

ELECTRODE POTENTIAL ON THE BOUNDARY OF A MEDIUM IN AN ELLIPTIC NOTCH MADE IN A STRESSED CONDUCTING BODY

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We determine the dependences of the distribution of the electrode potential along the contour of a medium-filled elliptic defect in a stressed conducting body on the physical and chemical characteristics of the material and medium and the relations between the geometric parameters of the defect. We also indicate the points of separation of the anode and cathode zones on the contour of the defect.

The analytic description of corrosion processes in electrically conducting bodies deformed in electrochemically active media with due account not only of the Coulomb fields but also of the field of electric potential is a necessary condition for the investigation of physical aspects of corrosive fracture of materials and the analysis of the parameters of formation of their corrosion crack resistance [1–3].

It was experimentally established that electrochemical processes running with different activity near curvilinear notches are accompanied by the localization of the zones of intense dissolution of the metal in the most stressed places, by an increase in the difference between the electrode potentials of the anode and cathode regions, and, hence, by the intensification of corrosion [1].

By using the well-known method developed in [4, 5] and applied in [6–8], we construct the solution of the problem of distribution of electric potential along the contour of an elliptic defect in a stressed conducting body and determine the behavior of the potential in the analyzed metal–medium system.

Statement of the Problem and Basic Relations

We consider a two-dimensional problem for an elastic isotropic conducting body with density ρ_m and specific capacitance C_m containing a medium-filled elliptic (circular) notch. The body is subjected to the action of loads with intensity p directed along the minor semiaxis of the ellipse. On the boundary of contact between the body and the medium, we observe the formation of a double electric layer responsible for the appearance of the electrode potential between the metal and the medium (Fig. 1) [9].

We now write basic relations for the electric potentials in the body, double electric layer, and medium. We neglect the effects of polarization and magnetization of the body and, according to the linear thermodynamics of irreversible processes, use the hypothesis of local equilibrium. This enables us to determine the distribution of electric potential in the body by using the equation [10]

$$\nabla^2 \varphi_m = \chi_m^2 \varphi_m + \beta K \epsilon_\sigma \epsilon_0^{-1}. \quad (1)$$

According to the Debye–Hückel theory of solutions, the equations for the distributions of electric potentials in the double electric layer and medium have the form [9, 11]

$$\nabla^2 \varphi_{mc} = \chi_{mc}^2 \varphi_{mc} \quad \text{and} \quad \nabla^2 \varphi_c = \chi_c^2 \varphi_c, \quad (2)$$

respectively. Here, ∇^2 is the Laplace operator, φ_m , φ_{mc} and φ_c are the potentials of the metal, double electric layer, and medium, respectively, β is the electrostriction coefficient, K is the bulk modulus, ϵ_σ is a dilatation (re-

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lative cubic expansion), ε_0 is the electric constant, and χ_m , χ_{mc} and χ_c are constants of the metal, double electric layer, and medium, respectively, computed according to the following formulas [9–11]:

