

where  $\zeta = \exp(-i\pi\eta)$  ( $|\eta| \leq 1$ );  $2a$  is a linear parameter denoting, respectively, the length of a side of the square (for the first formula) or the length of the diagonal of the square (for the second formula) (Figs. 2 and 3). Taking symmetry of the problem into account, we obtain the stress intensity coefficients using an algorithm for numerically solving integral equations for open curvilinear cuts, as presented in [5].

Table 1 presents values of the dimensionless stress intensity coefficients  $Y^\pm = K_I^\pm \sqrt{\pi l}/F$  for the exterior ( $Y^-$ ) and interior ( $Y^+$ ) mutually admissible domains considered with a boundary crack. Data appearing above and below the horizontal line segments correspond to the cases of Figs. 2 and 3. Here  $\lambda = l/a$ ;  $\varepsilon = \xi/l$ .

If the parametric equations of the boundaries of the squares are chosen as in [4],

$$t_1 = 168c[\zeta^{-1} - \zeta^3/6 + \zeta^7/56]/143; \quad t_1 = 168c[\zeta^{-1} + \zeta^3/6 + \zeta^7/56]/199,$$

then the resulting numerical values of  $Y_*^\pm = K_I^\pm \sqrt{\pi l}/F$  differ little from the values of  $Y^\pm = K_I^\pm \sqrt{\pi l}/F$ , given in Table 1; this follows, respectively, from Table 2, where values of the indicated intensity coefficients are given for various values of  $\varepsilon$ .

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#### STATIC TRAJECTORIES OF THE DEVELOPMENT OF TWO STAGGERED PARALLEL CRACKS

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UDC 539.375

The problem of the development of two staggered parallel cracks in a finite plate subjected to uniaxial static tension at infinity is examined on the basis of the method of singular integral equations. The trajectories are calculated by the step-by-step method using the criterion of maximum circumferential stresses. In this case, a segment of the trajectory is approximated by a third-degree polynomial in each step. The results of numeric calculations are presented.

Investigation of the laws of crack propagation and determination of the stress-intensity factors (SIF) along the trajectories of developing cracks is the initial stage in calculating the strength and longevity of structural components. The development of general methods of solving two-dimensional problems of the theory of elasticity for arbitrary regions with smooth and piecewise-smooth curvilinear cuts (cracks), for example, the method of singular integral equations [1, 2], will permit the effective solution of the problem of calculating trajectories by the step-by-step method, when the shape of the crack is determined with allowance for stress redistribution during its development [2-5]. In this case, one of the local failure criteria is used in each step to determine the direction of the increase in the crack at its tip.

Let us examine the problem of the development of two staggered parallel cracks in a plate subjected to static tension at infinity. In this case, we can use the  $\sigma_\theta$ -criterion,

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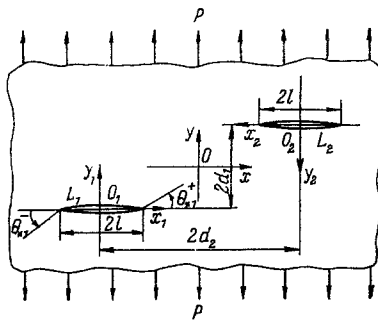


Fig. 1

which predicts the direction of crack development most adequately for the majority of materials where the stress field is asymmetric with respect to the crack [4, 6]. According to this criterion, the crack begins to grow in the direction perpendicular to the maximum circumferential tensile stress  $\sigma_\theta$ , when the factor  $K_{I0} = \lim_{r \rightarrow 0} \sqrt{2\pi r} \sigma_\theta$  reaches the critical value  $K_{IC}$ .

Let there be two rectilinear cracks of equal length  $2l$ , which are parallel to the  $Ox$  axis, in an infinite elastic isotropic plane referenced to an  $xOy$  coordinate system. The contours  $L_1$  and  $L_2$  of the cracks are linked with the local coordinate systems  $x_1O_1y_1$  and  $y_2O_2y_2$ , whose origins are positioned symmetrically with respect to the origin of the basic  $xOy$  coordinate system (Fig. 1). The mutual arrangement of the cracks is characterized by the parameters  $\varepsilon_1 = d_1/l$  and  $\varepsilon_2 = d_2/l$ . The problem consists of determining the trajectories of crack development under the given conditions.

