

EFFECT OF ELASTIC STRAINS ON THE ELECTRIC FIELD NEAR THE TIP OF A CRACKLIKE DEFECT IN A METALLIC BODY

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We derive an analytic relationship between the characteristics of electric voltages and mechanical stresses in the vicinity of the tip of a notch filled with medium. This relationship is expressed in terms of the introduced coefficient of electric-voltage intensity. The boundaries of the anodic zone in the vicinity of the tip of the notch filled with electrolyte are determined. A dependence of the size of the anodic zone on the physical characteristics of the metal and medium, the size of the defect, and the value of the mechanical load applied is found. This enables one to evaluate the corrosion conditions near the tips of cracklike stress concentrators.

In the present work, we obtain analytic relations for the calculation of the shift of the electrode potential caused by deformations and for the determination of the boundaries of the anodic zone in the vicinity of the tip of a notch filled with electrolyte. The corresponding formulas were deduced on the basis of known results [1–5]. The relationship between the electrical and mechanical fields is expressed in terms of the introduced coefficient of electric-voltage intensity.

Preliminary Information and Results

Consider an isotropic conducting nonferromagnetic body with density ρ_m and specific capacitance C_m that has an elliptic notch with semiaxes a (major) and b (minor). Tensile loads of intensity p oriented along the minor semiaxis of the ellipse act on the body. The distributions of electric potentials in the body φ_m and in the medium φ_c are determined from the expressions [5]

$$\begin{aligned} \varphi_m(r, \theta) = & \frac{\alpha}{\chi_m^2} \sqrt{\frac{R_0}{R_m}} e^{\chi_m(R_0 - R_m)} \left\{ \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} e^- - \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} e^+ + 2 \left[\left(\frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} + \frac{2 \sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m \chi_{mc}} R_0} \right) e^- \right. \right. \\ & \left. \left. - \left(\frac{2}{R_0 \sqrt{\chi_m \chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) e^+ \right] \frac{(r^4 + \tau^2 r_0^4) \cos 2\theta + 2 \tau r^2 r_0^2}{r^4 + 2 \tau r^2 r_0^2 \cos 2\theta + \tau^2 r_0^4} \right\} : \\ & \left[\left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) e^- - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) e^+ \right] \\ & - \frac{\alpha}{\chi_m^2} \frac{r^4 - \tau^2 r_0^2 - 2 \tau^2 r_0^4 + 2 r^2 r_0^2 \cos 2\theta}{r^4 - 2 \tau r^2 r_0^2 \cos 2\theta + \tau^2 r_0^4} \quad (r \geq r_0), \end{aligned} \quad (1)$$

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$$\varphi_c(r, \theta) = \frac{\alpha}{\chi_m^2} \sqrt{\frac{R_0}{R_m}} e^{\chi_c(R_1 - R_c)} \left[\sqrt{\frac{\chi_m}{\chi_c}} + 2 \left(\sqrt{\frac{\chi_m}{\chi_c}} - \frac{2}{R_0 \sqrt{\chi_m \chi_c}} \right) \frac{(r^4 + \tau^2 r_0^4) \cos 2\theta + 2\tau r^2 r_0^2}{r^4 + 2\tau r^2 r_0^2 \cos 2\theta + \tau^2 r_0^4} \right] : \\ \left[\left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) e^- - \left(\sqrt{\frac{\chi_m}{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) e^+ \right] \quad (r \leq r_1), \quad (2)$$

where $e^- = e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}$, $e^+ = e^{-\chi_{mc}\lambda} + e^{\chi_{mc}\lambda}$.

For the derivation of formulas (1) and (2), we used the conformal mapping

$$z = \omega(\zeta) = R \left(\frac{\zeta}{r_0} + \frac{\tau r_0}{\zeta} \right), \quad z = x + iy, \quad \zeta = r e^{i\theta}, \quad (3)$$

$$(R = 0.5(a+b), \quad 0 \leq \tau = (a-b)(a+b)^{-1} \leq 1)$$

of the exterior of a circle of radius $r = r_0$ in the plane of the complex variable $\zeta = \zeta(r, \theta)$ onto the exterior of the ellipse with semiaxes a and b in the plane of the complex variable $z = z(x, y)$ and took into account the boundary conditions.

