

CONTACT PROBLEM FOR A HALF PLANE WITH CRACKS SUBJECTED TO THE ACTION OF A RIGID PUNCH ON ITS BOUNDARY

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We consider a contact problem of the theory of elasticity for an isotropic half plane with a system of curvilinear cracks and a rigid punch pressed into the half plane. We assume that the base of the punch has an arbitrary convex shape, that either the punch interacts with the half plane via friction forces or they are rigidly engaged, and that the crack lips are under the conditions of smooth contact. The problem is reduced to singular integral equations in unknown derivatives of singular displacements on the crack contours and contact forces under the punch. These equations are solved by the method of mechanical quadratures. We present the results of numerical analysis of the stressed state of the half plane with internal vertical crack under pressure of a flat punch without friction on the boundary of the half plane.

Although the problem of interaction of a punch with crack-like defects of a body in the region of contact is of great importance for engineering practice, the results of its investigation cannot be regarded as adequate. It is well known that the process of contact interaction of solid bodies often leads to the appearance and propagation of sub-surface cracks which decrease the serviceability of structural elements. The major part of theoretical investigations is devoted to the analysis of the interaction of a punch with a vertical rectilinear crack in a semiinfinite plate [1–7]. A periodic system of vertical cracks and a system of rectilinear arbitrarily oriented cracks were studied in [8] and [9], respectively. In the present paper which is a continuation of [10], we consider a more general case of a half plane weakened by a system of curvilinear cracks and take into account the possibility of smooth contact of crack lips under the action of a punch.

Statement of the Problem

Consider a contact problem of a rigid punch with base of an arbitrary convex shape pressed into the boundary of an elastic isotropic plate with M curvilinear cuts (cracks) L_n ($n = \overline{1, M}$). In the half plane, we introduce a coordinate system xOy . The cuts are related to local coordinate systems $x_nO_ny_n$ (see Fig. 1). These local coordinate systems are connected with the principal system by the relations $z = z_n e^{i\alpha_n} + z_n^0$, where $z = x + iy$, $z_n = x_n + iy_n$, z_n^0 is the affix of the point O_n in the principal coordinate system, and α_n is the angle between the x_n -axis and the x -axis.

The principal vector of forces acting upon the punch can be decomposed into the vertical P and tangential Q components. Outside the contact region $|x - x_0| \leq c$, the boundary of the half plane is unloaded. It is assumed that the loaded punch remains horizontal (its rotation is excluded). We consider the case where friction forces described by the Coulomb law act on the interface of the punch and the half plane and the case where they are rigidly engaged. Therefore, we obtain the following boundary conditions on the boundary of the half plane:

$$\frac{du_y(x)}{dx} = f'(x), \quad T(x) = \rho N(x), \quad |x - x_0| \leq c, \quad (1)$$

$$\frac{du_x(x)}{dx} = 0, \quad \frac{du_y(x)}{dx} = f'(x), \quad |x - x_0| \leq c, \quad (2)$$

In Eq. (6), the normal component of displacements of the crack lips L_n is denoted by v_n . Superscripts "+" and "-" correspond to the limiting values of stresses and displacements on approaching the crack lips from the left and from the right, respectively.

Boundaries of the regions of contact of the punch base and crack lips with the boundary of the half plane are not known in advance. They are determined from additional conditions in the process of the solution of the integral equations of the problem. The solution of the contact problem is constructed step-by-step by iteration. Thus, the contact regions on the cuts L_n , $n = \overline{1, M'}$, are determined from the condition that the mode I stress intensity factor is equal to zero at the ends of the intervals $a_{ns}^* b_{ns}^*$, namely,

$$K_{In}(a_{ns}^*) = 0, \quad K_{In}(b_{ns}^*) = 0, \quad n = \overline{1, M'}, \quad s = \overline{1, S_n}.$$

Therefore, the boundary conditions (1), (3), and (4)–(8) are satisfied in the case of friction forces on the interface of the punch and the half plane and conditions (2), (3), and (4)–(8) are satisfied in the case of their engagement. For convenience, in what follows, we write several curvilinear integrals in the operator form.

