

METHOD OF DETERMINING BENDING STRESS INTENSITY COEFFICIENTS FOR CRACKED FERROCONCRETE COMPONENTS

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Cracks in concrete elements in building structures may cause catastrophic failure under working loads. Ferroconcrete allows not only the formation and presence of cracks in the working state but also their growth on opening to significant extents. Then it would be promising to use methods from the mechanics of failure to forecast the working strength for ferroconcrete, but there are few papers on this because of various difficulties in developing the idea.

Here we propose a method of designing ferroconcrete components containing cracks, which is based on a model composed of blocks that are separated by cracks and linked together in a zone containing stretched reinforcement [1].

A block model for ferroconcrete has also been formulated [2, 3], which meets [1] the basic concepts in the mechanics of failure and the conditions for the attachment of the reinforcement to the concrete. There is [4, 5] a model for the bending of a ferroconcrete beam containing a crack, where the beam is considered as a strip composed of a system of wedges.

Japanese researchers [6] have extensively developed a model, which uses computer methods to describe the behavior of a crack in ferroconcrete under normal and shearing stresses and which deals with the frictional contact at the edges. The main shortcomings of these methods are the lack of consideration of the work of crack growth in the tensile zone in the concrete and the forces of attachment between the reinforcement and the concrete, in addition to the detachment of the concrete matrix from the reinforcement in the region of the crack in the tensile zone in the concrete.

Formulation. We consider the safety margin in a ferroconcrete element (plate, panel, or beam) containing cracks on bending by calculating a parameter in the mechanics of failure, namely the stress intensity coefficient K_I , from which one can derive the limiting load or the size of the permissible cracks.

Here we present a method of calculating the stress intensity coefficients SIC in bending for thick concrete plates reinforced in the tensile zone by long steel rods and damaged by a flat surface crack normal to the rods. We use a superposition method that represents the SIC for the ferroconcrete plate as the sum of SIC for the unreinforced plate $K_I^{(M)}$ and the SIC produced by the reinforcement $K_I^{(a)}$, which prevents crack growth:

$$K_I = K_I^{(M)} - K_I^{(a)}. \quad (1)$$

We consider a ferroconcrete plate (Fig. 1a) of thickness H bent by a moment M_0 , which the matrix in the plate approximated as a homogeneous linearly elastic medium reinforced by a planar periodic system of circular steel rods with diameter D and period h_1 , and damaged by a surface flat crack of length a . The distance of the rods from the crack vertex is h . The initial condition is that h is so small that $h/a \rightarrow 0$ and $h/H \rightarrow 0$. In that case, we have the moment $M = M_0/h_1$ (Fig. 1b), and we use contour integration as described in [7] to get

$$K_I^{(M)} = \frac{6M}{H\sqrt{H}} \frac{1.9879\sqrt{\lambda}}{(1-\lambda)^{3/2}\sqrt{1+7.9927\lambda}}, \quad \lambda = \frac{a}{H}. \quad (2)$$

The restraining effect of the reinforcement on the crack growth is represented by compressive forces P_0 applied to the crack edges. The SIC $K_I^{(a)}$ can be established by solving the planar problem with a crack in a strip subject to a force $P = P_0/h_1$ (Fig. 1b) applied to the edges of the crack.

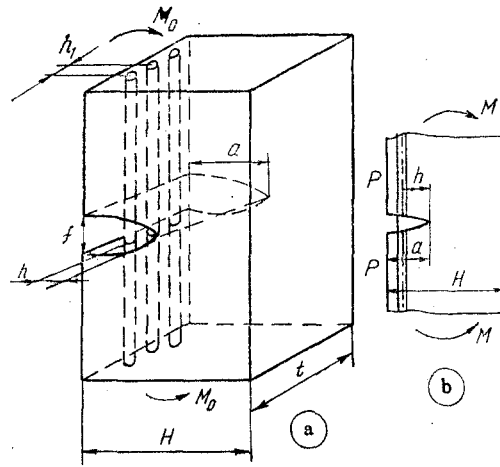


Fig. 1. Calculation scheme for a ferroconcrete plate containing a crack: a) reinforcing rods; b) reinforcing strip.

In the present case, the rod reinforcement is represented by a strip, and the planar SiC $K_{Ipl}^{(a)}$ is derived [8] from

$$K_{Ipl}^{(a)} = \frac{P_0}{h_1} \sqrt{\frac{2}{\pi h}}. \quad (3)$$

That solution is an approximation, and subsequently we consider its accuracy in relation to the solution for the three-dimensional treatment.

