

# Thermal and Thermomechanical Phenomena in Sliding Contact

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**Abstract**—The paper deals with the development of heat problems in friction, beginning from the works of H. Blok to the present day. Blok put forward the concept of flash temperature, which has become a fundamental one in frictional heating. The main models of frictional heating, developed by Blok, Jaeger, Archard, and Kuhlmann–Wilsdorf, are analyzed. The influence of different factors on flash temperature and its distribution over the friction surface is discussed. The effect of frictional heating on the mechanisms of material wear is considered.

**Key words:** thermotribology, heat problem, frictional heating, temperature, temperature distribution, thermal stress.

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## INTRODUCTION

Man has been familiar with the phenomenon of heat release during friction since ancient times, when he learned to apply this effect for fire production [1]. Today, people encounter frictional heating every day and there is an independent field of tribology devoted to studies of thermal phenomena in friction (in English, the term *thermotribology* is used). This field of study was founded by professor Harmen Blok (1910–2000), the 1973 recipient of the Tribology Gold Medal, who put forward the concept of flash temperature [2]. This concept has formed the basis for further studies in the given field. Even today, this physically elegant and mathematically plain concept remains the most suitable one for evaluating temperature in the operation of most tribojoints.

The development of thermotribology was stimulated by the necessity to solve numerous practical tasks. Frictional heating and friction-induced thermal and thermomechanical phenomena can produce significant effects on the service characteristics of friction units, primarily due to temperature-induced variations in the structure and chemical composition of the contact materials and in the properties of films on the friction surfaces. By way of example, with increasing temperature, such mechanical characteristics of materials as the modulus of elasticity and hardness decrease, which is of particular importance for polymer and polymer-based composites [3]. In addition, one may observe such effects as the melting of the surface layer of one of the mated materials [4], the destruction of polymers and polymer composites [5], the oxidation of metals and alloys [6, 7], the desorption of molecules of the

lubricant in the boundary layer and the rupture of oil films [8], etc. Frictional heating can also intensify various mechanisms of friction surface destruction, particularly thermal cracking of the working surfaces of brakes and face seals [9–12]. It causes sparking and increased wear of the brushes of electric machines [13], as well as the seizure of gears [2, 14]. In order to increase the reliability and service life of friction units, and consequently the service characteristics of machines and mechanisms, it is necessary to minimize the negative effect of thermal processes in the contact zone. The experimental methods used in thermotribology are analyzed in review [15]. The current article gives a brief review of theoretical studies of frictional heating and the associated thermal and thermomechanical phenomena in the friction zone.

## MAIN MODELS OF FRICTIONAL HEATING

In thermotribology it is common to distinguish three temperatures: the bulk temperature  $V_b$ , the average surface temperature  $T_s$ , and the flash temperature  $T_f$ . *Bulk temperature* is the temperature averaged over the bulk of one of the contact bodies; *surface temperature* is the temperature averaged over the thin surface layer of a body; and *flash temperature* is the local increment of temperature at the contact of microasperities on the rubbing surfaces.

The notion of flash temperature reflects the discrete nature of frictional contact. Since the area of real contact of bodies is three–four orders of magnitude lower than the nominal contact area, the heat generated in the friction zone is concentrated on small contact spots whose life varies from hundreds of microseconds to

several milliseconds. Owing to this, the heat-flow density on these spots can be very high and result in short-time temperature elevation by hundreds of degrees.

Chichinadze suggested finding the maximal temperature of frictional contact as the sum of bulk temperature, average surface temperature, and flash temperature [12]:

$$T_{\max} = T_b + T_s + T_f.$$

When solving the friction heat problem, model views are used since the solution of the thermal conductivity equation with the initial and boundary conditions, considering the real pattern of contact, the conditions of heat transfer to the environment, and variations in the mechanical and thermal characteristics of the materials, is a really intrinsic problem. The use of assumptions in models of frictional heating simplifies the solution of the heat problem, on the one hand, and, on the other, imposes certain limitations on the applicability of the solutions obtained.

**The Bowden model.** Bowden and Ridler [16] were the first to determine the friction zone temperature. While solving the problem of finding the pin-disc contact temperature, the following assumptions were made:

1. Contact bodies have smooth surfaces, that is, the contact occurs over nominal area;
2. The pin is a half-infinite rod in which the temperature field is taken as unidimensional;
3. According to Amontons' law, the coefficient of friction is independent of the load and sliding velocity;
4. Heat flow is taken as stationary;
5. On the side surface of the rod heat exchange with the environment occurs according to Newton's law with the known coefficient of heat exchange  $\sigma$ .