We introduce operators $A\{L_k\}$ and $B\{L_k\}$ by the formulas

$$A\{L_k\} \varphi_k(t_k) = \frac{1}{2\pi} \int_{L_k} \left\{ \left[\frac{1}{T_k - z} - \frac{1}{\bar{T}_k - z} \right] e^{i\alpha_k} \varphi_k(t_k) dt_k + \frac{2i \operatorname{Im} T_k}{(\bar{T}_k - z)^2} e^{-i\alpha_k} \overline{\varphi_k(t_k)} d\bar{t}_k \right\}, \quad (9)$$

$$B\{L_k\} \varphi_k(t_k) = \frac{1}{2\pi} \int_{L_k} \left\{ \left[\frac{1}{(\bar{T}_k - z)^2} - \frac{1}{(T_k - z)^2} \right] \bar{T}_k e^{i\alpha_k} \varphi_k(t_k) dt_k + \left[\frac{1}{T_k - z} - \frac{1}{\bar{T}_k - z} - 2i \operatorname{Im} T_k \frac{\bar{T}_k + z}{(\bar{T}_k - z)^3} \right] e^{-i\alpha_k} \overline{\varphi_k(t_k)} d\bar{t}_k \right\}, \quad (10)$$

where the index k indicates that the relevant quantity is written in the local coordinates connected with the contour L_k and the quantity T_k is given by the equation

$$T_k = t_n e^{i\alpha_k} + z_k^0.$$

The complex potentials of the problem under consideration can be represented in the form

$$\begin{aligned} \Phi(z) &= \Phi_0(z) + \Phi_1(z), \\ \Psi(z) &= \Psi_0(z) + \Psi_1(z). \end{aligned} \quad (11)$$

As follows from the boundary condition (3), the functions $\Phi_0(z)$ and $\Psi_0(z)$ characterizing the stressed state of the continuous half plane with loaded boundary [11] are given by the relations

$$\begin{aligned} \Phi_0(z) &= -\frac{1}{2\pi i} \int_{x_0-c}^{x_0+c} \frac{p_0(\zeta)}{\zeta - z} d\zeta, \\ \Psi_0(z) &= -\frac{1}{2\pi i} \int_{x_0-c}^{x_0+c} \left[\frac{p_0(\zeta)}{\zeta - z} - \frac{\zeta p_0(\zeta)}{(\zeta - z)^2} \right] d\zeta. \end{aligned} \quad (12)$$

In view of relations (9) and (10), the functions $\Phi_1(z)$ and $\Psi_1(z)$ which describe the stressed state induced by the discontinuities of displacements on the contours of the cuts L_n , $n = \overline{1, M}$, [12] can be represented in the form

$$\begin{aligned}\Phi_1(z) &= \sum_{k=1}^M A\{L_k\} g'_k(t_k), \\ \Psi_1(z) &= \sum_{k=1}^M B\{L_k\} g'_k(t_k).\end{aligned}\tag{13}$$

In relations (12), $p_0(x)$ are unknown contact stresses formed under the punch. Each unknown function $g'_n(\tau_n)$ in relations (13) can be represented as a sum of two functions [12], i.e.,

$$g'_n(\tau_n) = g'_{1n}(\tau_n) + g'_{2n}(\tau_n), \quad n = \overline{1, M'},$$

and the functions on the right-hand side of this relation are expressed in terms of the jumps of the normal v_n and tangential u_n components of the displacement vector on the contour of the cut L_n by the formulas

$$g'_{1n}(\tau_n) = \frac{2G}{1+\kappa} [v_n^+(\tau_n) - v_n^-(\tau_n)] \frac{d\tau_n}{ds_n},\tag{14}$$