Since the edges of the cracks are load-free in the initial state and during development, the boundary conditions assume the form

$$N^\pm(t) + iT^\pm(t) = 0, \quad t \in L_n (n = 1, 2). \quad (1)$$

Savruk [2] and Savruk and Osiv [3] propose an algorithm for the stepwise plotting of static trajectories of crack development with the same mechanical conditions at the ends of the cracks. Assuming the distances  $d_1$  and  $d_2$  to be such that the cracks develop simultaneously only from the inner (near) tips, and the further tips are stationary, we arrive at the conditions for the applicability of this algorithm. The range of the corresponding distances can be pre-established, analyzing the SIF and critical load values for the inner and outer tips, which are given in [1, 2] for two staggered parallel cracks.

Since the geometric-force diagram of the stated problem exhibits symmetry about the origin of the  $xOy$  coordinate system, it is sufficient to determine the trajectory of the development of one of the cracks, for example, the one referenced to the local system  $x_1O_1y_1$ .

According to Savruk [2] and Savruk and Osiv [3], the SIF can be determined in each step of trajectory plotting from solution of singular integral equations of the plane problem of the theory of elasticity for a given region with cuts. We can then find the angle of the initial direction of crack development and construct the next increment of the trajectory in this direction, which is approximated by a third-degree polynomial.

Using singular integral equations of the problem for two curvilinear cracks in an infinite plane with assigned boundary conditions [2] and assuming symmetry, we arrive at the solution of one equation in each step of the trajectory plotting, which in normalized form, is represented as

$$\int_{-1}^1 [R(\xi, \eta) \varphi(\xi) + S(\xi, \eta) \bar{\varphi}(\xi)] d\xi = \pi P(\eta), \quad |\eta| < 1, \quad (2)$$

where

$$R(\xi, \eta) = \operatorname{Re} \left[ \frac{\omega'(\eta)}{\omega(\xi) - \omega(\eta)} + \frac{\omega'(\eta)}{a(\xi, \eta)} \right]; \quad S(\xi, \eta) = \frac{1}{2} \frac{\partial}{\partial \eta} \left[ \frac{a(\xi, \eta)}{a(\xi, \eta)} - \frac{\omega(\xi) - \omega(\eta)}{\omega(\xi) - \omega(\eta)} \right]; \quad a(\xi, \eta) = \omega(\xi) + \omega(\eta) - i\varepsilon_1 - \varepsilon_2. \quad (3)$$

Here,  $\varphi(\xi) = g'(t) \omega'(\xi)$ ,  $g'(t)$  is the unknown density of complex Muskhelishvili potentials. The parametric equation of the contour of a developing crack in the  $j$ -th step is represented by the relationship

$$t = \omega(\xi) = x_1(\xi) + iy_1(x_1(\xi)), \quad |\xi| \leq 1, \quad (4)$$

where

$$x_{1j}(\xi) = \frac{(x_{1j} + l)\xi + x_{1j} - l}{2}, \quad x_{1j} = (j-1)h + l; \quad (5)$$

$x_{1j}$  is the abscissa of the end of the crack in the  $j$ -th step in the  $x_1O_1y_1$  coordinate system,  $h$  is the growth interval of the trajectory ( $h > 0$ ), and  $y_1(x_1(\xi))$  is the equation of the developing crack in the  $[-1, 1]$  interval, whose parameters are determined from the conditions of the trajectory's smoothness. The right side of Eq. (2) is calculated from the formula

$$P(\eta) = p(\eta)\omega'(\eta), \quad (6)$$

in which the quantity  $P(\eta)$  is determined from the relationship [2]:

$$p(\eta) = -\Phi_0'(t') - \overline{\Phi_0'(t')} - [t'\overline{\Phi_0'(t')} + \overline{\Psi_0'(t')}] \frac{dt'}{d\eta} = -p \operatorname{Re} \omega'(\eta) / \omega'(\eta), \quad t' = \omega(\eta), \quad |\eta| < 1. \quad (7)$$

Here,  $\Phi_0(z)$  and  $\Psi_0(z)$  are complex Muskhelishvili potentials, which characterize the stress state of the cut-free plane. The solution of Eq. (2) should satisfy the additional condition

$$\int_{-1}^1 \varphi(\xi) d\xi = 0, \quad (8)$$

which ensures the uniqueness of the displacements in tracing the boundary of the developing crack.