In expressions (1) and (2), χ_m , χ_{mc} , and χ_c are the functional characteristics of the metal, electric double layer formed at the metal–medium interface, and medium, respectively [5], σ_m , σ_{mc} , and σ_c are the conductances of the metal, electric double layer, and medium, respectively, β is the electrostriction coefficient of volumetric expansion of the metal, ε_0 is the permittivity constant,

$$R_0 = R \sqrt{1 + 2\tau \cos 2\theta + \tau^2}, \quad R_1 = R(r_1 r_0)^{-1} \sqrt{r_1^4 + 2\tau r_1^2 r_0^2 \cos 2\theta + \tau^2 r_0^4},$$

$$R_m = R(r_1 r_0)^{-1} \sqrt{r^4 + 2\tau r^2 r_0^2 \cos 2\theta + \tau^2 r_0^4} \quad (r \geq r_0), \quad \alpha = 2\beta(1+\nu)p[3\varepsilon_0(1+4\nu)]^{-1},$$

ν is Poisson's ratio, $\lambda = R_0 - R_1$ is the width of the electric double layer ($r < r_0$), and R_c is calculated in the same way as R_m but for $r \leq r_1$.

Analytic Relationship between the Deformation and Electric Characteristics near the Tip of the Crack

Let us pass to the variable z in expression (1) according to formula (3). As a result, we obtain

$$R_m = |z|, \quad R_0 = a, \quad \frac{(r^4 + \tau^2 r_0^4) \cos 2\theta + 2\tau r^2 r_0^2}{r^2 + 2\tau r^2 r_0^2 \cos 2\theta + \tau^2 r_0^2} = \frac{1}{2} \left(\frac{z}{\bar{z}} + \frac{\bar{z}}{z} \right), \quad (4)$$

$$\frac{r^4 - \tau^2 r_0^4 - 2\tau r_0^4 + 2r^2 r_0^2 \cos 2\theta}{r^4 - 2\tau r^2 r_0^2 \cos 2\theta + \tau^2 r_0^4} = 1 + \frac{(1+\tau)r_0}{2\sqrt{\tau}} \left(\frac{1}{\zeta - \sqrt{\tau} r_0} - \frac{1}{\zeta + \sqrt{\tau} r_0} + \frac{1}{\bar{\zeta} - \sqrt{\tau} r_0} - \frac{1}{\bar{\zeta} + \sqrt{\tau} r_0} \right).$$

In the last equation, according to the corresponding branch of mapping (3), we have

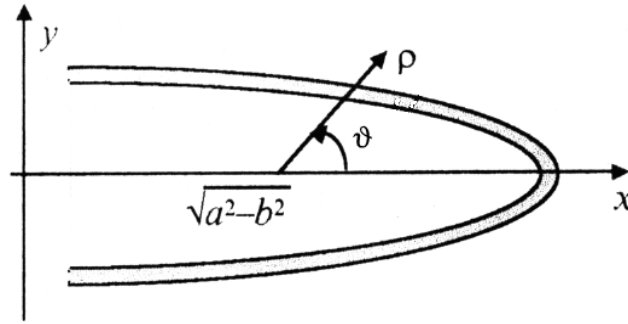


Fig. 1. Polar coordinate system (ρ, ϑ) centered at the focus of the ellipse.

$$\zeta = r_0(z + \sqrt{z^2 - 4\tau R^2})(2R)^{-1}.$$

We introduce a new coordinate system (ρ, ϑ) centered at the focus of the ellipse (Fig. 1). We denote

$$l \equiv 2\sqrt{\tau}R = \sqrt{a^2 - b^2} \quad \text{and assume that } z = l + z_1, \quad \text{where } z_1 = \rho e^{i\vartheta}. \quad (5)$$

In particular, it follows from expressions (5) that we obtain a point at the tip of the notch if $\tau \rightarrow 1$ (i.e., $b \rightarrow 0$) and $z_1 \rightarrow 0$.