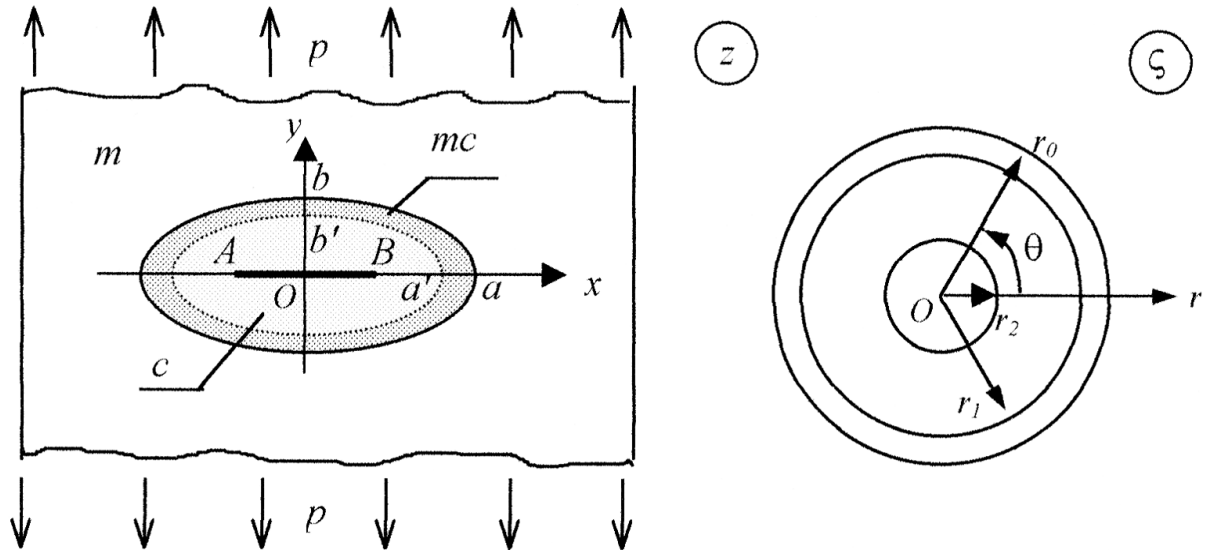


Fig. 1. Schematic diagrams of the problem in the physical plane z and auxiliary plane ζ (m is the metal, mc is the double electric layer, and c is the medium).

$$\chi_m^2 = \rho_m C_m \varepsilon_0^{-1}, \quad \chi_{mc}^2 = e^2 (kT \varepsilon_{mc} \varepsilon_0)^{-1} \sum z_i^2 n_i^2, \quad \text{and} \quad \chi_c^2 = e^2 (kT \varepsilon_c \varepsilon_0)^{-1} \sum z_i^2 n_i^2,$$

where ε_{mc} and ε_c are the relative electric permittivities of the double electric layer and medium, respectively, $e z_i$ is the charge ions of the i th sort, n is their number, and T is absolute temperature.

We specify boundary conditions of the problem by using the requirements of continuity of the normal components of currents on the line of contact of electrically conducting phases and continuity of the electric potentials in these phases [12–14]. Thus, on the line of contact of the metal with the double electric layer, we can write

$$\sigma_m \frac{\partial \varphi_m}{\partial n} = \sigma_{mc} \frac{\partial \varphi_{mc}}{\partial n}, \quad \varphi_m = \varphi_{mc}. \quad (3)$$

At the same time, on the line of contact of the double electric layer with the medium, we have

$$\sigma_{mc} \frac{\partial \varphi_{mc}}{\partial n} = \sigma_c \frac{\partial \varphi_c}{\partial n}, \quad \varphi_{mc} = \varphi_c, \quad (4)$$

where σ_m , σ_{mc} and σ_c are the conductivities of the metal, double electric layer, and medium, respectively, and $\partial/\partial n$ is the derivative along the normal to the contact line.

Solution of the Problem

We solve the problem of distribution of electric potentials inside and near the elliptic hole by using the conformal mapping of a disk onto an ellipse accompanied by the corresponding changes in the boundary conditions [15, 16]. We consider an ellipse with semi-axes a and b in the complex plane $z = z(x, y)$ and a disk of radius $r = r_0$ in the plane $\zeta = \zeta(r, \theta)$ (Fig. 1) and introduce a mapping

$$z = \omega(\zeta) = R \left(\frac{\zeta}{r_0} + \frac{mr_0}{\zeta} \right), \quad z = x + iy, \quad \zeta = re^{i\theta}, \quad (5)$$

where

$$R = 0.5(a + b), \quad 0 \leq m = (a - b)(a + b)^{-1} \leq 1. \quad (6)$$

In the two-dimensional problem of the theory of elasticity, the dilatation ε_σ is computed as follows:

$$\varepsilon_\sigma = 2(1 + \nu)[3K(1 + 4\nu)]^{-1}(\varepsilon_1 + \varepsilon_2) = 4(1 + \nu)[3K(1 + 4\nu)]^{-1} \left[\frac{\phi'(\zeta)}{\omega'(\zeta)} + \overline{\frac{\phi'(\zeta)}{\omega'(\zeta)}} \right],$$

where ε_1 and ε_2 are the components of the first invariant of the strain tensor and, according to the data presented in [15] and relation (5), we have

$$\phi(\zeta) = 0.25pR \left[\frac{\zeta}{r_0} + \frac{r_0(2 + m)}{\zeta} \right]$$

for the case of uniaxial tension of a plate with elliptic hole in the direction normal to the x -axis.

Therefore, in relation (1), we get

$$\varepsilon_\sigma = \frac{2(1 + \nu)p}{3K(1 + 4\nu)} \frac{r^4 - m^2 r_0^4 - 2mr_0^4 + 2r^2 r_0^2 \cos 2\theta}{r^4 - 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4}, \quad (7)$$

where ν is Poisson's ratio.