Integral equation (2) can be solved numerically by the method of Gauss-Chebyshev mechanical quadratures [1, 2]. The solution will assume the form

$$\varphi(\eta) = \frac{u(\eta)}{\sqrt{1-\eta^2}}, \quad (9)$$

where  $u(\eta)$  is a function that is continuous in the  $[-1, 1]$  interval and that is represented by a Lagrangian interpolation polynomial in terms of Chebyshev nodes. From relationships (2) and (8), we therefore arrive at the following system of algebraic equations for determination of the unknowns  $u(\xi_k)$  ( $k = 1, N$ ):

$$\begin{aligned} \sum_{k=1}^N [R(\xi_k, \eta_m) u(\xi_k) + S(\xi_k, \eta_m) \overline{u(\xi_k)}] &= NP(\eta_m), \quad m = \overline{1, N-1}; \\ \sum_{k=1}^N u(\xi_k) &= 0, \end{aligned} \quad (10)$$

where the nodal points  $\xi_k, \eta_m$  are determined by the relationships

$$\xi_k = \cos \frac{2k-1}{2N} \pi \quad (k = \overline{1, N}); \quad \eta_m = \cos \frac{\pi m}{N} \quad (m = \overline{1, N-1}).$$

Based on the solution of system (10), we can calculate the SIF for the right (+) and left (-) tips of the developing crack from the equation [2]

$$K_I^\pm - iK_{II}^\pm = \mp \frac{\sqrt{\pi} \omega'(\pm 1)}{\omega'(\pm 1)} u(\pm 1). \quad (11)$$

Then, from the equation of limiting equilibrium [1, 6]

$$\cos^2 \frac{\theta_*}{2} \left( K_I - 3K_{II} \tan \frac{\theta_*}{2} \right) = K_{Ic}, \quad (12)$$

where the angle of initial crack development

$$\theta_* = f(K_I, K_{II}) = 2 \arctan \frac{K_I - \sqrt{K_I^2 + 8K_{II}^2}}{4K_{II}} \quad (13)$$

is determined in accordance with the  $\sigma_\theta$ -criterion, we find the values of  $\theta_*^\pm$  and the limiting values of the external load  $p_*^\pm$  for both crack tips.

The critical loads  $p_*^+$  and  $p_*^-$  were determined in each step of the trajectory plot to confirm the condition

$$p_*^+ < p_*^-. \quad (14)$$

When this condition was satisfied, it was assumed that the crack develops only from the right side.

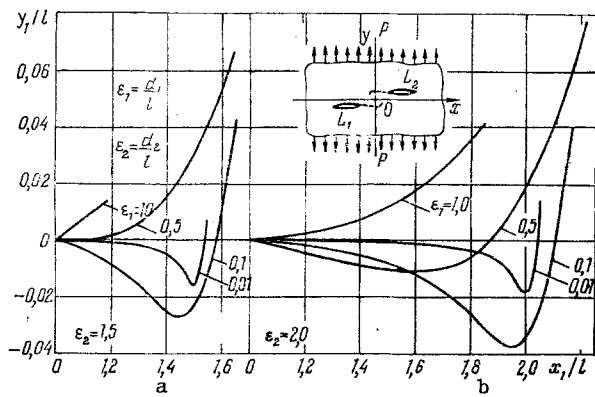


Fig. 2

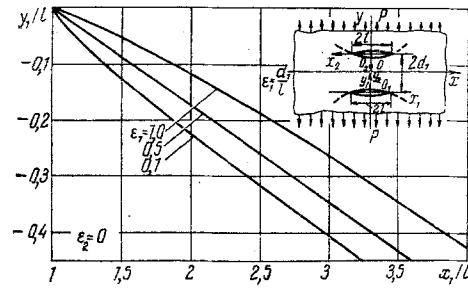


Fig. 3

The static trajectories of initial-crack development from the right tip when  $x_1/l \geq 1$  are shown in Fig. 2 for different relative distances between cracks along the vertical ( $\epsilon_1 = d_1/l$ ) and horizontal ( $\epsilon_2 = d_2/l$ ). Here, Fig. 2a and b correspond to  $\epsilon_2$  values equal to 1.5 and 2.0, respectively. The trajectories of crack development in Fig. 2 terminate at points for which condition (14) is violated.