$$g'_{2n}(\tau_n) = -\frac{2iG}{1+\kappa} [u_n^+(\tau_n) - u_n^-(\tau_n)] \frac{d\tau_n}{ds_n},\tag{15}$$

where G is the shear modulus, $\kappa = 3 - 4\mu$ for plane deformation, $\kappa = (3 - \mu)/(1 + \mu)$ for the generalized plane stressed state, μ is Poisson's ratio, and s_n is the arc abscissa of a point τ_n on the contour L_n . Hence, according to the boundary condition (6), the final expressions for the complex potentials $\Phi_1(z)$ and $\Psi_1(z)$ take the form

$$\begin{aligned}\Phi_1(z) &= \sum_{k=1}^{M'} [A\{L_k^*\} g'_{1k}(t_k) + A\{L_k\} g'_{2k}(t_k)] + \sum_{k=M'+1}^M A\{L_k\} g'_k(t_k), \\ \Psi_1(z) &= \sum_{k=1}^{M'} [B\{L_k^*\} g'_{1k}(t_k) + B\{L_k\} g'_{2k}(t_k)] + \sum_{k=M'+1}^M B\{L_k\} g'_k(t_k).\end{aligned}\tag{16}$$

Punch Pressed into the Half Plane in the Presence of Friction

Assume that the punch is in the state of limiting equilibrium with respect to shear and, for the horizontal component of shearing forces, we have $Q = \rho P$ (see Fig. 1). For the problem under consideration, boundary conditions can be represented in the form (1), (3), (4)–(8).

Let $p(x)$ be unknown pressure under the punch in the contact region $|x - x_0| \leq c$. Then, by virtue of (9), we can write

$$p_0(x) = -(1 + i\rho)p(x), \quad |x - x_0| \leq c.$$

By substituting the complex potentials (11), (12), and (16) in the boundary conditions (4), (5), and (8), we arrive at the following system of $\sum_{n=1}^{M'} S_n + 2M - M'$ singular integral equations:

$$\operatorname{Re} [D(\tau_n) - P_n(\tau_n)] = 0, \quad \tau_n \in L_n^*, \quad n = \overline{1, M'}, \quad (17)$$

$$\operatorname{Im} [D(\tau_n) - P_n(\tau_n)] = 0, \quad \tau_n \in L_n, \quad n = \overline{1, M'}, \quad (18)$$

$$D(\tau_n) - P_n(\tau_n) = 0, \quad \tau_n \in L_n, \quad n = \overline{M' + 1, M}, \quad (19)$$

in the unknown functions $g_{1n}(\tau_n)$ on the open intervals $a_{ns}^* b_{ns}^*$ ($n = \overline{1, M'}, s = \overline{1, S_n}$) of the cuts, $g_{2n}(\tau_n)$ on the contours L_n ($n = \overline{1, M'}$), and $g'_n(\tau_n)$ on the contours L_n ($n = \overline{M' + 1, M}$). Here,

$$D(\tau_n) = \sum_{k=1}^{M'} [I_n\{L_k^*\} g'_{1k}(t_k) + I_n\{L_k\} g'_{2k}(t_k)] + \sum_{k=M'+1}^M I_n\{L_k\} g'_k(t_k),$$

$$P_n(\tau_n) = \int_{x_0-c}^{x_0+c} \left\{ \operatorname{Im} \frac{1-i\rho}{\zeta - \bar{T}'_n} + \frac{\operatorname{Im}[(1+i\rho)(\zeta - \bar{T}'_n)]}{(\zeta - \bar{T}'_n)^2} \frac{d\bar{\tau}_n}{d\tau_n} e^{-2i\alpha_n} \right\} p(\zeta) d\zeta,$$

$$T'_n = \tau_n e^{i\alpha_n} + z_n^0,$$

and the operator $I_n\{L_k\}$ is defined by the formula

$$I_n\{L_k\} \varphi_k(t_k) = \int_{L_k} [R_{nk}(t_k, \tau_n) \varphi_k(t_k) dt_k + S_{nk}(t_k, \tau_n) \overline{\varphi_k(t_k)} d\bar{t}_k],$$