For a round-section rod of radius  $r$ , the temperature increment is found from the following expression:

$$T - T_0 = \frac{(1 - \alpha_{\text{tf}})fNv}{\pi r \sqrt{2\sigma\lambda r}},$$

where  $T_0 = \text{const}$  is the ambient temperature;  $\alpha_{\text{tf}}$  is the heat partition factor, which Bowden took as being 0.5;  $f$  is the coefficient of friction;  $N$  is the normal load;  $v$  is the sliding velocity; and  $\lambda$  is the thermal conductivity of the rod material.

Initially, Bowden and Ridler measured experimentally the friction surface temperature by means of a natural thermocouple, and then Bowden formulated and solved the aforementioned heat problem. The linear dependence of temperature on sliding velocity and load agreed well with the experimental results both in the cases of dry and lubricated friction. On the contrary, the temperature versus thermal conductivity dependence did not agree with the experimental data. The cause for this lies in Bowden's assumption about the ideal smoothness of the mated surfaces. In addition, in the given formulation, the heat problem is stationary: the

heat source was taken as stationary, when in reality it moves. Therefore, the Bowden model is applicable to the calculation of average surface temperature rather than flash temperature. Nevertheless, the Bowden study represented the first step in thermotribology and encouraged its further development.

**Blok model.** Blok was the first to suggest the flash temperature theory, which, in contrast to the Bowden model, allowed for calculating the temperature of local heat sources and taking into account their motion [2]. His theory has been used by tribologists for decades.

The main propositions of the model of frictional heating were formulated in the following way:

1. There are no films or deposits on the friction surfaces;
2. The size of a single contact spot is small as compared to the sizes of the contact bodies, which allows for taking the latter as infinitely large;
3. The heat generated during friction is totally absorbed by the friction bodies and heat transfer to the environment is absent;
4. Friction follows Amontons' law and the coefficient of friction is constant;
5. The specific heat capacities of the contact body materials are infinitely large;
6. The source intensity, defined as the heat generated on the unit area per unit time, is distributed over its surface by the same law as the contact pressure.

Blok considered different-shaped sources (band, circular, rectangular) without taking into account their physical nature. He solved the unidimensional heat problem with a heat flow directed normally to the source plane. The solutions for stationary and moving sources were extended to the case when a single heat-isolated friction contact spot was a heat source.

Blok's works were analyzed in detail and systematized by Shchedrov [17]. Below, the solutions of the heat problems are considered only for the main cases.

*1. A stationary asperity (body 2) contacts a half-space surface (body 1). The contact spot is a circle within which the density of heat flow  $q$  is uniformly distributed.* The rotation of a round thrust journal bearing against the plane can serve an example. The flash temperature at the contact spot is calculated as follows:

$$T_f = \frac{qr}{\lambda_1 + \lambda_2},$$

where  $q = fNv/\pi r^2$  is the specific intensity of heat release (heat-flow density) from the contact spot of radius  $r$  and  $\lambda_1$  and  $\lambda_2$  are the thermal conductivities of bodies 1 and 2, respectively.

2. Contact conditions are similar to case 1, but the heat flow density is distributed over the source surface according to parabolic law:

$$q = q_0 \left( 1 - \frac{x^2}{r^2} \right),$$

where  $q_0 = fp_0v$  is the heat flow density in the source center;  $p_0$  is the contact pressure in the source center;  $x$  is the distance from the source center to the point where the temperature is found.

Maximal temperature achieved in the center of the contact spot is found by the formula:

$$T_f = \frac{2}{3} \frac{q_0 r}{\lambda_1 + \lambda_2}.$$

3. The asperity forming a circular contact spot with the half-space surface is sliding on it at a velocity meeting the condition  $v \leq \frac{4a_2}{25r}$  (low sliding velocities;  $a_2$  is the thermal diffusivity of asperity material). In the given case, flash temperature can be found by the aforementioned formulas for a stationary source.

4. An asperity forming a circular contact spot with the half-space surface is sliding on it with a velocity meeting the condition  $v \geq \frac{4a_2}{r}$  (high sliding velocities).

Then the temperature increment in the contact center on the asperity surface is found as

$$T_{f2} = \frac{qr\psi}{\sqrt{2}(\lambda_1\psi + \sqrt{\pi}\lambda_2)}.$$

And on the half-space surface as

$$T_{f1} = \frac{qr\psi}{\lambda_1\psi + \sqrt{\pi}\lambda_2},$$

where  $\psi = \sqrt{\frac{4a_2}{vr}}$  is the parameter depending on sliding velocity and heat source size.