Analysis of the behavior of the curves indicates deviation, of the near tips of the developing cracks from the lines of their initial disposition in the direction of the neighboring crack, [observed] for small intercrack distances along the vertical ( $\epsilon_1 = 0.01, 0.1$ , and  $0.5$ ); this agrees with familiar conclusions in the literature [1, 2] and corresponds to the phenomenon of crack "repulsion," which is observed in experimental investigations [7]. Thereafter, these deviations in the trajectories attain the maximum (for each value of the parameter  $\epsilon_2$ ), and the trajectories then turn in the direction of the neighboring crack. For small values of the parameter  $\epsilon_1$  ( $\epsilon_1 = 1.0$ ), each of the cracks begins to develop immediately in the direction of the neighboring crack; however, (Fig. 2) the increase in the trajectory is relatively small prior to the point at which condition (14) is violated. Thereafter, the cracks develop from both tips simultaneously (this case is not considered here).

If the initial cracks are closer to one another along the horizontal ( $\epsilon_2 = 1.5$ ), condition (14) will be violated sooner than in the case when they are more remote ( $\epsilon_2 = 2.0$ ), although for both cases, the character of the trajectories of crack development is similar for fixed values of the parameter  $\epsilon_1$ .

The results obtained indicate that when an infinite plane with two staggered parallel cracks is subjected to static tension perpendicular to the lines of the cracks when failure initiative belongs to the internal tips, each of the cracks is directed not toward the tip of the neighboring crack, but to the near edge of its contour, attempting to bypass the tip. Even for an insignificant deviation ( $\epsilon_1 = 0.01$ ) of colinear cracks ( $\epsilon_1 = 0$ ) from their initial disposition, the observed "repulsion" phenomenon suggests the instability of a rectilinear trajectory for the development of colinear cracks.

Corresponding SIF values  $K_I^\pm$  ( $K_I^\pm = \rho\sqrt{\pi l} K_I^\pm$ ) as a function of the abscissa  $x_1/l$  for the right and left tips of developing crack  $L_1$  are given in Table 1. The numeric data indicate that the overlapping of contours of developing cracks significantly affects the character of SIF behavior. For small differences along the vertical ( $\epsilon_1 = 0.01, 0.1, 0.5$ ), the SIF for the internal tips increase monotonically and reach maximum values when the tips of the developing cracks approach the median line (Oy-axis), and then decrease monotonically in the region of overlap.

For large distances between initial rectilinear cracks along the vertical ( $\epsilon_1 = 1.0$ ), the near tips of the developing cracks do not reach the median line, i.e., there is no overlap zone of the contours, and the SIF  $K_I^\pm$  increases monotonically near both tips.

Note that in the vicinity of the overlap of the crack contours, the values of the SIF  $K_I^\pm$ , which correspond to the relative distance  $\epsilon_2 = 2.0$ , are larger than similar  $K_I^\pm$  values when  $\epsilon_2 = 1.5$ , since the dimensions of the crack will be large in the first case.

It should be pointed out that the stress-strain state in the neighborhoods of the tips of developing cracks is characterized primarily by the factors  $K_I^\pm$ , or the factors  $K_I^\pm$  are small (fluctuate about the zero value). The inequality  $K_I^+ > K_I^-$  can therefore be used in place of condition (14).

