous number of elements in the plate to obtain accurate results when the skin depth δ becomes much shorter than the thickness d . The present method does not suffer from this problem.

4.2 Axisymmetric problems

The first problem for this case is similar to the first problem in the previous section, in which the cylindrical shell is simply replaced by a spherical shell. The rotational (z -) axis of the sphere is chosen to be parallel to the applied uniform magnetic field. The computed frequency characteristics of the shielding factor, evaluated at the center of the sphere, are again in good agreement with the analytic values [6], as shown in Figure 6.

In the next problem a cylindrical shell immersed in a uniform magnetic field parallel to the rotational axis

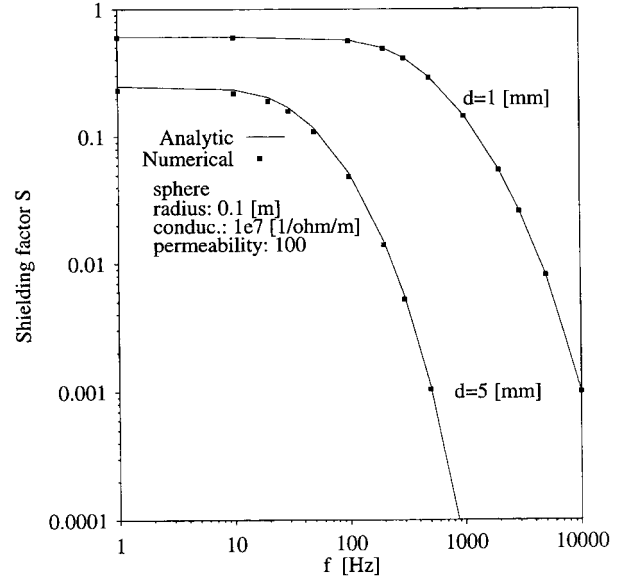


Fig. 6. Frequency characteristic of shielding factor S for a spherical shell.

of the cylinder is considered. This system can also be considered to be axisymmetric. The parameters for this problem are taken as follows: the radius of the cylinder $r = 0.1$ [m], height $h = 0.2$ [m], thickness $d = 1$ [mm], $\sigma = 1 \times 10^7$ [$\Omega^{-1}\text{m}^{-1}$] and $\mu = 1000\mu_0$.

Figure 7 shows the magnetic surfaces on which ψ is constant. The magnetic field is parallel to these surfaces as seen in equation (28). These contour lines are drawn using the following formula to regularize the quasi-singularity in the integral [8],

$$\psi_i(\mathbf{x}) = \frac{1}{\eta} \int_{\Gamma} \left[\frac{\partial \psi_i(\mathbf{x}')}{\partial n'} G(\mathbf{x}, \mathbf{x}') - \psi_i(\mathbf{x}') \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right] \frac{ds'}{r'} + \frac{\psi_{si}(\mathbf{x})}{\eta}, \quad (42)$$

where $\eta = C(\mathbf{x})$ and $\eta = C(\mathbf{x}) - 1$ for the interior Ω_1 and exterior Ω_2 of the shell, respectively, and

$$C(\mathbf{x}) = \frac{2}{r^2} \int_{\Gamma} \left[\frac{\partial r'}{\partial n'} G(\mathbf{x}, \mathbf{x}') - \frac{r'}{2} \frac{\partial G(\mathbf{x}, \mathbf{x}')}{\partial n'} \right] ds'. \quad (43)$$

When the applied field is almost static ($f = 1$ [Hz], $\delta/d \simeq 5$), the magnetic field lines tend to be strongly attracted to the shell. The field lines going into the shell pass through it without penetration, and then go out of it. When the frequency is set to 1 [kHz] ($\delta/d \simeq 0.16$), there are magnetostatical attraction of the field lines to the shell as well as repulsion due to the eddy currents induced in the shell. These effects balance to give the magnetic configuration shown in (b) of Figure 7. In both cases no field lines drawn here penetrate the shell.