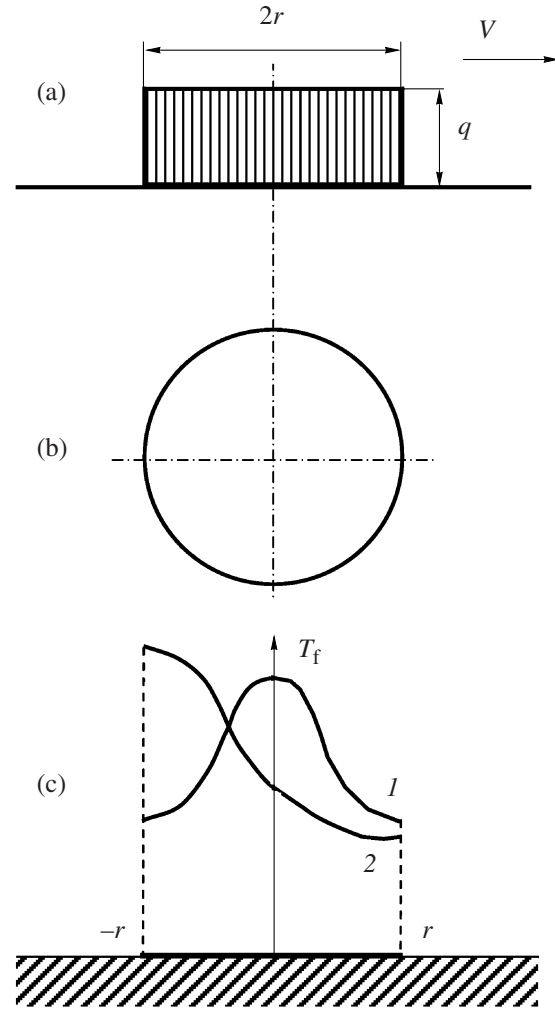
The maximal temperature on the contact surface is found as the arithmetic mean of  $T_{f1}$  and  $T_{f2}$ .

$$T_{\max} = \frac{1 + \sqrt{2}}{2} \frac{qr\psi}{\lambda_1\psi + \sqrt{\pi}\lambda_2}.$$

5. A circular heat source slides on the half-space surface and the thermal conductivity of one of the bodies is negligibly small compared to that of the other. At uniform distribution of the source intensity and low sliding velocities the flash temperature is

$$T_f = \frac{2qr}{\sqrt{\pi}\lambda}.$$

For a rectangular heat source with uniformly distributed intensity moving on the half-space surface, the for-



Temperature distribution at a single contact spot along the sliding direction: a, b—plane circular heat source (single contact spot); c—temperature distribution along the source; 1—distribution symmetrical relative to the contact spot center (stationary source); 2—distribution with maximum at the trailing edge of contact spot (source moving at a high velocity).

mulas for a circular source are applicable wherein  $r$  should be replaced with the half-width of the contact area.

$$T_f = \sqrt{2} \frac{qr\psi}{\sqrt{\pi}\lambda}.$$

Analysis of the Blok formulas for flash temperature allows for constructing temperature distribution over the contact spot in the sliding direction. In particular, the distributions for stationary and moving sources with uniformly distributed intensity  $q$  are shown in the Figure. If the heat source is stationary or moves at a low velocity, the temperature distribution is symmetrical relative to its center (Figure, curve 1). With rising velocity, the temperature maximum is shifted in the opposite to the sliding direction. At high velocities, the

point of maximal temperature is at the trailing edge of the contact spot (curve 2).

As in all models of frictional heating, the Blok model idealizes the generation and distribution of friction heat since it is based on assumptions that are far from reality (juvenile state of friction surfaces, absence of mutual effect of heat sources, absence of convective heat exchange from the source surface to environment). For example, the assumption of the small size of heat sources is valid, generally speaking, for elastic contact of bodies. However, under high pressures, saturated plastic contact occurs and the mutual effect of spots emitting heat energy becomes more essential. Therefore, in the given case the Blok formulas give low values for flash temperatures.

Blok postulated the law of distribution of the intensity of a heat source over its surface. In reality, however, this law is not known a priori and is governed by the shape of the actual contact spots and their distribution over the nominal contact area [17].

In most cases the Blok theory gives satisfactory evaluation of flash temperature.

**Jaeger model.** In essence, Jaeger's work [18] does not contradict the propositions of the Blok theory and can be considered as its development. Following Blok, Jaeger took the contact surfaces as planar; however, he formulated the heat problem as stationary. He found the temperature distribution for rectangular and band sources with constant intensity, stationary or moving on the half-space surface at a constant velocity. His work represented progress compared to the results of Blok since it became possible to find temperature distributions not only at high and low, but also at intermediate Peclet numbers with account for the mutual effect of single sources within the contact area and even to obtain the temperature distribution over half-space depth.