TABLE 1

| $\frac{x_1}{l}$       | $\varepsilon_1$ |         |         |         |         |         |         |         |
|-----------------------|-----------------|---------|---------|---------|---------|---------|---------|---------|
|                       | 0,01            |         | 0,1     |         | 0,5     |         | 1,0     |         |
|                       | $k_1^+$         | $k_1^-$ | $k_1^+$ | $k_1^-$ | $k_1^+$ | $k_1^-$ | $k_1^+$ | $k_1^-$ |
| $\varepsilon_2 = 1,5$ |                 |         |         |         |         |         |         |         |
| 1,0                   | 1,112           | 1,051   | 1,117   | 1,052   | 1,126   | 1,061   | 1,056   | 1,048   |
| 1,1                   | 1,179           | 1,088   | 1,188   | 1,089   | 1,188   | 1,099   | 1,086   | 1,080   |
| 1,2                   | 1,270           | 1,129   | 1,290   | 1,132   | 1,245   | 1,141   |         |         |
| 1,3                   | 1,412           | 1,176   | 1,462   | 1,182   | 1,306   | 1,185   |         |         |
| 1,4                   | 1,720           | 1,238   | 1,790   | 1,251   | 1,356   | 1,232   |         |         |
| 1,5                   | 3,9             | 1,400   | 2,168   | 1,354   | 1,381   | 1,281   |         |         |
| 1,6                   |                 |         | 1,955   | 1,477   | 1,370   | 1,329   |         |         |
| $\varepsilon_2 = 2,0$ |                 |         |         |         |         |         |         |         |
| 1,0                   | 1,048           | 1,028   | 1,049   | 1,028   | 1,057   | 1,032   | 1,049   | 1,032   |
| 1,1                   | 1,084           | 1,057   | 1,086   | 1,058   | 1,096   | 1,062   | 1,082   | 1,063   |
| 1,2                   | 1,124           | 1,088   | 1,126   | 1,088   | 1,139   | 1,095   | 1,116   | 1,094   |
| 1,3                   | 1,168           | 1,119   | 1,170   | 1,120   | 1,187   | 1,127   | 1,151   | 1,124   |
| 1,4                   | 1,218           | 1,152   | 1,223   | 1,152   | 1,240   | 1,161   | 1,184   | 1,156   |
| 1,5                   | 1,276           | 1,186   | 1,286   | 1,187   | 1,300   | 1,198   | 1,217   | 1,187   |
| 1,6                   | 1,354           | 1,222   | 1,368   | 1,224   | 1,365   | 1,235   | 1,246   | 1,219   |
| 1,7                   | 1,457           | 1,261   | 1,487   | 1,265   | 1,435   | 1,277   | 1,273   | 1,252   |
| 1,8                   | 1,624           | 1,308   | 1,691   | 1,315   | 1,503   | 1,320   | 1,294   | 1,284   |
| 1,9                   | 1,990           | 1,370   | 2,063   | 1,384   | 1,561   | 1,368   |         |         |
| 2,0                   | 4,5             | 1,537   | 2,5     | 1,487   | 1,593   | 1,416   |         |         |
| 2,1                   |                 |         | 2,3     | 1,612   | 1,588   | 1,467   |         |         |
| 2,2                   |                 |         |         |         | 1,532   | 1,516   |         |         |

Analysis of the behavior of the curves (Fig. 2) of SIF values for all  $\varepsilon_1$  and  $\varepsilon_2$  examined, including those for colinear cracks, makes it possible to propose that the linking of two staggered parallel cracks at one defect prior to the start from the outer tips occurs only in the case when the cracks are located on the same line.

In conclusion, let us examine the special case (Fig. 3), when the cracks are positioned strictly one above the other ( $\varepsilon_2 = 0$ ). In this case, there is an additional symmetry condition [ $u(\xi) = -\bar{u}(-\xi)$ ], which ensures a similar state at both tips of each of the cracks. Thus, the cracks will develop from both ends simultaneously symmetrically about the axis of ordinates of the corresponding local coordinate systems, and the values of the SIF  $K_I^+$  and the limiting loads will be the same for both ends.

Considering the problem's symmetry for even  $N$ , the number of algebraic equations of system (10) is halved:

$$\sum_{m=1}^{N/2} [R(\xi_k, \eta_m) - S(-\xi_k, \eta_m)] u(\xi_k) + [S(\xi_k, \eta_m) - R(-\xi_k, \eta_m)] \bar{u}(\xi_k) = NP(\eta_m), \quad m = \overline{1, N/2-1}; \quad (15)$$

$$\sum_{k=1}^{N/2} [\operatorname{Re} \{ [R(\xi_k, 0) - S(-\xi_k, 0)] u(\xi_k) + [S(\xi_k, 0) - R(-\xi_k, 0)] \bar{u}(\xi_k) \} + i \operatorname{Im} u(\xi_k)] = \operatorname{Re} P(0).$$

