

EFFECTS OF CRACKS AT AND NEAR EDGES ON THE DEFORMATION OF A MATERIAL BEARING A THIN COATING

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In the simulation of failure in a material bearing a thin coating, it is usual in a two-dimensional formulation to represent the coating as a thin elastic layer that is attached, for example, at the edge of an elastic half-plane (the substrate). Elastic attachment to a homogeneous elastic half-plane has been considered in detail [1], and studies have been made also on the effects of absolutely rigid attachment on the behavior of cracks at and near edges in an elastic half-plane [2]. Some additional results have been given in other papers [3-6]. Here we consider the effects of a thin elastic coating on the stress patterns around cracks at and near edges in the presence of mechanical stresses.

We consider a half-plane characterized by the elastic constants G , E , and μ (shear modulus, Young's modulus, and Poisson's ratio correspondingly), which contains a crack of length $2l$ perpendicular to the boundary (Fig. 1). Along the line $y = 0$, the half-plane is reinforced by a layer of elastically coupled material having the small height h and the elastic constants G_s , E_s , and μ_s . That layer can be considered as a thin elastic plate having bending rigidity [1]. The horizontal surface of the coating is free from stress.

On these assumptions,

$$\sigma_{yy}^{(s)}(x, 0) = 0; \quad \sigma_{xx}^{(s)}(x, 0) = 0. \quad (1)$$

The substrate is loaded at infinity by uniformly distributed forces σ_x^∞ . The cracks in the substrate are free from stress. One needs to evaluate the stress intensity concentration around the vertices of cracks at and near the edges, which interact with the thin coating.

The solution can be represented as the superposition of those for two cases: a half-plane with coating without cracks and a half-plane with coating and a crack loaded by internal pressure (absent in the first case). The solution to the first is known [1]. We consider the solution for the elastic equilibrium of an isotropic half-plane reinforced by a coating and containing a crack near the edge (Fig. 1). The external stresses in the half-plane are only tensile stresses with intensity $p(y)$ applied to the edges of the crack. One needs to derive the stress intensity coefficients around the two ends of the crack in relation to their distance from the interface between the coating and the half-plane and also in relation to the physicomaterial properties of the materials.

The boundary conditions are

$$\varepsilon_x = \varepsilon_x^{(s)} \text{ at } y = 0. \quad (2)$$

At $x = 0$

$$\tau_{xy}^{(s)}(0, y) = 0, \quad -h < y < 0, \quad \tau_{xy}(0, y) = 0, \quad 0 < y < \infty; \quad (3)$$

$$\sigma_{yy}(0, y) = p(y), \quad a < y < b, \quad u^{(s)}(0, y) = 0, \quad -h < y < 0. \quad (4)$$

Here $\tau_{xy}^{(s)}$, τ_{xy} , σ_{yy} are the stress-tensor components, $u^{(s)}$ is a component of the displacement vector, and ε_x is the relative deformation in the direction of the Ox axis.

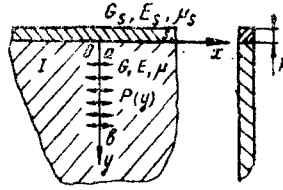


Fig. 1

The equilibrium equation for the reinforced element at $y = 0$ takes the form [1]

$$\eta_* \frac{d\sigma_{xx}}{dx} + h\tau_{xy}^{(s)} = 0, \quad (5)$$

in which $\eta_* = E_s F_s / E$ (F_s is the cross-sectional area of the stringer).