In addition, Jaeger solved the plane heat problem for a moving source with the intensity distributed by different laws and found that the maximal and average temperatures of the contact surface depended weakly on the law of heat-source distribution.

Jaeger considered two contacts that are essential for tribology in practice. In the first case, an asperity on a rigid half-space surface (body 2) slides on the surface of another rigid half-space (body 1), forming a contact area shaped as a square with the side  $2l$ . The second case corresponds to sliding of an infinitely long rod of square cross section (body 2) on a stationary half-space surface (body 1); heat exchange with the environment occurs on the side rod surface with heat exchange coefficient  $\sigma$ .

Jaeger's solutions differed from those of Blok in that the heat partition factor  $\alpha_{\text{tr}}$  was found from the condition of equality of average surface temperatures rather than maximal ones. This was due to the simple procedure of finding  $T_s$  compared to  $T_{\text{max}}$  in the experiment.

In the first case, the average temperature on the contact surface is found in the following way:

$$T_{\text{av}} = \begin{cases} 0.946 \frac{ql}{\lambda_1 + \lambda_2} & \text{for low velocities;} \\ \frac{1.064ql\sqrt{a_1}}{1.25\lambda_2\sqrt{a_1} + \lambda_1\sqrt{l\nu}} & \text{for high velocities.} \end{cases}$$

In the second case, we have

$$T_{\text{av}} = \begin{cases} \frac{0.946ql}{\lambda_1 + 1.338\sqrt{l\lambda_2}\sigma} & \text{for low velocities;} \\ \frac{1.064q\sqrt{a_1}l}{\lambda_1\sqrt{\nu} + 1.504\sqrt{a_1}\lambda_2\sigma} & \text{for high velocities.} \end{cases}$$

One of the most important differences of the Jaeger model from the Blok one is that Jaeger solved the problem of the temperature distribution over the surface layer thickness of one of the contact bodies. He obtained the descending dependence of temperature on depth under the friction surface, the temperature dropping rather quickly, indicating friction heat penetration by only a small depth. Nonuniform heating of the thin surface layer causes a high temperature gradient along the normal to the surface, which may lead to the appearance of significant thermal stresses. Thermal and thermal-stress states of the surface layer can be controlled by modifying its characteristics, e.g., by applying coatings with different thermal conductivity.

**Archard model.** Archard developed a model of flash temperature which is distinguished by mathematical simplicity [19]. He solved the heat friction problem in the same formulation as Blok and Jaeger, that is, under the assumptions of a single asperity on the surface of one body sliding on the surface of the other one. The contact spot is a circular heat source; heat flows to both bodies are unidimensional; the heat partition factor is found from the equality of average temperatures of the contact surfaces within the contact spot; and convective heat transfer to the environment is absent.

Archard used an electrical analogy; i.e., he interpreted the heat flow as a thermal current passing through a region of a certain thermal resistance [20, 21]. With respect to the moving asperity, the heat flow is stationary; the flash temperature (average temperature within the region occupied by the source) is found by the formula

$$T_{\text{f2}} = \frac{Q_2}{4r\lambda_2},$$

where  $Q_2$  is the heat flow directed into the asperity and  $r$  is the contact-spot radius.

The formula is also valid for a moving source (body 1) at low sliding velocities ( $\text{Pe} < 0.4$ ) and at high velocities

( $Pe > 20$ ) the flash temperature is calculated in the following way:

$$T_{f1} = \frac{0.31 Q_1}{\lambda_1 r} \sqrt{\frac{a_1}{v r}},$$

where  $Q_1$  is the heat flow directed to the half-space.

Then the temperature on the friction contact is found from the following condition:

$$\frac{1}{T_{\max}} = \frac{1}{T_{f1}} + \frac{1}{T_{f2}}.$$

Archard also took into account the dependence of the contact area on normal load and proposed an approximate theory of friction temperature at elastic and plastic contacts. He assumed that the contact bodies had the same mechanical and thermal properties and at low sliding velocities friction heat was distributed between them in equal proportions, while at high sliding velocities it was totally absorbed by body 1 (the half-space). In the given case the contact-spot temperature depends on the load, velocity, and material properties in the following way:

—plastic contact at low sliding velocities ( $Pe < 0.4$ )

$$T_f = \frac{f(\pi\sigma_s)^{\frac{1}{2}}}{8\lambda} N^{\frac{1}{2}} v,$$

where  $f$  is a coefficient of friction;  $\sigma_s$  is yield strength of material;

—plastic contact at high sliding velocities ( $Pe > 20$ )

$$T_f = \frac{f}{3.25} \frac{(\pi\sigma_s)^{\frac{3}{4}}}{(\lambda\rho c)^{\frac{1}{2}}} N^{\frac{1}{4}} v^{\frac{1}{2}};$$