The region lying outside the circle of radius

$$r_2 = r_0 \sqrt{\frac{a - b}{a + b}}$$

is mapped onto the region lying outside the interfocal segment $AB = k[-\sqrt{a^2 - b^2}, \sqrt{a^2 - b^2}]$. Hence, in order that the transition to a circular notch be correct, one must impose an additional boundary condition of continuity of the electric field in the interfocal segment of the ellipse, i.e.,

$$\vec{E}^+ = \vec{E}^-, \quad (8)$$

where $\vec{E}^+$ and $\vec{E}^-$ are the electric-field intensities on approaching the boundary from the upper and lower half planes, respectively.

In a polar coordinate system introduced, for the case of a circular notch ($m = 0$), by the formulas

$$x = Rrr_0^{-1} \cos \theta \quad \text{and} \quad y = Rrr_0^{-1} \sin \theta, \quad (9)$$

the solution of equations (1), (2) can be represented via modified cylindrical functions $I_k(\cdot)$ and $K_k(\cdot)$ [17] by taking into account the symmetry conditions: $\phi(r, \theta) = \phi(r, -\theta)$ and $\phi(r, \theta) = \phi(r, \pi - \theta)$, the properties of these functions, and the boundary conditions.

If we assume that the radius of the defect r_0 is such that

$$\min(\chi_m r_0, \chi_m r_1, \chi_c r_1) > 2, \quad (10)$$

where

$$r_1 = r_0(b - \lambda + \sqrt{a^2 - 2b\lambda + \lambda^2})(a + b)^{-1},$$

and λ is the width of the double electric layer, then we can represent the solutions obtained for a circular defect in the asymptotic form {this is possible in view of condition (10) from [17]}, pass to the elliptic coordinates introduced by relation (5):

$$x = R \left(\frac{r}{r_0} + \frac{mr_0}{r} \right) \cos \theta \quad \text{and} \quad y = R \left(\frac{r}{r_0} - \frac{mr_0}{r} \right) \sin \theta, \quad (11)$$

and perform the required changes in the boundary conditions. As a result, we obtain the following formulas for the electric potentials in the analyzed metal–medium system:

$$\begin{aligned} \varphi_c(r, \theta) &= a_0^c I_0(\chi_c R_c) + a_2^c I_2(\chi_c R_c) \frac{(r^4 + m^2 r_0^4) \cos 2\theta + 2mr^2 r_0^2}{r^4 + 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \\ &= \frac{2\alpha}{\chi_m^2} \sqrt{\frac{R_0}{R_c}} e^{-\chi_c(R_1 - R_c)} \\ &\quad \times \frac{\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + 2 \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} - \frac{2}{R_0 \sqrt{\chi_m \chi_c}} \right) \frac{(r^4 + m^2 r_0^4) \cos 2\theta + 2mr^2 r_0^2}{r^4 + 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4}}{\left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) (e^{-\chi_{mc} \lambda} - e^{\chi_{mc} \lambda}) - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda})}} \\ &\quad (r_2 < r < r_1), \end{aligned} \quad (12)$$

$$\begin{aligned} \varphi_{mc}(r, \theta) &= a_0^{mc} I_0(\chi_{mc} R_{mc}) + b_2^{mc} K_2(\chi_{mc} R_{mc}) \\ &\quad + [a_2^{mc} I_2(\chi_{mc} R_{mc}) + b_2^{mc} K_2(\chi_{mc} R_{mc})] \frac{(r^4 + m^2 r_0^4) \cos 2\theta + 2mr^2 r_0^2}{r^4 + 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \\ &= \frac{\alpha}{\chi_m^2} \sqrt{\frac{R_0}{R_{mc}}} \left\{ \frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} (e^{-\chi_{mc}(R_{mc} - R_1)} + e^{\chi_{mc}(R_{mc} - R_1)}) - \frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} (e^{-\chi_{mc}(R_{mc} - R_1)} - e^{\chi_{mc}(R_{mc} - R_1)}) \right. \\ &\quad + 2 \left[\left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} - \frac{2}{R_0 \sqrt{\chi_m \chi_c}} \right) (e^{-\chi_{mc}(R_{mc} - R_1)} + e^{\chi_{mc}(R_{mc} - R_1)}) \right. \\ &\quad \left. \left. + \left(\frac{2\sigma_c \sqrt{\chi_c}}{\sigma_{mc} \chi_{mc} \sqrt{\chi_m} R_0} - \frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} \right) (e^{-\chi_{mc}(R_{mc} - R_1)} + e^{\chi_{mc}(R_{mc} - R_1)}) \frac{(r^4 + m^2 r_0^4) \cos 2\theta + 2mr^2 r_0^2}{r^4 + 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \right] \right\} \\ &\quad \times \left[\left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) (e^{-\chi_{mc} \lambda} - e^{\chi_{mc} \lambda}) - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda}) \right]^{-1} \end{aligned}$$