The following is the solution in terms of displacements in the planar theory of elasticity [3]:

$$\begin{aligned} u(x, y) &= \frac{2}{\pi} \int_0^\infty \{A_1 + A_2 [y + (x+1)/2\alpha]\} e^{-\alpha y} \sin(\alpha x) d\alpha + \\ &+ \frac{2}{\pi} \int_0^\infty B(u) u^{-1} [(x+1)/2 + ux] e^{-ux} \cos(yu) du; \\ v(x, y) &= \frac{2}{\pi} \int_0^\infty \{A_1 + A_2 [y + (x+1)/2\alpha]\} e^{-\alpha y} \cos(\alpha x) d\alpha - \\ &- \frac{2}{\pi} \int_0^\infty B(u) u^{-1} [(x+1)/2 - ux] e^{-ux} \sin(yu) du, \end{aligned} \quad (6)$$

in which $\kappa = (3 - \mu)/(1 + \mu)$ for the conditions of the general strain state, $\kappa = 3 - 4\mu$ for planar strain, and μ is Poisson's ratio. Hooke's law gives the stress tensor components. Obedience to the boundary conditions (1)-(3) enables one to express the unknowns A_j ($j = 1, 2$) in terms of $B(u)$. Then from (4) we get the dual integral equation for the unknown function $B(u)$, which is related to $c(t)$ by

$$B(u) = \frac{2}{\kappa + 1} \int_a^b c(t) \sin(ut) dt \quad (7)$$

which reduces to the singular integral equation

$$\frac{1}{\pi} \int_a^b \frac{c(t) dt}{t-y} - \frac{1}{\pi} \int_a^b \left[\frac{1}{t+y} + K(y, t) \right] c(t) dt = 0,25 (1 + \kappa) p(y) / (\pi G) \quad (8)$$

$$y \in [a, b].$$

Then the kernel $K(y, t)$ is given by

$$K(y, t) = 2 \int_0^\infty \alpha \left[t + \frac{(y\alpha - 2)(ht + 2\eta)}{2\eta\alpha + h} \right] e^{-\alpha(y+t)} d\alpha, \quad (9)$$

in which $\eta = G_s/G$, with G_s and G the shear moduli for the coating and matrix correspondingly. Equation (9) is a singular integral equation with kernel of Cauchy type. The solution can be obtained by numerical methods [7]. We solve (8) by means of mechanical quadratures, which leads to replacement by the discrete analog of algebraic equations, in which the number of interpolation points is chosen such that any further increase in the number does not influence the final result. The condition for single-valued displacements is met for defects near the edges [2, 7], while the case of an edge crack is represented by the condition for the stress intensity coefficient at the vertex being zero, as the crack emerges at the edge of the half-plane. The

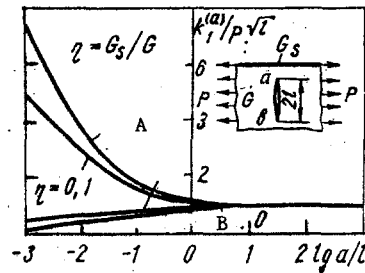


Fig. 2

Fig. 2. A) Crack-free edge; B) crack-rigid edge.

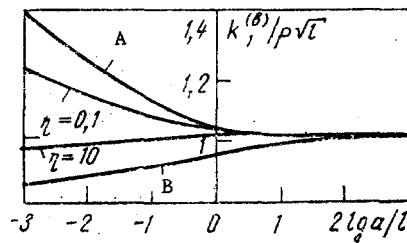


Fig. 3

Fig. 3. A) Crack-free edge; B) crack-rigid edge.

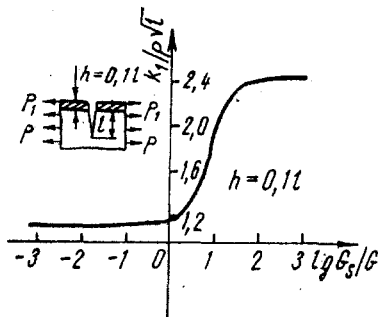


Fig. 4

Fig. 4

Fig. 5. A) Crack-free edge; B) crack-rigid edge.