—elastic contact at low sliding velocities ( $Pe < 0.4$ )

$$T_f = \frac{f}{8.8\lambda} \left(\frac{E}{R}\right)^{\frac{1}{3}} N^{\frac{2}{3}} v,$$

where  $E$  is the modulus of elasticity of the material and  $R$  is the curvature radius of the spherical asperity (body 2) in the nondeformed state;

—elastic contact at high sliding velocities ( $Pe > 20$ )

$$T_f = \frac{f}{3.8} \left(\frac{E}{\lambda\rho c R}\right)^{\frac{1}{2}} N^{\frac{1}{2}} v^{\frac{1}{2}},$$

**Kuhlmann-Wilsdorf model.** In the models analyzed above, heat sources of regular geometrical shape were considered. However, as experimental study results show, heat sources actually have a shape close to elliptical; the large ellipse axis is parallel to the sliding direction [22–24]. Kuhlmann-Wilsdorf modified the Blok and Jaeger models in order to calculate the flash temperature at the elliptical contact spot [25–27]. For a

plane elliptical heat source, the flash temperature is found as

$$T_f = \frac{qR'}{\frac{\lambda_1}{\Phi_1(v_1)\Phi_2(e, v_1)} + \frac{\lambda_2}{\Phi_1(v_2)\Phi_2(e, v_2)}},$$

where  $R'$  is the characteristic contact spot size  $R' = \sqrt{\bar{A}/\pi}$ ;  $\bar{A}$  is the average contact spot area;  $e$  is the ratio of the length of the large contact-spot axis parallel to the sliding direction to that of the small axis (ellipticity);  $\Phi_1$  is the function describing the temperature–velocity dependence and varying from 0.1 to 2;  $\Phi_2$  is the function describing the temperature versus contact spot ellipticity;  $v_1$  and  $v_2$  are the relative velocities of the first and second bodies, respectively,  $v_{1,2} = \frac{\rho_{1,2} c_{1,2} v R}{\lambda_{1,2}}$ ; and  $v$  is the sliding velocity.

At low sliding velocities, the spots elongated in the sliding direction have the highest flash temperature. At high velocities ( $Pe = 1–10$ ), the spots with ellipticity  $e = 4–10$  have the maximal temperature. The temperature of the spots strongly elongated in the sliding direction ( $e > 10$ ) or spots strongly elongated in the normal direction is lower than that of circular spots.

#### Other theoretical studies of frictional heating.

Blok's concept of flash temperature was stimulated by studies of seizure in involute gearings. However, it considered thermal processes at a single contact spot, and the transition from the heat problem for a local spot to that for a real tribojoint caused definite difficulties. In particular, one should take into account that the densities of heat flows are variables in a real contact. Also, the correct choice of boundary conditions of the heat problem is of extreme importance. In reviews [28–30] the heat problem was shown to be extended to the case of a real friction unit. In this connection one should note the works by Chichinadze and his followers dealing with thermal processes in brakes, frictional clutches, and other friction units, as well as the development of frictional materials and methods for their testing [12, 31–34]. In these works, fundamental dependences of the coefficient of friction and wear rate on the temperature characteristics of the contact were derived.

Korovchinskii [35, 36] reviewed the investigation results concerning the heat generated during friction and cutting. The solutions of the heat problem obtained by Korovchinskii himself are applicable to moving heat sources of various shapes (square, elliptic, etc.) with a constant and variable intensity. The results were the basis for solving the problems on heat distribution in various friction units.

In works [29, 30] the results of application of the modified Blok model to the heat problem for involute gearings were generalized. The use of the given model allowed the authors to determine the arrangement of local wear zones or “weak zones” on the tooth surface.

This is of practical importance in terms of allowing for the possibility of local thermal treatment of machine parts, in particular, for increasing their wear resistance and life.

The Blok and Jaeger models were based on using the heat-source method that was used by other researchers as well. Thus, Barber extended Jaeger's solution for a single source to a multiple contact [37]. Marsher published the calculation results of surface temperature resulting from the action of several sources whose mutual effect produced unidimensional and three-dimensional heat flows [38].

One should stress once again the limited character of the methods based on heat sources. They are only applicable when thermal processes are considered in half-restricted bodies. This consideration encouraged the development of techniques based on integral transforms and realized by numerical methods since such complicated heat problems could not be solved analytically. Most of the works in this area were performed by Ling with coauthors. Their results are outlined in monograph [39]. In particular, they proposed a stochastic model wherein a finite number of small contact spots distributed over the nominal area were considered. The positions of spots varied randomly. The contact-spot temperature was stated to significantly exceed the average surface temperature [40], and the average temperature distribution agreed well with the experimental data. Integral transforms were also used by Floquet and coauthors in the solution of three-dimensional problems for real friction units [41, 42].