$$\begin{aligned}
& + \frac{\alpha}{\chi_m^2} \sqrt{\frac{R_0}{R_{mc}}} \frac{e^{-\chi_{mc}(R_{mc}-R_1)} - e^{\chi_{mc}(R_{mc}-R_1)}}{e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}} 2m [3 + m + 3m^2 + m^3 + (m+1)(m^2-1) \cos 2\theta \\
& - 2(2+m+m^2) \cos^2 2\theta] (1 + 2m^2 + m^4 - 4m^2 \cos^2 2\theta)^{-1} \quad (r_1 \leq r < r_0), \quad (13)
\end{aligned}$$

$$\begin{aligned}
\varphi_m(r, \theta) &= a_0^m K_0(\chi_m R_m) + a_2^m K_2(\chi_m R_m) \frac{(r^4 + m^2 r_0^4) \cos 2\theta + 2mr^2 r_0^2}{r^4 + 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \\
&- \frac{\alpha}{\chi_m^2} \frac{r^4 - m^2 r_0^4 - 2mr_0^4 + 2r^2 r_0^2 \cos 2\theta}{r^4 - 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \\
&= \frac{\alpha}{\chi_m^2} \sqrt{\frac{R_0}{R_m}} e^{\chi_m(R_0-R_m)} \left\{ \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} (e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}) - \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} (e^{-\chi_{mc}\lambda} + e^{\chi_{mc}\lambda}) \right. \\
&+ 2 \left[\left(\frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} + \frac{2\sigma_c \sqrt{\chi_c}}{\sigma_{mc} \sqrt{\chi_m} \chi_{mc} R_0} \right) (e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}) \right. \\
&- \left. \left(\frac{2}{R_0 \sqrt{\chi_m \chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc}\lambda} + e^{\chi_{mc}\lambda}) \right] \frac{(r^4 + m^2 r_0^4) \cos 2\theta + 2mr^2 r_0^2}{r^4 + 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \Big\} \\
&\times \left[\left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) (e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}) - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc}\lambda} + e^{\chi_{mc}\lambda}) \right]^{-1} \\
&- \frac{\alpha}{\chi_m^2} \frac{r^4 - m^2 r_0^4 - 2mr_0^4 + 2r^2 r_0^2 \cos 2\theta}{r^4 - 2mr^2 r_0^2 \cos 2\theta + m^2 r_0^4} \quad (r > r_0). \quad (14)
\end{aligned}$$

The deviation of the electrode potential $\Delta\varphi$ from the standard chemical potential $\Delta\varphi_0$ along the contour of the defect is given by the formula

$$\begin{aligned}
\Delta\varphi - \Delta\varphi_0 &= \varphi_m(r, \theta) - \varphi_c(r, \theta) = a_0^m K_0(\chi_m R_0) - a_0^c I_0(\chi_c R_1) \\
&+ a_2^m K_2(\chi_m R_0) [(1+m^2) \cos 2\theta + 2m] (1 + 2m \cos 2\theta + m^2) \\
&- a_2^c I_2(\chi_c R_1) \frac{(r_1^4 + m^2 r_0^4) \cos 2\theta + 2mr_1^2 r_0^2}{r_1^4 + 2mr_1^2 r_0^2 \cos 2\theta + m^2 r_0^4} \\
&- \alpha \chi_m^{-2} (1 - m^2 - 2m + 2 \cos 2\theta) (1 - 2m \cos 2\theta + m^2)^{-1} \\
&= \frac{\alpha}{\chi_m^2} \left\{ \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} (e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}) - \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} (e^{-\chi_{mc}\lambda} + e^{\chi_{mc}\lambda}) \right. \\
&+ 2 \left[\left(\frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} + \frac{2\sigma_c \sqrt{\chi_c}}{\sigma_{mc} \sqrt{\chi_m} \chi_{mc} R_0} \right) (e^{-\chi_{mc}\lambda} - e^{\chi_{mc}\lambda}) \right.
\end{aligned}$$