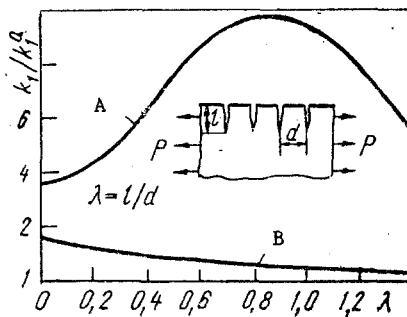


Fig. 5

numerical solutions to (8) indicated how the stress intensity coefficients vary around the two vertices of a crack near the edge for particular materials ($\eta = G_s/G = 0.1-10$, with the crack at an absolutely rigid edge, and a crack at a free edge, Figs. 2 and 3). When the layer is 10 times less rigid than the main material ($\eta = 0.1$), the stress intensity coefficient around the vertex $k_1^{(a)}$, close to the boundary as the crack approached the boundary between the materials, was replaced by analogy with the case of an internal crack in a half-plane with an edge free from stress [7]. Finally, when the layer was 10 times more rigid than the substrate material ($\eta = 10$), $k_1^{(a)}$ was altered in the same way as in the case of an internal crack with a rigidly gripped edge [2]. A similar situation occurs with the stress intensity coefficient around the remote vertex of the crack b (Fig. 3). For $\eta \in]0; 0.1[\cup]10, \infty[$, one can assume with an error of less than 5-10% in real calculations on the strength of coated materials that the relative values of the stress intensity coefficients will be similar for any form of loading or for a rigidly clamped edge (Figs. 2 and 3). For physically possible characteristics satisfying $0.1 < \eta < 10$, the strength calculations should be based on numerical solutions to (8).

Figure 4 shows the stress intensity coefficients $k_1/p\sqrt{l}$ as functions of G_s/G for composites with broken edges (mathematically, this means that the order of the load singularity at the point of emergence of the crack on the broken edge is less than $1/2$, which amounts to the condition $k_1^{(a)} \equiv 0$). The intensity coefficient increases with the rigidity of the coating for $G_s/G = 0.1$, and there is an inflection at $G_s/G = 10$ and a transition to the limiting case of the stress intensity coefficient for a half-plane containing a crack with an absolutely rigidly clamped edge. As the rigidity of the coating decreases, so does the stress intensity coefficient, and the numerical value for $\eta \in G_s/G < 0.1$ uniformly approaches the relative values of the stress intensity coefficients with the following values of edge loading:

G_s/G	0,0001	0,1	1,0	10	∞
$k_1^{(b)}/k_1^{(a)}$	0,88	1,08	1,12	2	4,1

In the case of an absolutely rigid undamaged coating, the stress intensity coefficient is reduced by a factor of 4.1 by comparison with the previous case, which agrees with the particular case considered previously [2]. The above data also agree with published ones [6].

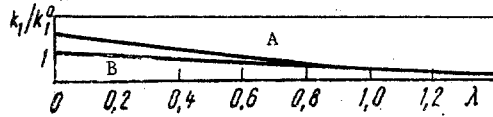


Fig. 6

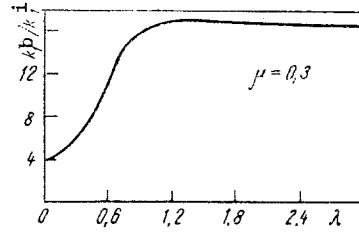


Fig. 7

Fig. 6. A) Crack-free edge; B) crack-rigid edge.
Fig. 7

If one compiles a system of singular integral equations describing the state of stress and strain in a half-plane reinforced at a rectilinear boundary by an absolutely rigid stringer and weakened by a system of arbitrarily oriented cracks [2], one can specify a perpendicular orientation for the crack system to the half-plane boundary (Fig. 5), which gives two singular equations of the form