According to (11),

$$K_I^+ - iK_{II}^+ = \frac{1}{N} \frac{\sqrt{\pi_1 \omega'(1)}}{\omega'(1)} \sum_{k=1}^{N/2} (-1)^k \left[ u(\xi_k) \cot \frac{2k-1}{4N} \pi + \bar{u}(\xi_k) \tan \frac{2k-1}{4N} \pi \right]; \quad K_I^- = K_I^+; \quad K_{II}^- = -K_{II}^+, \quad (16)$$

in this case.

The trajectories of the development of the lower crack from the right tip with respect to the local coordinate system  $x_1 O_1 y_1$  for different values of the parameter  $\varepsilon_1$  when  $x_1/l \geq 1$  are shown in Fig. 3. The trajectories are smooth curves and have a characteristic break only at the very start of initial-crack growth, after which they develop in directions opposite to one another. In this case, the closer the initial cracks, the larger the break in the trajectories.

The factors  $K_I^+/(p\sqrt{\pi l})$  at the start for  $\varepsilon_1$  of 0.1, 0.5, and 1.0 assume the values 0.700, 0.773, and 0.843, respectively, and then increase monotonically.

It should be pointed out that in calculating static trajectories of crack development on a computer as a function of the computational process, the interval of trajectory growth was selected within the limits from 0.1l to 0.1l.

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## DYNAMIC REACTION OF A PSEUDO-CERAMIC HALF-SPACE WITH A LONGITUDINAL SHEAR CRACK TO CONCENTRATED FORCES

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UDC 539.3

The dynamic problem of electroelasticity for a piezoelectric half-space with tunnel cavities-cuts is examined. The problem is reduced to the solution of an integrodifferential singular equation in terms of the amplitude of the jump in displacements at the cut. Equations are obtained for the dynamic mechanical-stress intensity factor. The influence exerted by the curvature of the crack, its orientation, and the normalized wave number on the intensity factor is studied numerically.

Let us examine a semi-infinite piezoceramic medium  $x_3 \geq 0$  (a crystal of the hexagonal system 6mm, piezoceramics PZT-4 and PZT-5), which is referenced to a crystallophysical coordinate system  $(x_1, x_2, x_3)$  and which is weakened by curvilinear cracks-cuts  $L_j$  ( $j = 1, 2, \dots, k$ ) that tunnel along the  $ox_3$ -axis (Fig. 1).

Let the shear load  $q = \text{Re}\{Q \exp(-i\omega t)\}$ , which is constant along the  $ox_3$ -axis and which varies harmonically with time, be applied to the lines  $x_1 = x_{10}$ ,  $x_2 = 0$ ,  $-\infty < x_3 < \infty$  of the boundary of the half-space. The boundary of the half-space and the surfaces of the cavities-cuts are free of forces. Let us assume that  $L_j$  ( $j = 1, 2, \dots, k$ ) are simple broken arcs with curvatures satisfying the condition H [1].

Under these conditions, coupled wave fields of mechanical longitudinal shear stresses  $\tau_{13}(x_1, x_2)$  and  $\tau_{23}(x_1, x_2)$ , the vectors of electrical and magnetic intensity  $E_1(x_1, x_2)$ ,  $E_2(x_1, x_2)$ , and  $H_3(x_1, x_2)$ , and the electric induction  $D_1(x_1, x_2)$  and  $D_2(x_1, x_2)$  act in the body.

The complete system of equations of the problem under consideration assumes the form of the equation of state of a piezomedium [2]

$$\begin{aligned} \tau_{23} &= c_{44} \partial_2 w - e_{15} E_2, \quad D_1 = e_{15} \partial_1 w + \epsilon_{11} E_1; \\ \tau_{13} &= c_{44} \partial_1 w - e_{15} E_1, \quad D_2 = e_{15} \partial_2 w + \epsilon_{11} E_2; \end{aligned} \quad (1)$$

the equation of motion

$$\partial_1 \tau_{13} + \partial_2 \tau_{23} = \rho \frac{\partial^2 w}{\partial t^2}; \quad (2)$$

and the Maxwell equation

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