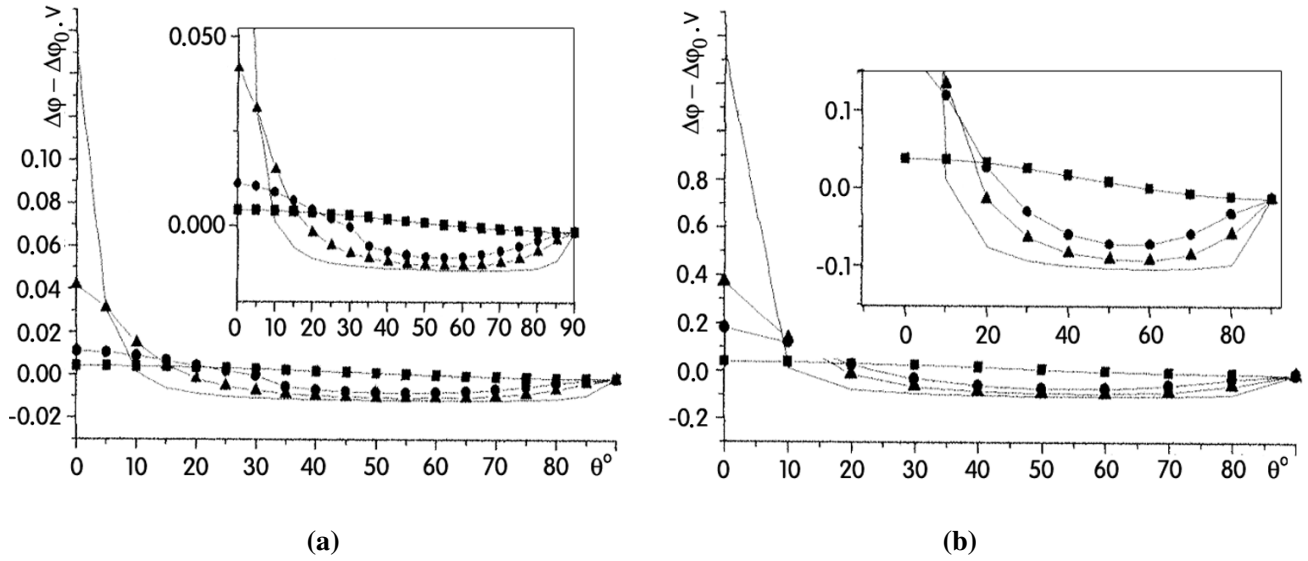


Fig. 2. Variation of the electrode potentials along the boundary of the medium-filled elliptic defect in stressed copper (a) and steel (b) plates ($\Delta\phi - \Delta\phi_0$ is the deviation of the electrode potential $\Delta\phi$ from the standard potential $\Delta\phi_0$, θ is a polar angle, (■) $m = 0$, (●) $m = 0.5$, (▲) $m = 0.7$, and (—) $m = 0.9$).

$$\begin{aligned}
 & - \left(\frac{2}{R_0 \sqrt{\chi_m \chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda}) \left] \frac{(1 + m^2) \cos 2\theta + 2m}{1 + 2m \cos 2\theta + m^2} \right. \\
 & - \left. 2 \sqrt{\frac{R_0}{R_1}} \left[\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + 2 \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} - \frac{2}{R_0 \sqrt{\chi_m \chi_c}} \right) \frac{(r_1^4 + m^2 r_0^4) \cos 2\theta + 2m r_1^2 r_0^2}{r_1^4 + 2m r_1^2 r_0^2 \cos 2\theta + m^2 r_0^4} \right] \right\} \\
 & \times \left[\left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) (e^{-\chi_{mc} \lambda} - e^{\chi_{mc} \lambda}) - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda}) \right]^{-1} \\
 & - \frac{\alpha}{\chi_m^2} \frac{1 - m^2 - 2m + 2 \cos 2\theta}{1 - 2m \cos 2\theta + m^2}.
 \end{aligned} \tag{15}$$

In relations (12)–(15), we denote

$$\begin{aligned}
 R_0 &= R \sqrt{1 + 2m \cos 2\theta + m^2}, \quad R_1 = R(r_1 r_0)^{-1} \sqrt{r_1^4 + 2m r_1^2 r_0^2 \cos 2\theta + m^2 r_0^4}, \\
 R_m &= R(r r_0)^{-1} \sqrt{r^4 + 2m r^2 r_0^2 \cos 2\theta + m^2 r_0^4} \quad (r \geq r_0), \\
 R_{mc} &= R(r r_0)^{-1} \sqrt{r^4 + 2m r^2 r_0^2 \cos 2\theta + m^2 r_0^4} \quad (r_1 \leq r < r_0), \\
 R_c &= R(r r_0)^{-1} \sqrt{r^4 + 2m r^2 r_0^2 \cos 2\theta + m^2 r_0^4} \quad (r_2 < r < r_1), \quad \alpha = \frac{2\beta(1 + \nu)p}{3\varepsilon_0(1 + 4\nu)}.
 \end{aligned} \tag{16}$$