$$\begin{aligned} & \frac{1}{d} \int_0^l \operatorname{Re} g'(t) \{ \kappa^{-1} [0.5(\kappa^2 + 1) - 1] \operatorname{cth} [\pi(t-x)/d] - \\ & - 0.5 \pi(t-x) \operatorname{csch}^2 [\pi(t-x)/d] - \pi \kappa^{-1} d^{-1} [t-x - \\ & - 4 \pi x t d^{-1}] \operatorname{cth} [\pi(t-x)/d] \operatorname{csch}^2 [\pi(t+x)/d] \} dt = 0.5 (\kappa + 1) \operatorname{Im} p(x), \\ & \frac{1}{d} \int_0^l \operatorname{Im} g'(t) \{ 2 \operatorname{cth} [\pi(t-x)/d] - \pi d^{-1} (t-x) \operatorname{csch}^2 [\pi(t-x)/d] + \\ & + \pi \kappa^{-1} d (3t+x) \operatorname{csch}^2 [\pi(x+t)/d] - \kappa^{-1} [0.5(\kappa^2 + 1) + 1 + \\ & + 4 \pi^2 t x d^{-2} \operatorname{csch}^2 [\pi(t+x)/d]] \operatorname{cth} [\pi(t+x)/d] \} dt = 0.5 (\kappa + 1) \operatorname{Re} p(x), \end{aligned} \quad (10)$$

in which d is the distance between the cracks of length l , $g'(t) = \operatorname{Re} g'(t) + i \operatorname{Im} g'(t)$ is an unknown function of the stress increment, and $p(t) = \operatorname{Re} p(t) + i \operatorname{Im} p(t)$ is a function of the given stresses at the crack surfaces, which is expressed uniquely in terms of the mechanical forces defined locally or at infinitely remote points in the half-plane [2].

System (10) describes the elastically deformed state in a half-plane with a free boundary ($\kappa = -1$) or a rigidly clamped boundary together with a periodic system of weakening cracks at or near the boundaries. Numerical realization in the case of cracks near the boundaries can be based on a system of algebraic equations supplemented with the discrete analog for the condition for unique displacement:

$$\int_0^l g'(t) dt = 0, \quad (11)$$

and in the case of edge cracks (cracked edge in a half-plane), condition (11) is replaced by the condition for the stress intensity coefficients being zero at the vertices of cracks near the edges, which emerge on the surface of the stringer. That additional condition means that the order of the singularity in the stress function term differs from $-1/2$ at the points at which the defects emerge on the edge of the half-plane. The dependence of the relative stress intensity coefficients k_1/k_1^0 on $\lambda = l/d$ shows that there is a qualitative difference between the reinforced and free forms of edge loading (k_1^0 is the stress intensity coefficient for an isolated crack in an infinite elastic plane), and also one can use the calculated result for the reinforcement of a half-plane by elastic coatings (Fig. 5). One can use these numerical data to interpolate the effective stress intensity coefficient for the case of a uniformly distributed loading on an elastic coating. Also, when the number of cracks near the edge increases (when $\lambda > 0.8$), the stress intensity coefficient approaches zero.

In the case of a system of cracks near the edge and perpendicular to the boundary of the half-plane, there is no substantial difference between free and gripped edges for the case of a single isolated crack in an infinite plane for the vertices far from the boundary (Fig. 6). The numerical data (Figs. 2-6) have been used with mechanical quadratures here [2, 7]. The number of interpolated points was 20, since any further increase did not alter the stress intensity coefficients. In the case of internal cracks in a half-plane, the discrete analogs of the singular-equation system were supplemented with a condition of type

(11) for single-valued displacements; condition (11) was replaced for edge defects by the condition for the stress intensity coefficient being zero at the vertex closest to the boundary.

These equations and the numerical data can be used to estimate the viability of thin coatings. For example, the numerical data show that the performance of a coating can be estimated from the graph in Fig. 7, in which k_1^b is the stress intensity coefficient when the coating breaks as a result of the system of defects, while k_1^i is the stress intensity coefficient in the case of internal defects, which shows by what factor the stringer increases the stress intensity in a system of periodic defects of crack type with the most unfavorable orientation (perpendicular to the boundary between the coating and the substrate). When there is a single crack, a rigid stringer reduces the stress intensity by about a factor of four. The curve shows that there is a boundary value for the defect composition ($\lambda \approx 0.9$) above which there is no substantial change in the elastic and limiting equilibrium of the body.

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