Three-Dimensional Solution. Consider an infinite body having a circular crack of radius a , at the edges of which forces P_0 parallel to the z axis are applied at the point (ξ, η) (Fig. 2a and b) in a right-angled cartesian coordinate system Oxyz with origin at the crack vertex. The normal stresses on the continuation of the crack are derived by means of integral equations [7]:

$$\sigma_z = \frac{P_0}{\pi^2} \frac{\sqrt{a^2 - (\xi + a)^2 - \eta^2}}{\sqrt{(x + a)^2 + y^2 - a^2} [(x - \xi)^2 + (y - \eta)^2]}. \quad (4)$$

We allow a to tend to infinity to get the stresses for an infinite crack (Fig. 2c):

$$\sigma_{za \rightarrow \infty} = \frac{P_0}{\pi^2} \frac{\sqrt{-\xi}}{[(x - \xi)^2 + (y - \eta)^2] \sqrt{x}}. \quad (5)$$

We transfer to an infinite body containing a semiinfinite crack at the edges of which there is a system of localized forces P_0 with period h_1 at a distance h from the crack tip. In that case, we use the (4) solution to determine the loads on the continuation of the crack $\sigma_z^{(a)}$ as affected by the pressure of the reinforcement on the edges of the crack. We make the substitutions $\xi = -h$, $\eta = ih_1$, ($i = 0, \pm 1, \pm 2, \dots$) and sum to infinity with respect to i to get

$$\sigma_z^{(a)} = \frac{P_0 \sqrt{h}}{\pi h_1 (x + h) \sqrt{x}} \left[\frac{\text{sh} \left(\frac{2\pi(x + h)}{h_1} \right)}{\text{ch} \left(\frac{2\pi(x + h)}{h_1} \right) - \cos \frac{2\pi y}{h_1}} \right]. \quad (6)$$

The relative spatial SIC will be (Fig. 2d)

$$K_{Ipl}^{(a)} = \lim_{x \rightarrow 0} \sqrt{2\pi x \sigma_z^{(a)}}. \quad (7)$$

We substitute (6) into (7) to get for $y = ik_1 \pm h_2$, $0 \leq h_2 \leq h_1$ that

$$K_{Ipl}^{(a)} = \sqrt{\frac{2}{\pi}} \frac{P_0}{h_1} \frac{\text{sh } 2\pi\gamma}{\text{ch } 2\pi\gamma - \cos \frac{2\pi y}{h_1}}, \quad \gamma = \frac{h}{h_1}. \quad (8)$$

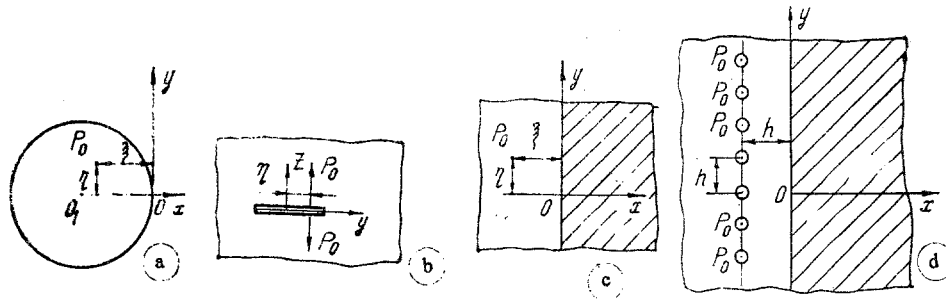


Fig. 2. Calculation scheme for simulating the restraining effect of reinforcement rods on crack growth: a) circular planar crack in infinite body in the x, y plane; b) the same in the z, y plane; c) plane of semiinfinite crack; d) section of plate in plane of crack.

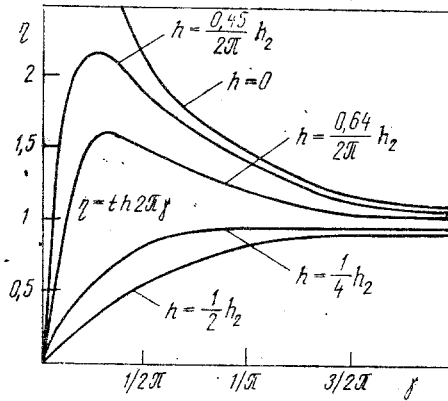


Fig. 3. The γ dependence of the ratio η of the spatial and planar SIC.

We take the ratio of $K_{Isp}^{(a)}$ and $K_{Ipl}^{(a)}$:

$$\eta(\gamma) = \frac{K_{Isp}^{(a)}}{K_{Ipl}^{(a)}} = \frac{\text{sh } 2\pi\gamma}{\text{ch } 2\pi\gamma - \cos \frac{2\pi y}{h_1}}. \quad (9)$$

The calculations show that the error in calculating the SIC from the planar treatment relative to that from the spatial one is not more than 2% for $\gamma > 5/2\pi$ (Fig. 3).

