

EXTENSION OF THE LOCALIZED PLASTIC FLOW ZONE IN SIMPLE SHEAR

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UDC 539.4

Propagation of localized plastic shear in a thin layer of metal was studied. An increase in the rate of propagation of the localized shear when moving away from the site of its initiation was established. The propagation rate of strains of a given level approaches the elastic shear-wave velocity in metal when moving away from the site of initiation of the localized shear.

Keywords: strain localization, numerical simulation, pure shear, dynamic loading, plastic strain.

Notation

v_x, v_y	– rate along the axes x and y , respectively
v_0	– given value of rate
ε_{ij}	– components of the total strain tensor
e_{ij}^e	– components of the elastic strain deviator
ε_{ij}^p	– components of the plastic strain tensor
ε_i^p	– intensity of plastic shear
ε_v	– volume strain
A, n	– strain hardening coefficients
δ_{ij}	– Kronecker's delta
μ	– viscosity coefficient of metal
ν	– Poisson's ratio
G_0	– initial shear modulus of metal
c_v	– volumetric heat capacity of metal
α	– coefficient of linear expansion
K	– bulk modulus
ρ	– density of metal
H	– thickness of the metal layer
F_c	– cross-sectional area in the center of the metal layer
T	– current temperature
T_c	– critical temperature of the metal softening
$\bar{\sigma}$	– volumetric stress
σ_{ij}	– total stress tensor

Institute of Problems of Strength, National Academy of Sciences of Ukraine, Kiev, Ukraine. Translated from Problemy Prochnosti, No. 4, pp. 75 – 82, July – August, 2002. Original article submitted September 19, 2001.

S_{ij}	– active stress tensor
σ_Y	– yield point of the metal under static loading
σ_i	– stress intensity
σ_{st}	– static shear resistance
i, j	– current value of the node along the x and y axes, respectively
t_k	– current value of the time step
dx, dy	– distance between the nodes of the grid along the x and y axes, respectively
dt	– time step
N_x, N_y	– number of nodes along the axes x and y , respectively

Introduction. Due to the wave processes, dynamic loading of a material results in a nonuniform stress-strain state. In the case of the high intensity of plastic strain, the nonuniformity coupled with adiabatic heating of a material can cause the localization of plastic strain and formation of adiabatic shear bands.

In some works, the localization of plastic flow was investigated using numerical simulation [1–3]. In [4], the effect of displacement rates at the boundaries of a thin layer of a metal on the stationary velocity of the adiabatic shear band was studied mainly with regard to the experimental data in [5]. Earlier [6], the development of localized plastic deformation and the stress and strain distribution resulting from plastic shear in the material were examined as a one-dimensional problem with regard to the nonlinear viscosity, damage, and temperature. However, the dynamics of expansion of the shear localization zone has not been adequately explored.

In this paper, the intent is to study the dynamics of propagation of the localized plastic strain in pure shear. The investigation was conducted using the numerical simulation of plane strain in a thin layer of metal, at the center of which the localized shear initiates and develops.

Model. We considered pure shear in a thin layer ($H = 6.3 \cdot 10^{-3}$ m) of isotropic metal of infinite length (axis x) whose loading scheme is presented in Fig. 1.

At the initial instant of time, the metal of the layer is in the zero stress-strain state:

$$v_x = v_y = 0, \quad \sigma_x = \sigma_y = \sigma_{xy} = 0.$$

Boundary conditions are defined by preset displacement rates:

$$\begin{aligned} v_x(t) &= +v_0(t), \quad v_y(t) = 0 \quad \text{for} \quad y = 0, \\ v_x(t) &= -v_0(t), \quad v_y(t) = 0 \quad \text{for} \quad y = H. \end{aligned}$$

We write the equations of motion as follows:

$$\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right), \quad (1)$$

$$\frac{\partial v_y}{\partial t} = \frac{1}{\rho} \left(\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{xy}}{\partial x} \right). \quad (2)$$

Let us assume the material of the layer to be elastoviscoplastic. In the case of the plane strain state, the constitutive equations for such a material look like

$$\sigma_{ij} = 2G(T)e_{ij}^e + \bar{\sigma}\delta_{ij}, \quad (3)$$

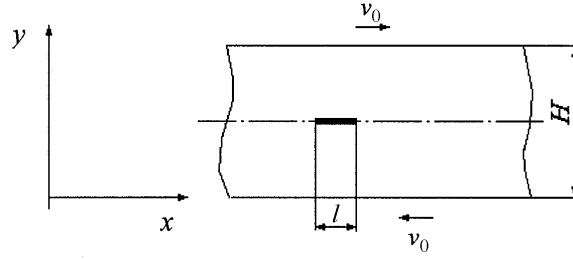


Fig. 1. Loading scheme for a layer of metal.

