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Theoretical and Applied Fracture Mechanics 32 (1999) 203–207

theoretical and
applied fracture
mechanics

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Energy release rate for interface debonding with prestress and friction

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Abstract

The constituents in a composite tend to reorganize during service. This gives rise to internal friction and slippage at the interfaces due to misfit. Energy release rate expressions are derived to quantify debonding. Two examples are provided to illustrate the method of solution. © 1999 Published by Elsevier Science Ltd. All rights reserved.

1. Introduction

Interface debonding is common in fiber reinforced composites. When the matrix is cracked the fibers may still be intact, but interface debonding near a matrix crack is unavoidable, as the matrix crack opens. Frictional sliding of the debonded interface would then follow. For such a complex system, there is the need to better understand the debonding behavior.

A debonding criterion was proposed in [1] on the basis of fracture mechanics by including the effect of interface friction. A more detailed analysis of fracture of fiber composites was made in [2] in which fiber contraction was considered.

While the energy approach in [1] considered prestress but the formulation contained unnecessary assumptions. Moreover, the reference state for computing the displacements were not clearly stated. These shortcomings are overcome in this

work and the results are compared with those in [3].

2. Potential energy and its release rate

Consider an elastic body that contains different constituents with internal misfits. This introduces the stress σ_0 . In the initial state (0), the body also contains cracks that is characterized by l_0 . S_u denotes the surface on which the displacements are specified as shown in Fig. 1(a). The potential energy for state (0) is

$$P_0 = \frac{1}{2} \int_V \sigma_0 : \varepsilon_0 dV \quad (1)$$

in which ε_0 is the strain corresponding to σ_0 .

When the external load T is applied on the surface S_T , the body changes to the state (*), as shown in Fig. 1(b), and the cracks grow by an amount Δl . The corresponding increments of stress and strain are denoted by $\Delta\sigma$ and $\Delta\varepsilon$, respectively while u is the displacement. On the crack surface S_F , frictional stress τ exists to resist the relative

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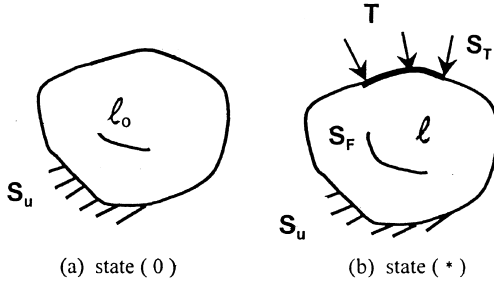


Fig. 1. Two states of the material. (a) state (0). (b) state (*).

sliding s . The total stress and strain are denoted by σ and ε ,

$$\sigma = \sigma_0 + \Delta\sigma, \quad \varepsilon = \varepsilon_0 + \Delta\varepsilon. \quad (2)$$

Then the potential energy for State (*) is

$$P = \frac{1}{2} \int_V \sigma : \varepsilon dV - \int_{S_T} T \cdot u dS_T. \quad (3)$$

Since T , τ and σ belong to an equilibrium force system and u, s and $\Delta\varepsilon$ belong to a compatible deformation field, it follows from the principle of virtual work that

$$\int_V \sigma : \Delta\varepsilon dV = \int_{S_T} T \cdot u dS_T - \int_{S_F} \tau s dS_F. \quad (4)$$

The product $\tau \cdot s$ is always taken to be positive. It can be noted from Eq. (2) that $\Delta\sigma : \varepsilon_0 = \Delta\varepsilon : \sigma_0$. Now, Eqs. (3) and (4) can be combined to give

$$P = \frac{1}{2} \int_V (\sigma_0 : \varepsilon_0 - \Delta\sigma : \Delta\varepsilon) dV - D_F, \quad (5)$$

$$D_F = \int_{S_F} \tau \cdot s dS_F, \quad (6)$$

where D_F is the dissipated energy by sliding with friction on internal surface. From state (0) to state (*), the released energy to form new crack surface is

$$R = P_0 - P - D_F = \frac{1}{2} \int_V \Delta\sigma : \Delta\varepsilon dV. \quad (7)$$

The energy release rate is

$$G = \frac{\partial R}{\partial l} = \frac{1}{2} \frac{\partial}{\partial l} \int_V \Delta\sigma : \Delta\varepsilon dV. \quad (8)$$

Eqs. (7) and (8) are the result as those given in [1], without the additional assumptions as in [1]. The significant feature of Eqs. (7) and (8) is that the quantities $\sigma_0, \varepsilon_0, T, u, \tau, s$ do not show up explicitly. Only the additional stress and strain are involved.