To calculate $K_{Ipl}^{(a)}$ for any plate thickness and also the crack length, we use boundary interpolation [7] and consider two limiting cases and relative SIC [9]:

1) the plate thickness is much greater than the crack length ($a/H \rightarrow 0$):

$$K_{Ipl0}^{(a)} = \frac{P_0}{h_1} \sqrt{\frac{2c}{\pi a \left(1 - \frac{a-h}{a}\right)^c}}, \quad (10)$$

in which $c = 2\pi^2/(\pi^2 - 4)$;

2) the plate thickness and crack length are comparable ($a/H \rightarrow 1$):

$$K_{Ipl1}^{(a)} = \frac{P_0}{h_1} \sqrt{\frac{2c(H-a-h)^c}{(H-a)[(H-a+h)^c - (H-a)^c]}}. \quad (11)$$

Then the SIC for any a and H is

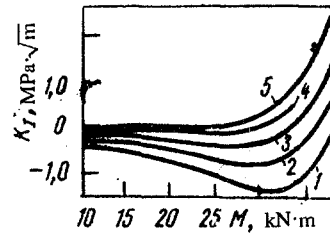


Fig. 4. Dependence of K_I on bending load M for a ferroconcrete plate containing a crack.

$$K_{Ipl}^{(a)} = \frac{P_0}{h_1} \sqrt{\frac{2c}{\pi} \left[\frac{6a}{H} + 1 \right]} \sqrt{\frac{(b+h)^c}{b[(b+h)^c - b^c] + a \left[\frac{6a}{H} + 1 \right]^{-2} (b-h)^c (1 - \delta_0^c)}}, \quad (12)$$

in which $\delta_0 = (a-h)/a$, $b = H - a$.

Forces in the Reinforcement. One can calculate the force P_0 in the reinforcement, which hinders crack opening, from the strain ϵ in a reinforcing rod:

$$P_0 = \frac{\pi D^2}{4} \epsilon E_s. \quad (13)$$

Here E_s is the elastic modulus of the reinforcement, $\epsilon = \delta_a/h_0$, δ_a the crack opening at points where the reinforcement passes through, and h_0 the distance between the points of application of the forces P_0 to the rods. The displacement of the crack edges at the level of the reinforcement is calculated as the sum of two contributions δ_1 and δ_2 . One derives δ_1 from the bending of an unreinforced plate with a crack, while δ_2 is calculated as the closure of the crack edges caused by the forces P_0 in the reinforcing rods.

Then the opening at the level of the reinforcement is

$$\delta_a = \delta_1 + \delta_2. \quad (14)$$

We calculate the crack opening in a concrete plate on bending by moment M from the solution for the bending of a strip containing a crack [10] and using the interpolation method [11]:

$$\delta_1 = -\sigma_t h \{ \Gamma(a_p, h, a) - \Gamma(a_p, h, -a) \} / \pi E_b, \quad (15)$$

$$\Gamma(a_p, h, a) = \frac{a_p^3 - (a-h)a - \sqrt{(a_p^3 - (a-h)^2)(a_p^3 - a^2)}}{a_p^3 - (a-h)a + \sqrt{(a_p^3 - (a-h)^2)(a_p^3 - a^2)}}. \quad (16)$$

$$d = a \left(\cos^{-1} \left(\frac{\pi}{2} \right) \left(\frac{P_e}{\sigma_p} \right) - 1 \right), \quad a_p = a + d, \quad (17)$$

$$P_e = 4.2 \lambda_* M (H)^{-3/2} \sqrt{\left[\left(1 - \frac{a}{H} \right)^{-3} - \left(1 - \frac{a}{H} \right)^3 \right] / \pi a}, \quad (18)$$

in which σ_t is the temporary strength of the concrete in tension, while λ_* is taken from [10], and E_b is the elastic modulus of concrete.

To calculate δ_2 , we use the solution for the crack opening in a disk [12] when the forces P are applied symmetrically and normally to the edges of the crack, while the distance between the points of application is h_0 . We allow the radius of the disk to tend to infinity to get [12]

$$\delta_2 = \frac{4Ph_0}{\pi E_1 t^2 (x+1)} \left(\frac{\sqrt{a^2 - x_1^2}}{(a^2 + h_0^2)(x_1^2 + h_0^2)} + \right. \\ \left. + \left(\frac{2h_0^2}{(a^2 + h_0^2)^{3/2}} + \frac{x-1}{\sqrt{a^2 + h_0^2}} \right) \ln \left(\frac{\sqrt{a^2 + h_0^2} + \sqrt{a^2 - x_1^2}}{\sqrt{a^2 + h_0^2} - \sqrt{a^2 - x_1^2}} \right) \right), \quad (19)$$

$$E_1 = \frac{4E_b}{(1+\mu)(x+1)}, \quad x_1 = a - h, \quad x = 3 - 4\mu, \quad (20)$$

in which t is plate width and μ is Poisson's ratio for concrete.