$$\bar{\sigma} = K(\epsilon_v - 3\alpha T), \quad (4)$$

$$S_{ij} = 2G(T)e_{ij}^e, \quad (5)$$

$$\dot{\epsilon}_v = \dot{\epsilon}_{xx} + \dot{\epsilon}_{yy}, \quad (6)$$

$$\dot{e}_{ij}^e = \dot{\epsilon}_{ij} - \dot{\epsilon}_{ij}^p - \frac{\dot{\epsilon}_v}{3} \delta_{ij}, \quad (7)$$

$$\dot{\epsilon}_{xx} = \frac{\partial v_x}{\partial x}, \quad \dot{\epsilon}_{yy} = \frac{\partial v_y}{\partial y}, \quad \dot{\epsilon}_{zz} = \dot{\epsilon}_{xz} = \dot{\epsilon}_{yz} = 0, \quad \dot{\epsilon}_{xy} = \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}. \quad (8)$$

Hereinafter, the dot over a symbol indicates the time derivative along the path of a particle of continuum.

Components of the plastic strain-rate tensor and plastic shear rate in the metal were deduced from Perzyna's equations [7]:

$$\dot{\epsilon}_{ij}^p = \dot{\epsilon}_i^p \frac{S_{ij}}{\sigma_i}, \quad (9)$$

$$\dot{\epsilon}_i^p = \frac{(\sigma_i - \sigma_{st})}{\mu}. \quad (10)$$

The viscosity coefficient was assumed to be constant. In view of the stability of counting, we did not introduce the artificial viscosity.

Taking into account strain hardening and thermal softening, we determine the static shear resistance from

$$\sigma_{st} = \sigma_Y [1 + A(\epsilon_i^p)^n] [1 - (T/T_c)^{1/2}]. \quad (11)$$

As the result of the temperature rise in the metal of the layer, a change in the elastic shear modulus occurs, which is described by the formula

$$G(T) = G_0 (1 - T/T_c). \quad (12)$$

The rate of temperature increase is governed only by the work of plastic strain:

$$\frac{dT}{dt} = \frac{\dot{\epsilon}_i^p \sigma_i}{c_v}. \quad (13)$$

To initiate inhomogeneous plastic flow in the layer of metal and its further localization, we include a linear defect of size l ($1.5 \cdot 10^{-4}$ m) in the layer (Fig. 1).

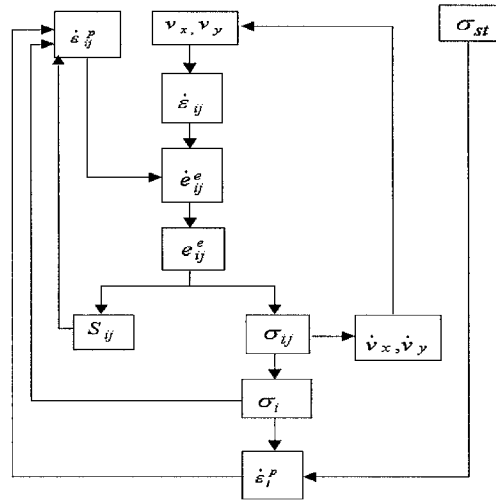


Fig. 2. Simplified computational algorithm.

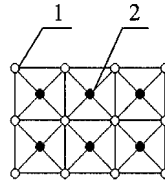


Fig. 3. Scheme of partition of the layer of the deformed metal: 1 – node, 2 – internode.

Calculation Procedure. The layer is presented in the form of a rectangular grid of 200×21 nodes, where $N_x = 200$ and $N_y = 21$. The time step was chosen to satisfy the Courant stability condition.

The displacement rate at the boundaries of the layer grows up to a given constant value in $2.5 \mu s$ ($t_k = 50$) and is defined by the formula

$$v_x(t) = v_0(1 - \exp(-t_k/3)). \quad (14)$$

The simplified computational algorithm is presented in Fig. 2.