where [13]

$$\begin{aligned} R_{nk}(t_k, \tau_n) &= \frac{e^{i\alpha_k}}{2} \left\{ \frac{1}{T_k - T'_n} - \frac{1}{\bar{T}_k - \bar{T}'_n} - 2i \frac{\operatorname{Im} T_k}{(T_k - \bar{T}'_n)^2} \right. \\ &\quad \left. - \left[\frac{1}{T_k - \bar{T}'_n} - \frac{1}{\bar{T}_k - \bar{T}'_n} - 2i \operatorname{Im} T_k \frac{T_k + \bar{T}'_n - 2T'_n}{(T_k - \bar{T}'_n)^3} \right] \frac{d\bar{\tau}_n}{d\tau_n} e^{-2i\alpha_n} \right\}, \\ S_{nk}(t_k, \tau_n) &= \frac{e^{-i\alpha_k}}{2} \left\{ \frac{1}{\bar{T}_k - \bar{T}'_n} - \frac{1}{T_k - T'_n} + 2i \frac{\operatorname{Im} T_k}{(\bar{T}_k - T'_n)^2} \right. \\ &\quad \left. - \left[\frac{T_k - T'_n}{(T_k - \bar{T}'_n)^2} + \frac{\bar{T}_k - T'_n}{(\bar{T}_k - \bar{T}'_n)^2} \right] \frac{d\bar{\tau}_n}{d\tau_n} e^{-2i\alpha_n} \right\}, \quad T'_n = \tau_n e^{i\alpha_n} + z_n^0. \end{aligned}$$

To complete system (17)–(19), one must add the following equations:

$$\operatorname{Im} \left[g_{1n}(t_n) \frac{d\bar{\tau}_n}{ds_n} \right] = 0, \quad \tau_n \in L_n^*, \quad n = \overline{1, M'}, \quad (20)$$

$$\operatorname{Re} \left[g_{2n}(t_n) \frac{d\bar{\tau}_n}{ds_n} \right] = 0, \quad \tau_n \in L_n, \quad n = \overline{1, M'}, \quad (21)$$

which immediately follow from relations (14) and (15), and an equation for the unknown pressure $p(x)$. The latter can be derived from the first boundary condition in (1). For this purpose, by using the Sokhotskii–Plemelj formula, we take the derivative of a complex combination of displacements on the boundary of the half plane with respect to x . This yields

$$2G \left[\frac{du_x(x)}{dx} + i \frac{du_y(x)}{dx} \right] = \frac{\kappa - 1}{2} p_0(x) + \frac{\kappa + 1}{\pi} i \left[W(x) + \frac{1}{2} \int_{x_0-c}^{x_0+c} \frac{p_0(\zeta)}{\zeta - x} d\zeta \right], \quad y = 0, \quad (22)$$

where

$$W(x) = \sum_{k=1}^{M'} [U\{L_k^*\} g'_{1k}(t_k) + U\{L_k\} g'_{2k}(t_k)] + \sum_{k=M'+1}^M U\{L_k\} g'_k(t_k)$$

and the operator $U\{L_k\}$ is defined by the formula

$$U\{L_k\} \varphi_k(t_k) = \int_{L_k} \left[\operatorname{Im} \left(\frac{1}{\bar{T}_k - x} \right) e^{i\alpha_k} \varphi_k(t_k) dt_k + \frac{\operatorname{Im} T_k}{(\bar{T}_k - x)^2} e^{-i\alpha_k} \overline{\varphi_k(t_k)} d\bar{t}_k \right].$$

As a result, we arrive at the following singular equation:

$$2 \operatorname{Re} W(x) - \pi \frac{\kappa - 1}{\kappa + 1} \rho p_0(x) - \int_{x_0-c}^{x_0+c} \frac{p(\zeta)}{\zeta - x} d\zeta = \frac{4\pi G}{\kappa + 1} f'(x), \quad |x - x_0| \leq c. \quad (23)$$