For the numerical evaluation of the electrode potentials, we used the following values of the parameters [10, 18, 19] (Fig. 2): $p = 235$ MPa, $\rho_m = 8940$ kg/m³, $C_m = 1.05$ kC/(kg · V), $\beta = -0.131$ mV⁻¹, and $\nu = 0.35$ in the copper plate, $p = 689$ MPa, $\rho_m = 7700$ kg/m³, $C_m = 1.0$ kC/(kg · V), $\beta = -1.0$ mV⁻¹, and $\nu = 0.28$ in the steel plate,

and $\rho_c = 1300 \text{ kg/m}^3$, $C_c = 10 \text{ kC/(kg} \cdot \text{V)}$, $\lambda = 1 \text{ nm}$, and $\sigma_c/\sigma_m = 10^{-6}$ in the medium. We also used the following formulas: $\chi_{mc} = 0.5(\chi_m + \chi_c)$ and $\sigma_{mc} = \sigma_m \cdot 10^{-3}$.

Separation of the Anode and Cathode Regions on the Contour of the Defect

Under loading, electric charges are nonuniformly distributed over the surface of a defect filled with an electrochemically active medium, which may noticeably affect the kinetics of corrosion processes. Thus, by using expression (15), we determine the angles θ for which the quantity $\Delta\phi - \Delta\phi_0$ changes its sign.

We denote

$$\begin{aligned}
 d &= \left(\frac{\sigma_c \sqrt{\chi_m \chi_c}}{\sigma_{mc} \chi_{mc}} + \frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} \right) (e^{-\chi_{mc} \lambda} - e^{\chi_{mc} \lambda}) - \left(\frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} + \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda}), \\
 \gamma_0 &= \frac{1}{d} \left[\frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} (e^{-\chi_{mc} \lambda} - e^{\chi_{mc} \lambda}) - \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda}) - 2 \sqrt{\frac{R_0}{R_1}} \frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} \right], \\
 \gamma_1 &= \frac{2}{d} \left[\left(\frac{\sigma_{mc} \chi_{mc}}{\sigma_m \sqrt{\chi_m \chi_c}} + \frac{2\sigma_c \sqrt{\chi_c}}{\sigma_{mc} \sqrt{\chi_m \chi_{mc}} R_0} \right) (e^{-\chi_{mc} \lambda} - e^{\chi_{mc} \lambda}) - \left(\frac{2}{R_0 \sqrt{\chi_m \chi_c}} - \frac{\sigma_c \sqrt{\chi_c}}{\sigma_m \sqrt{\chi_m}} \right) (e^{-\chi_{mc} \lambda} + e^{\chi_{mc} \lambda}) \right], \\
 \gamma_2 &= \frac{4}{d} \sqrt{\frac{R_0}{R_m}} \left(\frac{2}{R_0 \sqrt{\chi_m \chi_c}} - \frac{\sqrt{\chi_m}}{\sqrt{\chi_c}} \right), \quad \delta = \frac{r_0^2}{r_1^2}.
 \end{aligned} \tag{17}$$

Equating expression (15) to zero, we obtain

$$\begin{aligned}
 \gamma_0 + \gamma_1 \frac{(1+m^2)\cos 2\theta + 2m}{1+2m\cos 2\theta + m^2} + \gamma_2 \frac{(1+m^2\delta^2)\cos 2\theta + 2m\delta}{1+2m\delta\cos 2\theta + m^2\delta^2} \\
 - (1-m^2-2m+2\cos 2\theta)(1-2m\cos 2\theta + m^2)^{-1} = 0.
 \end{aligned} \tag{18}$$

For a circular notch ($m = 0$), equation (18) implies that

$$\theta = \pm 0.5 \arccos(1 - \gamma_0)(\gamma_1 + \gamma_2 - 2)^{-1} + 180^\circ k \quad (k = 0, 1). \tag{19}$$

For $m \neq 0$, the roots of equation (18) admit the following representation:

$$\begin{aligned}
 \theta &= \pm 0.5 \arccos(3\sqrt[3]{2}a_3)^{-1} \left\{ [9a_1a_2a_3 - 27a_0a_3^2 - 2a_2^3 + (108a_0a_2^3a_3^2 + 108a_1^3a_3^3 - 486a_0a_1a_2a_3^3 \right. \\
 &\quad \left. - 27a_1^2a_2^2a_3^2 + 729a_0^2a_3^4)^{1/2}]^{1/3} - [(9a_1a_2a_3 - 27a_0a_3^2 - 2a_2^3 - (108a_0a_2^3a_3^2 + 108a_1^3a_3^3 \right. \\
 &\quad \left. - 486a_0a_1a_2a_3^3 - 27a_1^2a_2^2a_3^2 + 729a_0^2a_3^4)^{1/2}]^{1/3} - \sqrt[3]{2}a_2 \right\} + 180^\circ k \quad (k = 0, 1),
 \end{aligned} \tag{20}$$

where

$$a_0 = \gamma_0(1+m^2)^2(1+m^2\delta^2) + (2\gamma_1m + m^2 + 2m - 1)(1+m^2)(1+m^2\delta^2) + 2\gamma_2m(1+m^2)^2\delta,$$