3. Discussion

To better understand Eqs. (7) and (8), we consider the three states, namely state (0) with σ_0, ε_0 ; state (1) with $\sigma_1, \varepsilon_1, u_1, s_1, T, \tau, l_1$ and state (2) with $\sigma_2, \varepsilon_2, u_2, s_2, T, \tau, l_2$. Let R_{01}, R_{02} denote the released energy from state (0) to state (1) and (2), respectively. According to Eq. (7), it is found that

$$\begin{aligned} R_{01} &= \frac{1}{2} \int_V \Delta\sigma_1 : \Delta\varepsilon_1 dV, \\ R_{02} &= \frac{1}{2} \int_V \Delta\sigma_2 : \Delta\varepsilon_2 dV \end{aligned} \quad (9)$$

in which

$$\begin{aligned} \Delta\sigma_1 &= \sigma_1 - \sigma_0, & \Delta\sigma_2 &= \sigma_2 - \sigma_0, \\ \Delta\varepsilon_1 &= \varepsilon_1 - \varepsilon_0, & \Delta\varepsilon_2 &= \varepsilon_2 - \varepsilon_0. \end{aligned} \quad (10)$$

If the released energy from state (1) to state (2) is denoted by R_{12} , then

$$R_{12} = R_{02} - R_{01} = \frac{1}{2} \int_V (\Delta\sigma_2 : \Delta\varepsilon_2 - \Delta\sigma_1 : \Delta\varepsilon_1) dV. \quad (11)$$

Eq. (11) is doubtlessly correct. However, the states (1) and (2) can also be regarded as the states (0) and (*) discussed earlier, hence Eq. (7) can be applied directly to give

$$R_{12} = \frac{1}{2} \int_V (\sigma_2 - \sigma_1) : (\varepsilon_2 - \varepsilon_1) dV. \quad (12)$$

Eq. (12) is equivalent to that given in [3]. Eqs. (11) and (12) appear so different, which deserve discussion. Since σ_0 is a self equilibrium stress system, both σ_1 and σ_2 tend to equilibrate T and τ , so $\Delta\sigma_1$ and $\Delta\sigma_2$ equilibrate with T and τ . Furthermore, $\Delta\varepsilon_1$ is compatible with u_1 , hence,

$$\begin{aligned} \int_V \Delta \sigma_1 : \Delta \varepsilon_1 dV \\ = \int_{S_T} T \cdot u_1 dS_T - \int_{S_F} \tau \cdot s dS_F, \end{aligned} \quad (13)$$

$$\begin{aligned} \int_V \Delta \sigma_2 : \Delta \varepsilon_1 dV \\ = \int_{S_T} T \cdot u_1 dS_T - \int_{S_F} \tau \cdot s dS_F. \end{aligned} \quad (14)$$

Eqs. (13) and (14) give

$$\int_V (\Delta \sigma_2 - \Delta \sigma_1) : \Delta \varepsilon_1 dV = 0. \quad (15)$$

Note that $\Delta \sigma_2 : \Delta \varepsilon_1 = \Delta \sigma_1 : \Delta \varepsilon_2$, Eqs. (11) and (15) give

$$R_{12} = \frac{1}{2} \int_V (\Delta \sigma_2 - \Delta \sigma_1) : (\Delta \varepsilon_2 - \Delta \varepsilon_1) dV \quad (16)$$

or

$$R_{12} = \frac{1}{2} \int_V (\sigma_2 - \sigma_1)(\varepsilon_2 - \varepsilon_1) dV.$$

Eq. (16) shows that Eqs. (12) and (11) are equivalent.

The derivation from Eqs. (13) to (16) uses only $\Delta \varepsilon_1$ and u_1 . Similarly, $\Delta \varepsilon_2$ and u_2 can also be used to give the derivation. Using the principle of virtual work, these, results

$$\begin{aligned} \int_V \Delta \sigma_2 : \Delta \varepsilon_2 dV \\ = \int_{S_T} T \cdot u_2 dS_T - \int_{S_F} \tau \cdot s dS_F. \end{aligned} \quad (17)$$

$$\begin{aligned} \int_V \Delta \sigma_1 : \Delta \varepsilon_2 dV + 2R_{12} \\ = \int_{S_T} T \cdot u_2 dS_T - \int_{S_F} \tau \cdot s dS_F. \end{aligned} \quad (18)$$

The second term of Eq. (18) is due to crack extension from state (1) to state (2) as shown in Fig. 1(b). Eqs. (17) and (18) can be combined to obtain

$$\int_V (\Delta \sigma_2 - \Delta \sigma_1) : \Delta \varepsilon_2 dV = 2R_{12}. \quad (19)$$

Eqs. (11) and (19) lead to Eq. (12). Therefore Eqs. (11) and (12) are equivalent.