In addition, the solutions of the system of singular integral equations (17)–(19), (20), (21), and (23) must satisfy the conditions of continuity of the displacements of crack lips at the ends of the contours of integration, i.e.,

$$\int_{a_{ns}^* b_{ns}^*} g'_{1n}(t_n) dt_n = 0, \quad n = \overline{1, M'}, \quad s = \overline{1, s_n}, \quad (24)$$

$$\int_{L_n} g'_{2n}(t_n) dt_n = 0, \quad n = \overline{1, M'}, \quad (25)$$

$$\int_{L_n} g'_n(t_n) dt_n = 0, \quad n = \overline{M' + 1, M}, \quad (26)$$

and the condition of equilibrium of the punch

$$\int_{x_0-c}^{x_0+c} p(\zeta) d\zeta = P.$$

Punch Engaged with the Half Plane

We now consider the case where a punch pressed into the boundary of a half plane with a system of curvilinear cuts is rigidly engaged with the half plane. Boundary conditions of the problem are given by Eqs. (2), (3), and (4)–(8). In this case, our aim is to determine contact forces under the punch, i.e.,

$$p_0(x) = N(x) + iT(x), \quad |x - x_0| \leq c,$$

and the derivatives of singular displacements on the contours of the cuts.

By substituting the complex potentials (11), (12), and (16) in the boundary conditions (4), (5), and (8), we arrive at the system of singular integral equations (17)–(19) with

$$P_n(\tau_n) = \int_{x_0-c}^{x_0+c} \left\{ \operatorname{Im} \frac{p_0(\zeta)}{\zeta - \bar{\tau}_n'} - \frac{\operatorname{Im}[(\zeta - \bar{\tau}_n') p_0(\zeta)]}{(\zeta - \bar{\tau}_n')^2} \frac{d\bar{\tau}_n}{d\tau_n} e^{-2i\alpha_n} \right\} d\zeta.$$

This system should also be supplemented with Eqs. (20) and (21). As in the previous case, by using the boundary condition (2) and relation (22), we arrive at the following integral equation:

$$2W(x) - \pi i \frac{\kappa - 1}{\kappa + 1} p_0(x) + \int_{x_0-c}^{x_0+c} \frac{p_0(\zeta)}{\zeta - x} d\zeta = \frac{4\pi G}{\kappa + 1} f'(x), \quad |x - x_0| \leq c.$$

Moreover, it is necessary to add conditions (24)–(26) and the condition of equilibrium of the punch

$$\int_{x_0-c}^{x_0+c} p_0(\zeta) d\zeta = -(P + iQ).$$

Numerical solutions of the resulting system of singular integral equations can be obtained by the method of mechanical quadratures. These solutions enable one to analyze the stressed states of the half plane with cuts under various conditions of contact of the punch with the boundary of the half plane.

Example

Consider a rigid punch with flat horizontal base pressed without friction into an elastic half plane weakened by a subsurface internal vertical crack of length $2l$ [14]. In the half plane, we introduce a coordinate system xOy whose y -axis coincides with the axis of the punch, and the crack is related to a local coordinate system $x'O'y'$ depicted in Fig. 2. The location of the crack is determined by the distance d_1 from its axis to the axis of the punch and the distance d_2 from the nearest crack tip to the boundary of the half plane. The punch is subjected to the action of a force P which is, actually, the resultant of pressure forces $p(x)$ formed in the contact region $|x| \leq c$. Since there is no friction between the punch and the half plane, tangential forces under the punch are equal to zero.