Let us pass to this new coordinate system. We can write

$$\frac{r_0}{\zeta - \sqrt{\tau}r_0} = \frac{2R}{z - \sqrt{z^2 - l^2} - l} = \frac{1}{2\sqrt{\tau}} \frac{\sqrt{2lz_1 + z_1^2} - z_1}{z_1} = \frac{1}{\sqrt{\tau}} \left[\frac{\sqrt{l}}{\sqrt{2z_1}} + \frac{\sqrt{z_1}}{4\sqrt{2l}} + O(\sqrt{z_1}) - \frac{1}{2} \right]. \quad (6)$$

Let us obtain analogous expressions for

$$\frac{r_0}{\bar{\zeta} - \sqrt{\tau}r_0}, \quad \frac{r_0}{\zeta + \sqrt{\tau}r_0}, \quad \text{and} \quad \frac{r_0}{\bar{\zeta} + \sqrt{\tau}r_0}.$$

Taking into account expressions (4)–(6) and performing corresponding transformations, we get

$$\begin{aligned} \varphi_m(\rho, \vartheta) = & \frac{\sqrt{a} \exp[\chi_m(a - \sqrt{4\tau R^2 + 4\sqrt{\tau}R\rho \cos \vartheta + \rho^2})]}{\sqrt[4]{4\tau R^2 + 4\sqrt{\tau}R\rho \cos \vartheta + \rho^2}} \left(A_1 + A_2 \frac{4\tau R^2 + 4\sqrt{\tau}R\rho \cos \vartheta + \rho^2 \cos 2\vartheta}{4\tau R^2 + 4\sqrt{\tau}R\rho \cos \vartheta + \rho^2} \right) \\ & + \frac{1}{\sqrt{2\rho}} \frac{\alpha \sqrt{2R}(3\tau + 1)}{\chi_m^2 \sqrt[4]{\tau^3}} \cos \frac{\vartheta}{2} + \frac{\alpha(\tau + 1)}{\chi_m^2 \tau} + O(\sqrt{\rho}), \end{aligned} \quad (7)$$

where

$$A_1 = \frac{\alpha}{\chi_m^2} \left[\frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} e^- - \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} e^+ \right] : d, \quad (8)$$

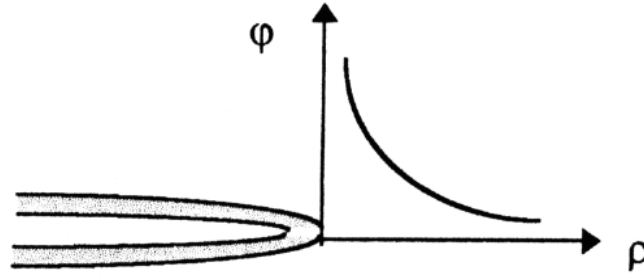


Fig. 2. Distribution of the electric potential in the vicinity of the end of the notch.

$$A_2 = \frac{4\alpha}{\chi_m^2} \left[\left(\frac{\sigma_{mc}\chi_{mc}}{\sigma_m\sqrt{\chi_m\chi_c}} + \frac{2\sigma_c\sqrt{\chi_c}}{\sigma_m\sqrt{\chi_m\chi_{mc}}} \right) e^- - \left(\frac{2}{a\sqrt{\chi_m\chi_c}} + \frac{\sigma_c\sqrt{\chi_c}}{\sigma_m\sqrt{\chi_m}} \right) e^+ \right]; d, \quad (9)$$

$$d = \left(\frac{\sigma_c\sqrt{\chi_m\chi_c}}{\sigma_{mc}\chi_{mc}} + \frac{\sigma_{mc}\chi_{mc}}{\sigma_m\sqrt{\chi_m\chi_c}} \right) e^- - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c\sqrt{\chi_c}}{\sigma_m\sqrt{\chi_m}} \right) e^+. \quad (10)$$

By using expression (7), for the vicinity of the tip of the notch (i.e., as $b \rightarrow 0$) we obtain

$$\begin{aligned} \varphi_m(\rho, \vartheta) = & \frac{\sqrt{a} \exp[\chi_m(a - \sqrt{a^2 + 2a\rho \cos \vartheta + \rho^2})]}{\sqrt[4]{a^2 + 2a\rho \cos \vartheta + \rho^2}} \left(A_1 + A_2 \frac{a^2 + 2a\rho \cos \vartheta + \rho^2 \cos 2\vartheta}{a^2 + 2a\rho \cos \vartheta + \rho^2} \right) \\ & - \frac{4\alpha\sqrt{2a}}{\chi_m^2\sqrt{2\rho}} \cos \frac{\vartheta}{2} + \frac{2\alpha}{\chi_m^2} + O(\sqrt{\rho}). \end{aligned} \quad (11)$$