4. Examples

4.1. Fiber pull out by a force

Consider a shear lag model for composites, the bar and the sleeve represent fiber and matrix, respectively, as shown in Fig. 2. The end of $x = L$ is fixed. Let R and r denote the radii of the matrix and the fiber, respectively. The volume fractions are C_m and C_f . The Young's module are E_m and E_f . This understood that the subscripts m and f refer, respectively, to the matrix and fiber. In the initial State (0), the debond length is zero. but the pre-stress in tension for the matrix and fiber are

$$\sigma_{m0} = \frac{\sigma_0}{C_m}, \quad \sigma_{f0} = -\frac{\sigma_0}{C_f}. \quad (20)$$

In the current State (*), the total load $\pi R^2 T$ acts on the fiber end at $x = 0$ with l being the debond length. The frictional stress on the debonded interface is τ . Let (σ_m, σ_f) denote the stresses and (u_m, u_f) the displacements of matrix and fiber, respectively. To calculate the potential energy for State (0) and State (*), the following contraction will be made:

$$\beta = C_f E_f / (C_m E_m), \quad E = C_f E_f + C_m E_m. \quad (21)$$

Further introduce the new variables $\bar{u}, \tilde{u}, \bar{\sigma}$ and $\tilde{\sigma}$ as

$$\bar{u} = \frac{1}{1 + \beta} (\beta u_f + u_m), \quad \tilde{u} = u_f - u_m, \quad (22)$$

$$\bar{\sigma} = C_f \sigma_f + C_m \sigma_m, \quad \tilde{\sigma} = E \left(\frac{\sigma_f}{E_f} - \frac{\sigma_m}{E_m} \right); \quad (23)$$

then

$$u_f = \bar{u} + \frac{\tilde{u}}{1 + \beta}, \quad u_m = \bar{u} - \frac{\beta}{1 + \beta} \tilde{u}, \quad (24)$$

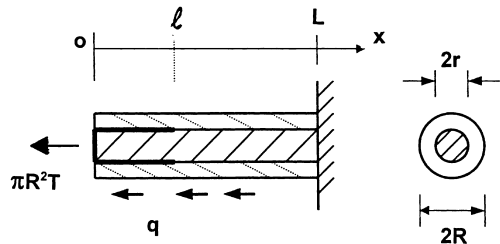


Fig. 2. Shear lag.

$$\sigma_f = \frac{E_f}{E} \left(\bar{\sigma} + \frac{\tilde{\sigma}}{1 + \beta} \right), \quad (25)$$

$$\sigma_m = \frac{E_m}{E} \left(\bar{\sigma} - \frac{\beta \tilde{\sigma}}{1 + \beta} \right),$$

$$C_f \sigma_f u_f + C_m \sigma_m u_m = \bar{\sigma} \bar{u} + \frac{\beta}{(1 + \beta)^2} \tilde{\sigma} \tilde{u}, \quad (26)$$

$$\frac{C_f}{E_f} \sigma_f^2 + \frac{C_m}{E_m} \sigma_m^2 = \frac{1}{E} \left[\bar{\sigma}^2 + \frac{\beta}{(1 + \beta)^2} \tilde{\sigma}^2 \right]. \quad (27)$$

Neglecting the Poisson's effect, the equilibrium conditions for fiber and matrix become

$$\begin{aligned} \pi r^2 \frac{d\sigma_f}{dx} + 2\pi r \tau &= 0, \\ \pi(R^2 - r^2) \frac{d\sigma_m}{dx} - 2\pi r \tau &= 0. \end{aligned} \quad (28)$$

Eqs. (25) and (28) can be combined to give

$$\frac{d\bar{\sigma}}{dx} = 0, \quad \frac{d\tilde{\sigma}}{dx} + \frac{\alpha}{\beta} (1 + \beta)^2 \tau = 0 \quad (29)$$

in which

$$\alpha = 2r/R^2. \quad (30)$$

At $x = 0$, $\sigma_m = 0$ and $\sigma_f = T/C_f$, the solution of Eq. (29) is thus obtained:

$$\bar{\sigma} = T, \quad \tilde{\sigma} = \frac{1 + \beta}{\beta} T - \frac{\alpha}{\beta} (1 + \beta)^2 \tau x. \quad (31)$$

Let

$$\tilde{\sigma}_0 = E \left(\frac{\sigma_{f0}}{E_f} - \frac{\sigma_{m0}}{E_m} \right) = -\frac{(1 + \beta)^2}{\beta} \sigma_0, \quad (32)$$

$$\Delta \sigma_f = \sigma_f - \sigma_{f0}, \quad \Delta \sigma_m = \sigma_m - \sigma_{m0}. \quad (33)$$