By analyzing the stressed state of a continuous half plane under the action of a punch, we discovered that normal stresses on the crack line are compressive. We assume that the effect of the crack on the distribution of pressure under the punch is negligible and that the crack lips are in contact along the entire length of the crack. As a result, by using relations (1), (3), and (4)–(8), we can write boundary conditions of the problem in the form

$$\frac{du_y(x)}{dx} = 0, \quad T(x) = 0, \quad |x| \leq c,$$

$$N(x) + iT(x) = 0, \quad |x| > c,$$

$$T^\pm(x') = 0, \quad |x'| < l,$$

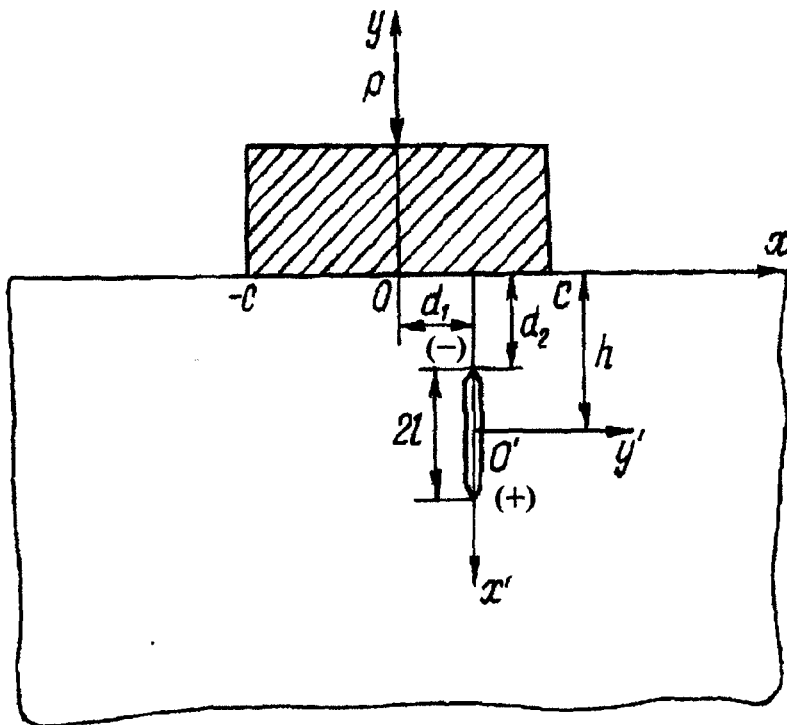


Fig. 2. Flat punch on the boundary of a half plane with internal vertical crack.

$$u_y^+(x') - u_y^-(x') = 0, \quad |x'| < l,$$

$$N^+(x') - N^-(x') = 0, \quad |x'| < l.$$

By virtue of relations (17)–(19), (20), (21), and (23) with $M = 1$ and $\rho = 0$, we obtain the singular integral equations of the problem

$$\int_{-l}^l R(t, \tau) \varphi_2(t) dt + \int_{-c}^c V(\zeta, \tau) p(\zeta) d\zeta = 0, \quad |\tau| < l, \quad (27)$$

$$\int_{-l}^l H(t, \tau) \varphi_2(t) dt - \int_{-c}^c \frac{p(\zeta)}{\zeta - x} d\zeta = 0, \quad |x| \leq c, \quad (28)$$

where

$$R(t, \tau) = \frac{1}{t - \tau} + \frac{1}{t + \tau + 2h} + 2 \frac{t + h}{(t + \tau + 2h)^2} - 4 \frac{(t + h)^2}{(t + \tau + 2h)^3},$$

$$V(\zeta, \tau) = -\operatorname{Im} \frac{\tau + h}{(\zeta - \bar{T}')^2}, \quad T' = d_1 - i(\tau + h),$$

$$H(t, x) = 2 \left[\operatorname{Im} \frac{1}{T - x} - \operatorname{Re} \frac{t + h}{(\bar{T} - x)^2} \right],$$

in the unknown functions

$$\varphi_2(t) = -\frac{2G}{1+\kappa} \frac{d}{d\tau} [u_x^+(\tau) - u_x^-(\tau)], \quad |\tau| < l,$$