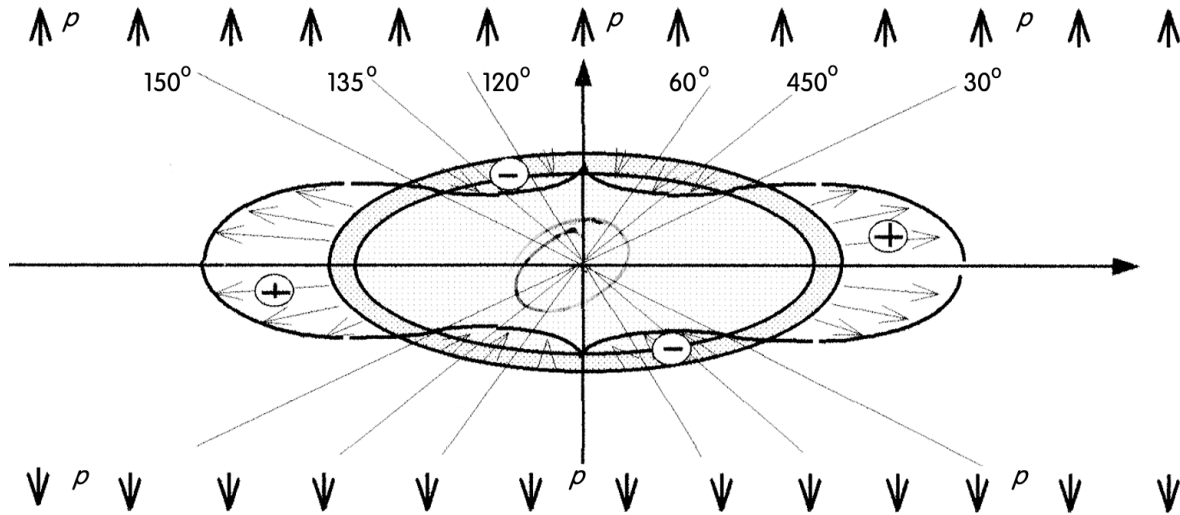


Fig. 3. Distribution of electric charges over the contour of the elliptic defect ($m = 0.3$).

$$\begin{aligned}
 a_1 &= -2\gamma_0 m(1+m^2)^2 \delta - \gamma_1 (1+m^2)^2 (1+m^2 \delta^2) \\
 &\quad - 4\gamma_1 m^2 (\delta + m^2 \delta - 1 - m^2 \delta^2) - \gamma_2 (1+m^2)^2 (1+m^2 \delta^2) \\
 &\quad - 2m\delta(1+m^2)(m^2+2m-1) - 2m(1+m^2 \delta^2)(m^2+2m-1) + 2(1+m^2)(1+m^2 \delta^2), \quad (21) \\
 a_2 &= 2m[2\gamma_0 m(1+m^2 \delta^2) - \gamma_1 (1+m^2)^2 \delta + \gamma_1 (1+m^2)(1+m^2 \delta^2) + 4m(\gamma_1 + \gamma_2)\delta \\
 &\quad - 2m\delta(m^2+2m-1) + 2(1+m^2)\delta + 2(1+m^2 \delta^2)], \\
 a_3 &= 4m^2[2\gamma_0 m\delta + \gamma_1 (1+m^2)\delta + \gamma_2 (1+m^2 \delta^2) + 2\delta].
 \end{aligned}$$

For the angles specified by expression (20), the anode zone on the surface of the defect turns into the cathode zone (Fig. 3).

CONCLUSIONS

Mechanical stresses affect the concentration of electric charges and their sign. Therefore, on the boundary of the anode and cathode zones on the surface of the defect, the electrochemical interaction of the body with the ambient medium is intensified. The comparison of the electrode potentials for circular and elliptic notches shows that the electromotive force of corrosive currents caused by mechanical tensile loads is more dangerous for stretched elliptic cavities.

The fact that the electrode potential in the vicinity of an elliptic defect in the steel specimen is much (by an order of magnitude) higher than in the copper specimen well agrees with the results presented in [4] for a circular stress concentrator.