We write the forces P_0 in the reinforcing rods as the limiting forces of attachment between the reinforcement and the concrete [13]:

$$P_0 = \pi D l_{fr} \tau_{fr}, \quad (21)$$

in which τ_{fr} is the average attachment stress [13] and l_{fr} is the necessary length of the steel rod in the concrete to provide attachment to the concrete with force τ_{fr} .

On the other hand, the force in the reinforcement can be calculated from (13). We equate the right-hand sides of (13) and (21) and use (19) to get

$$\pi D l_{fr} \tau_{fr} = \frac{\pi}{4} E_A D^2 \frac{\delta_1}{h_0} \left(1 - \frac{\frac{\pi}{4} E_A D^2 \delta_p}{h_0 + \frac{\pi}{4} E_A D^2 \delta_p} \right), \quad (22)$$

in which $\delta_p = \delta_2/P$.

The expression in parentheses is taken as close to one, which implies the additional condition $D \ll t$. Then we use (22) to get the distance between the points of application of the forces P_0 :

$$h_0 = \frac{\delta_1 E_A D}{4 l_{fr} \tau_{fr}}. \quad (23)$$

Solution Analysis. We derive K_I for a ferroconcrete plate containing a crack in bending by using (2) in which $K_I^{(M)}$ is derived from (1) and $K_{Ipl}^{(a)}$ from (12):

$$K_I = \frac{6M}{H\sqrt{H}} \frac{1,9879\sqrt{\lambda}}{(1-\lambda)^{3/2}\sqrt{1+7,9927\lambda}} - \frac{\text{sh } 2\pi\gamma \left[\frac{6a}{H} + 1 \right]}{\text{ch } 2\pi\gamma - \cos \frac{2\pi y}{h_1}} \times \\ \times \frac{\pi^{3/2} D \delta_a l_{fr} \tau_{fr}}{\delta_1 h_1} \sqrt{\frac{2c(b+h)^c}{b[(b+h)^c - b^c] + a \left[\frac{6a}{H} + 1 \right]^{-2} (b-h)^c (1-\delta_0^2)}}. \quad (24)$$

We derive δ_1 from (15) and δ_a from (14) to get a quantity for actual rod assemblies defining the crack opening at the level of the reinforcement. As an example, we calculate K_I as a function of the bending moment M for a ferroconcrete plate made of ordinary hardening class B20 concrete and reinforced with A-III class rods, which has the following physical and geometrical parameters: $a = 0.1$ m, $H = 0.32$ m; $E_b = 2.7 \cdot 10^{10}$ Pa; $\mu = 0.15$; $t = 1.8$ m; $\sigma_t = 1.4 \cdot 10^6$ Pa; $E_s = 2 \cdot 10^{11}$ Pa; $h = 30$ mm, $l_{fr} = 0.36$ m; $D = 18$ mm, $\tau_{fr} = 5 \cdot 10^6$ Pa; $h_1 = 0.8-0.12$ m.

Figure 4 shows K_I for various h_1 as a function of the loading, which indicates that K_I attains the experimental critical K_{Ic} level in this loading range only with $h_1 = 12$ mm.

To use the interpolation method [11] to establish the crack opening at the surface of the plate f (Fig. 1a), which applies in the case of opening δ_1 , one can derive an expression for the load M in terms of the opening f , which can be used in analyzing the working state of building construction elements:

$$M = \sqrt{\frac{a}{\pi}} \frac{\sigma_t H^{3/2}}{2.1 \lambda_* \sqrt{\left(1 - \frac{a}{H}\right)^3 - \left(1 - \frac{a}{H}\right)^3}} \arccos a / \sqrt{a_0}, \quad (25)$$

in which

$$a_0 = \frac{-8 x_1^2 a^2}{2((1-R)^2(a^2 + x_1^2)^2 - 4(R+1)^2 x_1^2 a^2)} \{(R+1)(a^2 + x_1^2) - \sqrt{(a^2 + x_1^2)(R+1)^2 + R^2[(1-R^2)(a^2 + x_1^2)^2 - 4(R+1)^2 x_1^2 a^2]}\}, \quad (26)$$

$$R = \exp\left(-\frac{(f + \delta_s) \pi E_s}{2 \sigma_t h}\right).$$

This method can be used to estimate the strength margin of a ferroconcrete plate containing a crack in bending by the use of the calculated K_I , which are equated to the critical stress intensity coefficient K_{Ic} , and can also be used in design to calculate the possible effects from cracks.

Summary. A method is proposed for calculating the stress intensity coefficient in a ferroconcrete element containing a crack in bending with reinforcement by rods. A three-dimensional solution has been obtained. The attachment of the reinforcement to the concrete is incorporated.

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