and $p(x)$, $|x| \leq c$. In this case, the solution of system (27), (28) must satisfy the conditions

$$\int_{-l}^l \varphi_2(t) dt = 0 \quad (29)$$

and

$$\int_{-c}^c p(\zeta) d\zeta = P. \quad (30)$$

By the change of variables

$$t = l\xi, \quad \zeta = c\xi, \quad |\xi| \leq 1,$$

$$\tau = l\eta, \quad x = c\eta, \quad |\eta| < 1,$$

one can rewrite the integral equations (27) and (28) and the additional conditions (29) and (30) in the following normalized form:

$$\int_{-1}^1 [R(\xi, \eta) \varphi_2(\xi) dt + V(\xi, \eta) p(\xi)] d\xi = 0, \quad |\eta| < 1, \quad (31)$$

$$\int_{-1}^1 \left[H(\xi, \eta) \varphi_2(\xi) - \frac{p(\xi)}{c\xi - c\eta} \right] d\xi = 0, \quad |\eta| < 1, \quad (32)$$

$$\int_{-1}^1 \varphi_2(\xi) d\xi = 0, \quad (33)$$

$$\int_{-1}^1 p(\xi) d\xi = P, \quad (34)$$

where, for the sake of convenience, we preserve the same notation of functions, i.e.,

$$R(\xi, \eta) = R(l\xi, l\eta), \quad V(\xi, \eta) = V(c\xi, c\eta), \quad H(\xi, \eta) = H(l\xi, l\eta),$$

$$\varphi_2(\xi) = l\varphi_2(l\xi), \quad p(\xi) = cp(c\xi).$$

Let us now represent the root singularities of the functions $\varphi_2(\eta)$ and $p(\eta)$ at the points $\eta = -1$ and $\eta = 1$ in the explicit form, i.e.,

$$\varphi_2(\eta) = \frac{u(\eta)}{\sqrt{1-\eta^2}} \quad \text{and} \quad p(\eta) = \frac{q(\eta)}{\sqrt{1-\eta^2}}, \quad (35)$$

where the functions $u(\eta)$ and $q(\eta)$ are continuous on the interval $[-1, 1]$.