Most methods based on integral transforms do not use the heat-partition factor. Instead of it, the condition of temperature continuity at the interface over the whole contact area is used [43]. Another approach, outlined in work [44], assumes that friction heat is generated in a so-called third body dividing the contact surfaces. Integral equations describing the heat flow in the third body were supplemented by the equations for friction bodies under the condition of temperature continuity on the interfaces of the bodies and the third body.

The development of numerical techniques encouraged progress in thermotribology, especially in the solution of heat problems for irregularly shaped bodies. In particular, in works [45–47] the finite element method is used to simulate temperature distribution in both stationary and moving friction members under transient and quasistationary operating conditions. Single and multiple heat sources were considered with the mutual effect of the sources within the contact area being taken into account. In addition, the temperature fields in both friction members were found, and were used in the finite-element analysis of the stress distribution.

The finite-element method was also used to study the influence of heat generation under the friction surface on the temperature distribution [45]. If the subsurface heat generation is taken into account, the temperature gradient in the surface layer is shown to significantly exceed

the gradient when all heat is generated on the friction surface. In the element that is stationary relative to the heat source, the arising subsurface temperature can be much higher than the surface temperature near the leading edge of the contact spot [45]. Rozeanu proved theoretically possible temperature peaks to be available under the friction surface [48, 49], which was justified experimentally in Balakin's research [50].

## THERMOMECHANICAL PHENOMENA DURING SLIDING FRICTION

Frictional heating results in so-called thermomechanical phenomena, including thermal deformations in the zone adjacent to the contact area, contact geometry variations due to thermal deformations and thermoelastic instability of the contact, and a thermal stress field in the vicinity of contact spots. These effects are essential in thermal cracking, wear, and other kinds of material damage during friction.

**Thermal deformations and thermoelastic instability of the friction contact.** The stress-strain and thermal states of the friction contact zone are related. The interrelation can manifest itself in so-called thermoelastic instability, transition to which is characterized by the appearance of "hot spots" at the nominal continuous contact. Pressure and temperature developing in the regions of the friction surface exceed nominal pressure and average surface temperature, which can intensify the wear of tribojoints with a high overlapping factor operating under heavy loads and at high sliding velocities.

Parker and Marshall were the first to directly prove the localization of frictional heating on a microscopic scale [51]. They used a low-temperature radiation pyrometer to measure the friction-surface temperature of a railway wheel during braking with block. The measurements showed the existence of heated regions on the brake block surface; the size of these regions was within an interval between the asperity spacing and nominal contact-area size. Similar results were later obtained by Sibley and Allen [52] and Santini and Kennedy [53] in disc brake testing.

Barber proved experimentally the existence of microscopic "hot spots" [54, 55]. By means of thermocouples, the temperature at different points of the railway brake block was measured. Each sensor recorded periodically repeating temperature fluctuations. This finding indicated the presence of an instability at which individual spots of the rubbing surface of microscopic scale were subjected to cyclic thermal loading. The mechanism of thermoelastic instability (TEI), according to Barber, is due to simultaneous thermal expansion and wear of the material running at the contact spots. Contact stability is governed by the ratio of the expansion rate and wear rate of the material of the stationary friction member.

Dow and Burton in work [56] considered a problem wherein a rectangular plate slides by its face against the surface of a rigid half-space in the normal to its surface direction. In contrast to the Barber model, an unstable thermoelastic state can exist even in the absence of wear in the given case. Removal of heat to the half-space by heat conduction takes the part of a heat expansion damper. TEI arises only at a certain sliding velocity, below which a stable regime occurs. The velocity is governed by thermal conductivity, thermal diffusivity, heat expansion, the moduli of elasticity of the contact materials, and the geometric dimensions of the moving body. The generalization of the given model to the non-zero wear case has shown the calculated critical velocity to be higher in this case than under wearless conditions [57]. Experimental verification of the given model by Dow and Stockwell [58] showed good agreement between the theoretical and experimental data.

The Dow–Burton model was developed in works [59, 60], with the focus on the influence of the thermal conductivity of the mated materials on the stability of systems of the face-seal type. It is shown that at the contact of materials having the same conductivity, the transition to TEI occurs only when the coefficient of friction exceeds unity. The calculated critical velocity is so high for the tribojoint that it exceeds the maximal velocities at which real seals work. A heat insulator–heat conducting material pair is, on the contrary, always unstable since at any coefficient of friction it has a certain critical sliding velocity.