Denote

$$L(\rho, \vartheta) = \frac{\sqrt{a} \exp[\chi_m(a - \sqrt{a^2 + 2a\rho \cos \vartheta + \rho^2})]}{\sqrt[4]{a^2 + 2a\rho \cos \vartheta + \rho^2}} \left(A_1 + A_2 \frac{a^2 + 2a\rho \cos \vartheta + \rho^2 \cos 2\vartheta}{a^2 + 2a\rho \cos \vartheta + \rho^2} \right). \quad (12)$$

Then

$$\varphi_m(\rho, \vartheta) = L(\rho, \vartheta) - \frac{4\alpha\sqrt{a}}{\chi_m^2\sqrt{2\rho}} \cos \frac{\vartheta}{2} + \frac{2\alpha}{\chi_m^2} + O(\sqrt{\rho}). \quad (13)$$

As $\rho \rightarrow 0$, the quantity $L(\rho, \vartheta) + 2\alpha\chi_m^{-2}$ tends to

$$A_1 + A_2 + 2\alpha\chi_m^{-2} = \frac{\alpha[7a\sigma_{mc}\chi_{mc}(\sigma_{mc}\chi_{mc}e^- - \sigma_c\chi_ce^+) + 2\sigma_m(4 + \alpha\chi_m)(\sigma_c\chi_ce^- - \sigma_{mc}\chi_{mc}e^+)]}{\chi_m^2\alpha[(\sigma_m\chi_m\sigma_c\chi_c + \sigma_{mc}^2\chi_{mc}^2)e^- - \sigma_{mc}\chi_{mc}(\sigma_m\chi_m + \sigma_c\chi_c)e^+]} \quad (14)$$

and is bounded (under the constant load p); furthermore, as ρ increases, we have $L(\vartheta) \xrightarrow{\vartheta \rightarrow 0} 0$. The third

and fourth terms in (13) are also bounded. Therefore, the behavior of the electric potential in the vicinity of the tip of the notch is determined by the behavior of the second term.

Since $p\sqrt{a} = K_I$ and $\chi_m^2 = \rho_m C_m \varepsilon_0^{-1}$, we can write

$$8(1+\nu)\beta[3(1+4\nu)\rho_m C_m]^{-1} K_I = K_e. \quad (15)$$

Performing the same transformations of expression (2) as in the case of expression (1), we can determine the electric potential φ_c in the medium at the tip of the cracklike defect of the material:

$$\varphi_c(0, 0) = \frac{2\alpha\sigma_{mc}\chi_{mc}(3a\sigma_m\chi_m - \sigma_m)}{\chi_m^2\alpha[(\sigma_m\chi_m\sigma_c\chi_c + \sigma_{mc}^2\chi_{mc}^2)e^- - \sigma_{mc}\chi_{mc}(\sigma_m\chi_m + \sigma_c\chi_c)e^+]}. \quad (16)$$

One can see that the electric potential in the medium near the tip of the crack is constant and depends on the physical characteristics of the metal and medium.

Let us find the value of the shift of the electrode potential (caused by mechanical loading) in the vicinity of the tip of the notch filled with the corrosive medium from the standard value $\Delta\varphi_m$. To do this, we subtract expression (16) from expression (13) and take into account estimate (14). As a result, we get

$$\Delta\varphi - \Delta\varphi_0 = \frac{2\beta(1+\nu)p}{3(1+4\nu)\rho_m C_m} \left(D - \frac{K_e}{\sqrt{2\rho}} \cos \frac{\vartheta}{2} \right), \quad (17)$$

where

$$D = \frac{\alpha}{\chi_m^2} \left\{ \frac{\sigma_{mc}\chi_{mc}(7\sigma_{mc}\chi_{mc}e^- - 7\sigma_c\chi_c - 6\sigma_m\chi_m e^+)}{[(\sigma_m\chi_m\sigma_c\chi_c + \sigma_{mc}^2\chi_{mc}^2)e^- - \sigma_{mc}\chi_{mc}(\sigma_m\chi_m + \sigma_c\chi_c)e^+]} + \frac{2\sigma_m(4 + a\chi_m)(\sigma_c\chi_c e^- - \sigma_{mc}\chi_{mc}e^+ + 4\sigma_{mc}\chi_{mc})}{a[(\sigma_m\chi_m\sigma_c\chi_c + \sigma_{mc}^2\chi_{mc}^2)e^- - \sigma_{mc}\chi_{mc}(\sigma_m\chi_m + \sigma_c\chi_c)e^+]} \right\}.$$