Substituting Eqs. (20) and (25) and (31)–(33) into (7), it is obtained that

$$\begin{aligned} R &= \frac{\pi R^2}{2E} \int_0^L \left[\bar{\sigma}^2 + \frac{\beta}{(1 + \beta)^2} (\tilde{\sigma} - \tilde{\sigma}_0)^2 \right] dx \\ &= \frac{\pi R^2}{2E} \left[L \bar{\sigma}^2 + \frac{\beta}{(1 + \beta)^2} \int_0^L (\tilde{\sigma} - \tilde{\sigma}_0)^2 dx \right]. \end{aligned} \quad (34)$$

Finally, Eq. (8) gives

$$G = \frac{\partial R}{\partial l} = \frac{\pi R^2}{2E} \frac{\beta}{(1 + \beta)^2} (\tilde{\sigma} - \tilde{\sigma}_0)^2 \Big|_{x=l}. \quad (35)$$

Combining Eqs. (31), (32) and (35), the final result is

$$G = \frac{\pi R^2}{2E\beta} [T + (1 + \beta)(\sigma_0 - \alpha \tau l)]^2. \quad (36)$$

4.2. Fiber debond under fixed displacement

Consider the same shear lag model as discussed earlier. At the end of $x = L$, the matrix and fiber are fixed, but at $x = 0$ the fiber is fixed while the matrix is traction free. In the initial state (0), debond length is zero. In the current state (*), the debond length is l . An additional force $\pi R^2 T$ acts on the fiber end at $x = 0$, while T is an unknown that can be determined by the constraint condition at $x = 0$, as

$$u_f(0) = \left(\bar{u} + \frac{\tilde{u}}{1 + \beta} \right)_{x=0} = 0. \quad (37)$$

When the Poisson's effect is neglected, there results

$$E_f \frac{du_f}{dx} = \sigma_f - \sigma_{f0}, \quad E_m \frac{du_m}{dx} = \sigma_m - \sigma_{m0}. \quad (38)$$

Eqs. (22)–(25) and (38) can be combined to give

$$\begin{aligned} \bar{u}|_{x=0} &= \frac{-1}{E} \int_0^L \bar{\sigma} dx, \\ \tilde{u}|_{x=0} &= \frac{-1}{E} \int_0^l (\tilde{\sigma} - \tilde{\sigma}_0) dx. \end{aligned} \quad (39)$$

The solution of Eq. (31) is still valid. From Eqs. (39), (37) and (31), it follows that

$$T = \frac{(1 + \beta)l}{\beta L + l} \left(\sigma_0 - \frac{\alpha \tau}{2} l \right). \quad (40)$$

The released energy R is still given by Eq. (34), then

$$\begin{aligned} G &= \frac{\partial R}{\partial l} = \frac{\pi R^2}{2E} \left[2LT \frac{\partial T}{\partial l} + \frac{\beta}{(1 + \beta)^2} (\tilde{\sigma} - \tilde{\sigma}_0)^2 \Big|_{x=l} \right. \\ &\quad \left. + \frac{2\beta}{(1 + \beta)^2} \int_0^l (\tilde{\sigma} - \tilde{\sigma}_0) \frac{\partial \tilde{\sigma}}{\partial l} dx \right]. \end{aligned} \quad (41)$$

According to Eqs. (31), (37), (39) and (40), the first and third terms in Eq. (41) cancel each other, such that

$$G = \frac{\pi R^2}{2E} \frac{\beta}{(1 + \beta)^2} (\tilde{\sigma} - \tilde{\sigma}_0)^2 \bigg|_{x=l}. \quad (42)$$

Eqs. (35) and (42) shown that the energy release rate for debonding depends on the value of $\tilde{\sigma} - \tilde{\sigma}_0$ in front of debonding.

5. Concluding remarks

- Although the composites with misfit stress and frictional interface could be complex to quantify nevertheless the released energy given by Eq. (7) is simple.
- The energy release rate to debond an interface depends only on the change of stress state near the front of debonding.

- Energy expressions in Eqs. (11) and (12) appear to be different but they are actually equivalent.

Acknowledgements

This work is supported partially by the National Science Foundation of China, and partially by the Hong Kong Polytechnic University.

References

- [1] Y.C. Gao, Debonding along the interface of composites, *Mech. Resear. Comm.* 14 (1987) 67–72.
- [2] Y.C. Gao, Y.W. Mai, B. Cotterell, Fracture of fiber-reinforced materials, *J. Appl. Math. Phys. ZAMP* 39 (1988) 550–572.
- [3] B. Budiansky, J.W. Hutchinson, A.G. Evans, Matrix fracture in fiber-reinforced ceramics, *J. Mech. Phys. Solids* 34 (1986) 167–189.