In the review on face seals [61], it is noted that most joints of this kind operate under hydrodynamic conditions characterized by the presence of a viscous liquid film between the rubbing surfaces. The film causes non-uniform heating and thermoelastic deformation of the surface, which results in a hydrodynamic effect. Work [62] investigated TEI during the sliding of a heat-conducting body on a heat insulator under hydrodynamic lubrication. The authors calculated the critical velocity as the function of viscosity and average thickness of the lubricating film. In the improved model [63, 64], the contact of a rotor with an initially wavy surface and an elastic unrigidly fixed stator was considered. It was noted that the average thickness of the lubricating film and amplitude of initial waviness increase with increasing sliding velocity. However, such a monotonous rise of waviness, in the author's opinion, is caused by the continuous variation in the stable system state, rather than by the TEI regime.

A more realistic 3D geometry of face seals was considered by Lebeck in works [65, 66] with account for heat transfer from the seal rotor to the surrounding fluid. The parameter characterizing the system stability is larger than that for the 2D scraper whose edges are supposedly heat insulated. The author attributes this to heat removal from the moving member of the seal, which compensates its heat expansion and increases the critical velocity.

According to the data available in [9, 67, 68], the TEI regime arises during the operation of friction pairs with a high overlapping factor under heavy loads or at high sliding velocities. Face seals and brakes are typical examples of these units. Work [67] presents data on the operation of seals having the elements of graphite and tungsten carbide preliminarily run-in up to the nominally plane state of the surfaces. The presence of graphite transfer films on different spots of the tungsten-carbide ring surface and its profile demonstrating the appearance of waviness in the course of operation indicate load localization caused by nonuniform heat deformation of the surfaces. Heavy mechanical and thermal loads experienced by individual contact regions result in “thermal cracks” occurring in the surface and subsurface layers of the material within these regions. Thermal cracking of the material is also noted by Bill and Ludwig [10] as applied to seals and Dow [9] as applied to block brakes.

Kennedy received the images of “hot spots” during the operation of a disc brake [60] and revealed the localization of high temperatures in a small part of the nominal contact area. He pointed to the relationship between thermal and thermomechanical processes occurring under sliding contact and thermal cracking and other kinds of friction surface damage [69].

Thermal instability of the sliding contact can occur not only under TEI conditions. In [70], for example, thermal instability of frictional discs is understood as their capacity to retain their initial (usually plane) geometric shape under the effect of thermal stresses induced by frictional heating.

The authors of [71] studied the stability of rubbing bodies with account for the dependence of the coefficient of friction on the temperature and the relative velocity of the mated surfaces. Instability occurs at certain ratios between the sliding velocity and the derivatives of the coefficient of friction with respect to velocity and temperature. It is revealed in the following forms: monotonous increase in body velocity and temperature; temperature elevation followed by body braking; increasing variations in velocity and temperature; variations in velocity and temperature at constant amplitude; and a series of discrete slip–rest states of the body.

Tribochemical processes running in the friction zone have been shown [72, 73] to contribute to the formation of the thermal state of friction contact. The heat generated in exothermal tribochemical reactions can have a noticeable effect both on the temperature distribution on the surface of single contact spots and on the flash temperature at these spots. This kind of instability was experimentally confirmed in work [74]. During friction of sapphire against an aluminum alloy disc, local heat sources were registered, the temperatures of which exceeded significantly the alloy's melting temperature (660°C), reaching 1700°C. It is assumed that aluminum is oxidized in the friction contact zones, resulting in the contact of sapphire with aluminum

oxide having a 2050°C melting temperature. Oxidation is followed by severe heat generation (1670 kJ/mol) and thus takes the part of an additional energy source contributing to flash temperature elevation. Flash temperature exceeding that of the metal's melting temperature is also registered in study of the heating of leading shell collars [75].

**Thermal stresses in the vicinity of the friction zone and thermally-induced modes of damage.** Localization of frictional heat at the friction contact spots and high temperature gradients can cause high thermal stresses arising in the vicinity of these spots. The field of thermal stresses is superimposed on the mechanical stress field and the resulting stresses can exceed the ultimate strength of the material. Work [76] was one of the first to deal with thermal stresses in the vicinity of the friction zone. Here, an integral transform was used to solve the problem for a band heat source moving at a high velocity on the elastic half-space surface. Yang [77] considered similar problems using integral transforms and Mercier and coauthors applied numerical methods [78]. Very high compressive thermal stresses were shown to arise in the vicinity of the heat source, reaching a maximum on the half-space surface.

In tribology, the problem of thermal stresses in the vicinity of the contact zone is of importance in terms of their effect on the wear of friction members, particularly thermal cracking of the working surfaces of brakes and face seals. The results of the research undertaken in this direction are reported in work [11], the authors of which measured the sizes of macroscopic "hot spots" on the worn seal surface and applied finite element analysis to calculate the thermal and thermoelastic stresses in the spot vicinity. The thermal stresses proved to be very high, exceeding the mechanical stresses acting in the zone. Similar results were obtained in work [79] for a heat source having uniformly distributed intensity and moving on the half-space surface. The authors of [80] found that thermal stresses were compressive on the contact surface and that tensile stresses operated beneath the surface.