As the ratio of the minor semiaxis of the ellipse to its major semiaxis decreases, the point where the electrode potential changes its sign shifts toward the vertex of the ellipse. Thus, for the copper specimen with $m = 0.5$, the angle of transition from the anode zone on the surface of the defect to the cathode zone is close to 30° . At the same time, for $m = 0.9$, this angle is approximately equal to 11° .

As the ellipse shrinks into a cut ($b \rightarrow 0$) and $\theta \rightarrow 0$, the quantity $\Delta\varphi - \Delta\varphi_0$ is an infinitely increasing function of the load p . This is explained by the fact that, according to the Griffith model, the dilatation of the material and, hence, the electrode potential become infinite at the tip of this notch (crack). Thus, in the computational models of actual cracks, it is necessary to take into account the plasticity of the prefracture zone, which is possible, e.g., within the framework of the Leonov–Panasyuk model.

REFERENCES

1. I. M. Dmitrakh and V. V. Panasyuk, *Influence of Corrosive Media on the Local Fracture of Metals Near Stress Concentrators* [in Ukrainian], Karpenko Physicomechanical Institute, Ukrainian Academy of Sciences, L'viv (1999).
2. M. H. Stashchuk, *Problems of the Mechanics of Elastic Bodies with Cracklike Defects* [in Russian], Naukova Dumka, Kiev (1993).
3. M. H. Stashchuk and V. H. Horopats'kyi, "Modeling of the accumulation of media near stress concentrators," *Fiz.-Khim. Mekh. Mater.*, **31**, No. 3, 62–69 (1995).
4. M. H. Stashchuk, "Evaluation of the electrode potentials and corrosion currents along the surfaces of stress concentrators," in: V. V. Panasyuk (editor), *Fracture Mechanics of Materials and Strength of Structures* [in Russian], 2nd Ed., Vol. 2, Kamenyar, Lvov (1999), pp. 213–219.
5. M. H. Stashchuk, M. M. Pokrovs'kyi, and A. M. Stashchuk, *Electromotive Force in Stressed and Strained Bodies Interacting with Electrolytes* [in Ukrainian], Certificate of State Registration of Author's Copyright, PA No. 2480, Ukrainian State Copyright Agency (1999).
6. M. H. Stashchuk, "Evaluation of the influence of elastic deformation on the electrode potential of a metal cylinder in the medium," *Fiz.-Khim. Mekh. Mater.*, **36**, No. 1, 47–50 (2000).
7. M. H. Stashchuk, "Electrode potential on the rectilinear boundary between a stressed metal and a medium," *Probl. Kor. Kor. Zakh. Mater.*, No. 1, 222–225 (2000).
8. M. H. Stashchuk and V. H. Horopats'kyi, "Computational model for the numerical analysis of the electrode potential on the boundary of a medium-filled elliptic cavity in a loaded metallic body," *Probl. Kor. Kor. Zakh. Mater.*, No. 1, 737–742 (2000).
9. L. I. Antropov, *Theoretical Electrochemistry* [in Ukrainian], Lybid', Kiev (1993).
10. Ya. I. Burak, B. P. Galapats, and B. M. Gnidgets', *Physical and Mechanical Processes in Conducting Bodies* [in Ukrainian], Naukova Dumka, Kiev (1978).
11. N. A. Izmailov, *Electrochemistry of Solutions* [in Russian], Khimiya, Moscow (1976).
12. W. R. Smythe, *Static and Dynamic Electricity*, New York (1950).
13. Yu. Ya. Iosel' and G. É. Klenov, *Mathematical Methods for the Numerical Analysis of the Electrochemical Corrosion of Metals and Protection Against This Type of Corrosion (A Handbook)* [in Russian], Metallurgiya, Moscow (1984).
14. Yu. Ya. Iosel', *Electric Fields of Direct Currents* [in Russian], Énergoatomizdat, Leningrad (1986).
15. N. I. Muskhelishvili, *Some Basic Problems of the Mathematical Theory of Elasticity* [in Russian], Nauka, Moscow (1966).
16. S. J. Farlow, *Partial Differential Equations for Scientists and Engineers*, Wiley, New York (1982).
17. V. Ya. Arsenin, *Methods of Mathematical Physics and Special Functions* [in Russian], Nauka, Moscow (1984).
18. V. V. Skorochelletti, *Theoretical Electrochemistry* [in Russian], Khimiya, Leningrad (1969).
19. H. Kuchling, *Physik*, VEB Fachbuchverlag, Leipzig (1980).