It follows from the last relations that $\Delta\varphi - \Delta\varphi_0 = 0$ for

$$\vartheta = \pm 2\arccos\sqrt{2\rho}D. \quad (18)$$

Moreover, $\Delta\varphi - \Delta\varphi_0 > 0$ if

$$-2\arccos\sqrt{2\rho}D < \vartheta < 2\arccos\sqrt{2\rho}D, \quad (19)$$

i.e., in the vicinity specified by inequalities (19), the action of tensile loading results in a shift of the electrode potential into the anodic zone. Let us determine the boundaries of the anodic and cathodic zones of the surface of the defect from the conditions $\Delta\varphi = 0$. We obtain

$$\vartheta^* = \pm 2\arccos\sqrt{2\rho}(D - \Delta\varphi_0 K_e^{-1}). \quad (20)$$

The last expression and relations (19) imply that the anodic zone is determined by the boundaries

$$-2\arccos\sqrt{2\rho}(D - \Delta\phi_0 K_e^{-1}) < \vartheta^* < 2\arccos\sqrt{2\rho}(D - \Delta\phi_0 K_e^{-1}). \quad (21)$$

The other part of the surface of the defect is the cathodic zone. The sizes of the anodic and cathodic zones, as follows from expression (21), depend on the physical characteristics of the metal and medium, the size of the notch, radius ρ , and mechanical loading. In particular, as ρ decreases, the size of the anodic zone also decreases. The presence of the coefficient K_e in expressions (20) and (21) indicates that, in the vicinity of the tip of the defect, the size of the anodic zone depends on the level of the load applied.

If we assume that $\lambda \rightarrow 0$ as $b \rightarrow 0$, then expressions (17)–(21) are simplified and, in particular, we have

$$\begin{aligned} \Delta\phi - \Delta\phi_0 &= \frac{2\beta(1+\nu)p}{3(1+4\nu)\rho_m C_m} \left(\frac{4\sigma_m + a(5\sigma_m\chi_m + 7\sigma_c\chi_c)}{a(\sigma_m\chi_m + \sigma_c\chi_c)} - 4 \frac{\sqrt{a}}{\sqrt{2\rho}} \cos \frac{\vartheta}{2} \right), \\ -2\arccos\sqrt{2\rho} \left[\frac{4\sigma_m + a(5\sigma_m\chi_m + 7\sigma_c\chi_c)}{4a\sqrt{a}(\sigma_m\chi_m + \sigma_c\chi_c)} - \frac{\Delta\phi_0}{K_e} \right] \\ &< \vartheta^* < 2\arccos\sqrt{2\rho} \left[\frac{4\sigma_m + a(5\sigma_m\chi_m + 7\sigma_c\chi_c)}{4a\sqrt{a}(\sigma_m\chi_m + \sigma_c\chi_c)} - \frac{\Delta\phi_0}{K_e} \right]. \end{aligned}$$

Relation (15) points to the relationship between the characteristics of the elastic and electric fields near the tip of the crack in the conducting body. The coefficient defined by (15) reflects the singularity of the mechanical stressed state in the vicinity of the tip of the crack and determines the intensity of the distribution of voltages of the electric field; it depends on the geometry of the defect, physicomaterial characteristics of the material, and mechanical loads applied and does not depend on the coordinates of a point located near the tip of the crack-like notch. The coefficient K_e is measured in the following units:

$$[K_e] = [\beta][K_I][[\beta][K_I]]^{-1} = V\sqrt{m}.$$

Therefore, similarly to the stress intensity factor, the coefficient defined by expression (15) can be interpreted as “the coefficient of electric-voltage intensity.” It enables one to determine how the electric potential varies under mechanical loading at the most stressed sites, namely, in the vicinity of the tips of cracklike defects, and to estimate the probability and rate of corrosion fracture of structural materials.

Expression (17) indicates that the shift of the electrode potential in a given medium for a given length of a cracklike defect of a metal is proportional to the value of voltage applied within the limits of elastic strains, which is corroborated by results of experimental investigations [6, 7].

In fact, in the process of transport of an elementary charge in the course of contact of a stressed metal with medium, chemical, electric, and mechanical works are done simultaneously. This is why only the total energy effect (but not its components) corresponding to changes in the electrochemical potential can be determined experimentally. Dependences (17)–(21) enable one to analytically calculate the value of deformation or stress on the basis of the shift of the electrode potential and determine the boundaries of the anodic zone at the tip of the cracklike defect. Thus, this analytic result is of great practical importance.

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