Also one should note the research by Evtushenko and coauthors [81–85] wherein the relation is studied of the temperature field in the contact zone and the thermal stresses due to nonstationary heating of the zone. The temperature field in the half-space was shown to be localized in the thin surface layer with increasing sliding velocity. Under nonstationary heating, the tensile thermal stresses exceed the compressive stresses in the case of isothermal heating. Further, in work [86], the influence of local frictional heating of the half-space surface on the stress intensity factor in the vicinity of internal and edge cracks and on the periodic system of the cracks was investigated. Work [87] studied thermal cracking of materials caused by frictional heating. High cross compressive stresses arise in the material surface layer. With time, they decrease and then their sign is reversed, i.e., they became tensile. Thermal cracking

occurs when these stresses exceed the material's ultimate strength. In work [88] the relation between thermal stresses in the friction contact zone and the wear of brittle and plastic materials was found, the construction of wear maps for several material combinations was outlined, and a criterion for thermomechanical wear was proposed. The criterion makes possible determination of the region of variations of the friction parameters within which thermomechanical wear occurs. Beyond the given region other wear modes can operate.

Thermal cracking should be mentioned once again in relation to its contribution to the accelerated wear of friction units, in particular, face seals and brakes [9–12]. Thermal cracks, as a rule, are normal to the sliding direction and approximately equidistant. The mechanism of their origination is as follows. Elevated temperatures on the contact surface cause very high compressive stresses favoring plastic flow of the material that, in turn, can cause tensile stresses. The tensile stresses can initiate thermal cracking through the damage of brittle inclusions in the material or through low-cycle fatigue as a result of the repeated action of thermal load over a single area of the contact surface.

In some instances several wear modes can operate at the same time. This situation can be exemplified by the experimental results on the abrasive wear of nonmetallic nonorganic materials widely used in the electronics and optics industries [89–92]. The required surface quality is achieved by using different abrasive treatment accompanied by severe frictional heating. The given problem is especially important in the case of brittle materials since during abrasion either plastic or brittle damage of the material occurs depending on the pulse heating intensity because the threshold of brittle damage is mostly governed by temperature and its gradient (see, e.g. [88]).

## CONCLUSIONS

The paper gives only a brief review of the main achievements in thermotribology. The study results are regularly published in *Tribology International*, *Tribology Letters*, *Wear*, *International Journal of Heat and Mass Transfer*, and *Transactions of ASME. Journal of Tribology* as well as in the international *Trenie i iznos* and *Trenie i smazka v mashinakh i mekhanizmaxh* journals. Collections of papers of specialized international symposia and conferences are released annually. Most information is nowadays accessible in electronic form. It should be mentioned that thermotribology remains a fast-developing field of the science of friction, wear, and lubrication; the field shows great promise in enhancing our understanding of the physical mechanisms of frictional heating. The foundation of the field is in the use of state-of-the-art thermography and promising numerical methods.

## DESIGNATIONS

$T_b$ —bulk temperature;  $T_s$ —average surface temperature;  $T_f$ —flash temperature;  $T_0$ —ambient temperature;  $T_{\max}$ —maximal temperature of the friction surface;  $\alpha_{tf}$ —heat partition factor;  $N$ —normal load;  $v$ —sliding velocity;  $\lambda$ —thermal conductivity;  $\sigma$ —coefficient of convective heat exchange;  $q$ —specific heat-release intensity (density of heat flow);  $r$ —contact-spot radius;  $q_0$ —heat-release intensity in the center of the heat source;  $a$ —thermal diffusivity;  $p_0$ —contact pressure in the source center;  $x$ —distance from the source center to the point of temperature determination;  $\psi$ —parameter depending on sliding velocity and heat-source size;  $l$ —half-width of a band heat source;  $Q$ —heat flow;  $Pe$ —Peclet number;  $\sigma_y$ —yield point;  $E$ —modulus of elasticity;  $R$ —curvature radius of a spherical asperity;  $R'$ —characteristic contact-spot size;  $\bar{A}$ —average contact spot area.  $e$ —ratio of the length of the large axis of the contact spot parallel to the sliding direction to that of the small axis (ellipticity);  $\Phi_1$ —function describing velocity–temperature relation;  $\Phi_2$ —function describing contact-spot ellipticity–temperature relation;  $v_1$  and  $v_2$ —relative velocities of the first and second bodies, respectively.

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