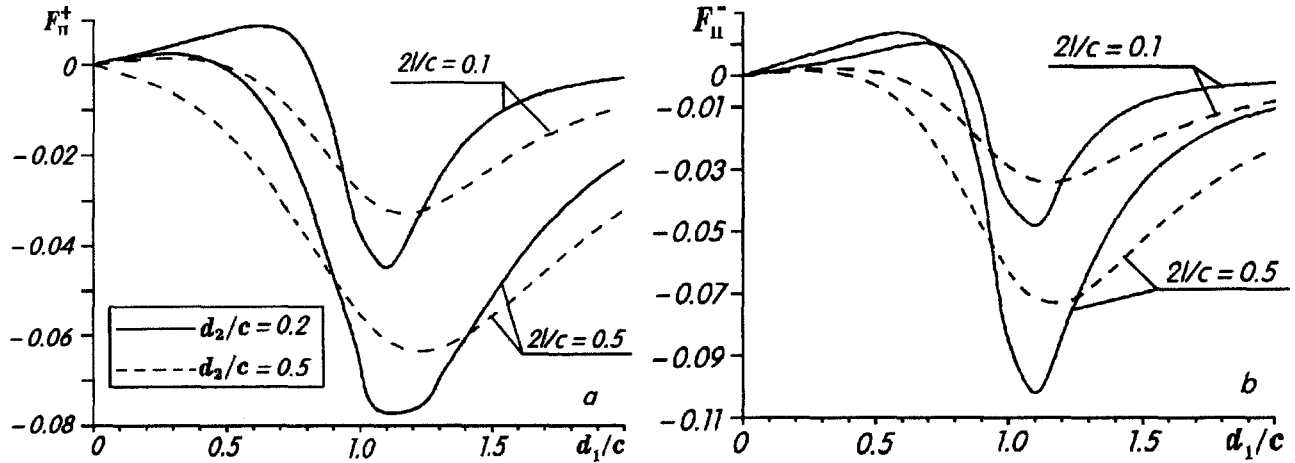


Fig. 3. Dependences of the normalized mode II stress intensity factor $F_{II}^{\pm} = K_{II}^{\pm} \sqrt{c} / (P \sqrt{\pi})$ on the relative distance d_1/c from the crack to the axis of the punch. Signs “+” and “-” correspond to the right and left tips of the crack, respectively.

By applying the Gauss quadrature formulas [12] to the integral equations (31)–(34) and taking relations (35) into account, we obtain a system of $2N$ linear algebraic equations in $2N$ unknowns $u(\xi_k)$ and $q(\xi_k)$, $k = \overline{1, N}$, namely,

$$\sum_{k=1}^N [R(\xi_k, \eta_m) u(\xi_k) + V(\xi_k, \eta_m) q(\xi_k)] = 0 \quad (m = \overline{1, N-1}),$$

$$\sum_{k=1}^N \left[H(\xi_k, \eta_m) u(\xi_k) - \frac{q(\xi_k)}{c\xi_k - c\eta_m} \right] = 0 \quad (m = \overline{1, N-1}),$$

$$\sum_{k=1}^N u(\xi_k) = 0,$$

$$\pi \sum_{k=1}^N q(\xi_k) = NP.$$

By using the solution of this system of equations and the formula for Lagrange interpolating polynomials presented in [12], we obtain the following expressions for the stress intensity factors at the crack tips:

$$K_{II}^{\pm} = \pm \sqrt{\frac{\pi}{l}} u(\pm 1)$$

and pressure in the contact region:

$$p(\eta) = -\frac{T_N(\eta)}{N\sqrt{1-\eta^2}} \sum_{k=1}^N \left[(-1)^k \frac{\sqrt{1-\xi_k^2}}{\eta-\xi_k} q(\xi_k) \right], \quad |\eta| < 1.$$

Plots of the normalized stress intensity factors $F_{II}^{\pm} = K_{II}^{\pm} \sqrt{c} / (P\sqrt{\pi})$ as functions of the relative distance d_1/c from the crack to the axis of the punch are displayed in Fig. 3a, b for various crack lengths and distances d_2 . The analysis of numerical data demonstrates that the intensity of tangential stresses at the crack tips is maximum if the crack is located near the edge of the punch. In particular, if the crack is oriented along the y -axis ($d_1 = 0$), then in view of the symmetry of the problem, the stress intensity factors K_{II} are equal to zero at both tips. In this case, the crack is no longer a stress concentrator and does not affect the distribution of pressure under the punch.

It is worth noting that a similar problem was studied in [2] for a special case of symmetrically located crack ($d_1 = 0$). However, the result obtained there is incorrect.

CONCLUSIONS

On the basis of the representation of complex stress potentials in terms of Cauchy-type integrals of contact stresses formed under the punch and derivatives of singular displacements on the crack lips, we deduced singular integral equations of the contact problem of the theory of elasticity for an isotropic half plane with a system of rectilinear cuts suffering on its boundary the action of a rigid punch with base of arbitrary convex shape. We consider the case of friction forces acting according to the Coulomb law on the interface of the punch and the half plane and the case where the punch is rigidly engaged with the half plane. It is assumed that crack lips are under the conditions of smooth contact in the compression zones.

In the case where a punch with flat horizontal base is pressed without friction into a half plane weakened by a subsurface internal vertical crack, it is shown that the crack lips are in contact along the entire length of the crack for all locations of the crack in the vicinity of the punch base. The intensity of tangential stresses at the crack tips is maximum when the crack is located near the edge of the punch. Cracks oriented along the axis of the punch lose the properties of stress concentrators and do not affect the distribution of pressure under the punch.

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