Figure 8 shows the magnetic surfaces for a cylindrical

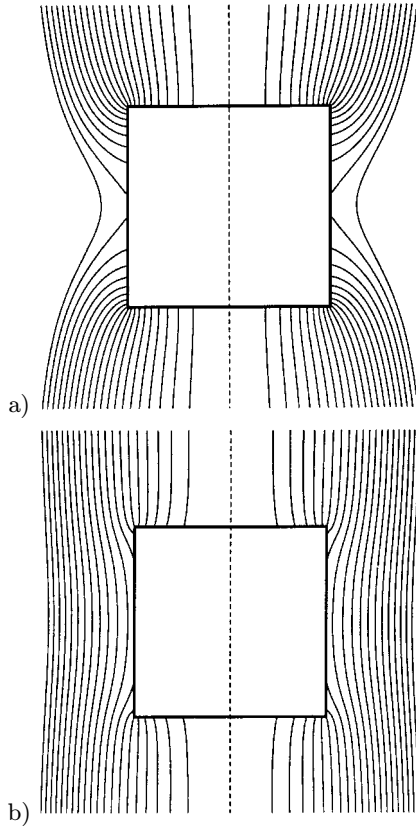


Fig. 7. Magnetic field around a cylinder. a) $f = 1$ [Hz], $\delta/d \simeq 5$; b) $f = 1$ [kHz], $\delta/d \simeq 0.16$.

shell without the upper surface. There are field lines going into the deep inside of the cylinder at 1 [Hz] while they tend to be absorbed into the fringe of the shell at 1 [kHz].

5 Conclusions

In this paper IBCs for magnetic conductive thin layers have been expressed in terms of the vector potential. The IBCs have been written in the form of a pair of first-order differential equations, which can be applied directly to the integral equations governing the magnetic field in the air region. This formulation does not have topological restriction, and allow directional dependence of IBCs for the treatment of edges and corners of a thin layer, unlike the formulation with the scalar magnetic potential. The IBCs have been applied to two-dimensional and axisymmetric test problems. The validity of the present method has been shown in these numerical examples.

The present formulation is valid when the thin layer has linear electromagnetic property. For non-linear materials, the one-dimensional equation for the electromagnetic field inside the layer would have to be numerically solved to get IBCs.

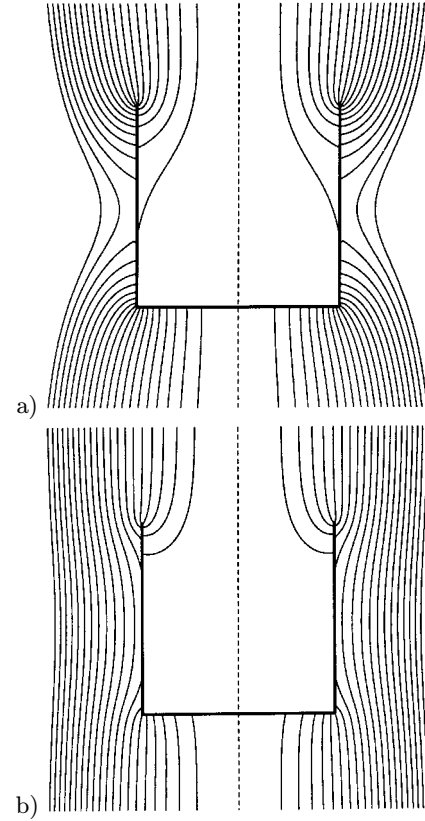


Fig. 8. Magnetic field around a cylinder without the upper surface. a) $f = 1$ [Hz], $\delta/d \simeq 5$; b) $f = 1$ [kHz], $\delta/d \simeq 0.16$.

The IBCs written in vector potential can be applied to three dimensional problems. Moreover, they can also be employed in an FEM formulation. These are the future works.

This work is supported by the Alexander von Humboldt foundation in part.

References

1. L. Krähenbühl, D. Muller, IEEE Trans. Magn. **29**, 1450 (1993).
2. I.D. Mayergoyz, G. Bedrosian, IEEE Trans. Magn. **31**, 1319 (1995).
3. C. Guérin, G. Tanneau, IEEE Trans. Magn. **31**, 1360 (1995).
4. O. Bíró, I. Bárdi, K. Preis, W. Renhart, K.R. Richter, IEEE Trans. Magn. **33**, 1173 (1997), presented at CEFC-Okayama, 1996.
5. O. Bíró, K. Preis, IEEE Trans. Magn. **25**, 3145 (1989).
6. J.F. Hoburg, IEEE Electromagn. Compat. **37**, 574 (1995).
7. L. Jänicke, *Finite Elemente Methode mit adaptiver Netzgenerierung für die Berechnung dreidimensionaler elektro-magnetischer Felder*, Ph.D. thesis, TU Berlin (1994).
8. H. Igarashi, T. Honma, J. Comp. Phys. **119**, 244 (1995).