The calculation procedure for each time step consisted of two stages, namely, “prediction” and “correction.” At the prediction stage, we calculated the displacement rates of nodes for the current time step (t_k) from the values of the displacement rate and acceleration of the nodes of the grid from two preceding steps ($t_k - 1$ and $t_k - 2$) as follows:

$$v_x(t_k) = v_x(t_k - 1) + [\dot{v}_x(t_k - 2) + \dot{v}_x(t_k - 1)]dt/2. \quad (15)$$

Using the obtained values of displacement rates at four adjacent nodes, we found the mean value of the total strain rate for the internode (Fig. 3). Then we determined the elastic strains, stresses, and plastic shear rate in the internodes employing the values of plastic strain rates and static shear resistance in the internodes from the preceding time step. Using the stress values in the four neighboring internodes, we defined the mean value of acceleration, which provided the mean displacement rate of the node central to them. The displacement rate of the boundary nodes ($y=0$ and $y=H$) is given by Eq. (14) with regard to the sign. As a result, using the obtained displacement rates of nodes for the current time step, we calculated refined values of static shear resistance, plastic strain rates, and displacement rates of the grid nodes for this time step.

At the correction stage, we repeated the indicated procedure using the refined values. Then we proceeded to the next time step.

To improve the accuracy of predictions, one can repeat the correction several times for each time step. For the purpose of saving the computer time and in view of the stability of counting, the authors restricted themselves to only one correction stage.

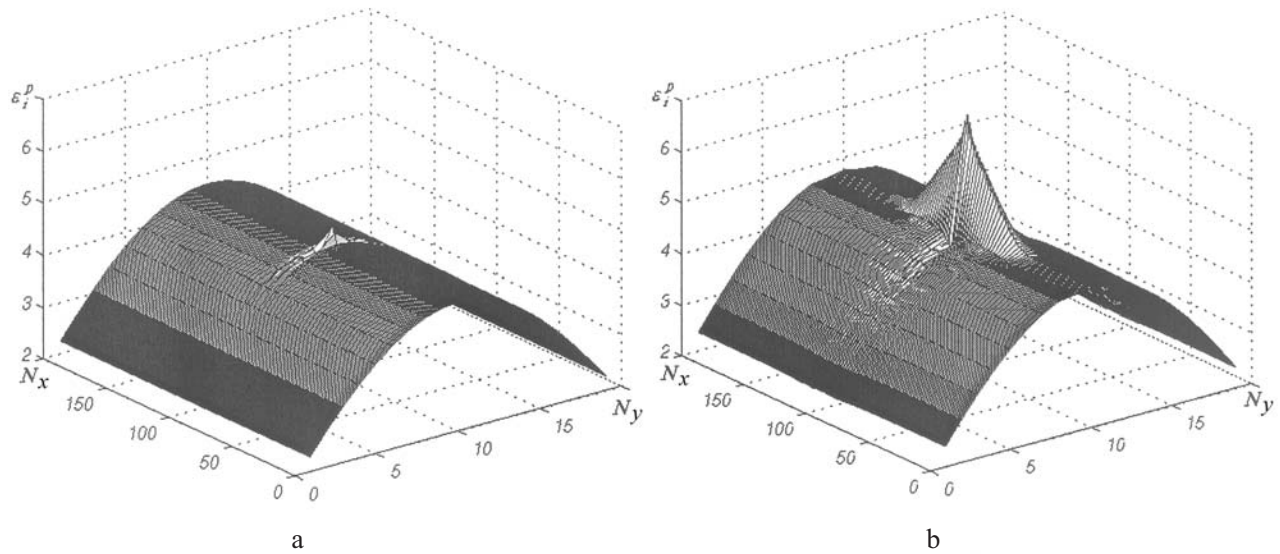


Fig. 4. Plastic strain distribution in the layer of metal.

To create an initial nonuniformity of the stress-strain state over the width of the plastic flow zone, the cross-sectional area of the layer was assumed to increase from the center to the boundaries of the deformed region:

$$F(y) = F_0 (1 + 0.8 (e^{-(j-1)/8} + e^{-(N_y-1)/8})), \quad (16)$$

$$F_0 = 1.45 F_c. \quad (17)$$

The coefficients in (16) were found by trial and error in such a way as to minimize the influence of the edge effects and the spread of the values of the cross-sectional area.

In the calculations, we assumed the elastic strain not to exceed the peak value corresponding to the theoretical strength:

$$e_{ij}^e \leq \frac{\sigma_Y}{G_0}. \quad (18)$$

The linear defect was presented as a portion of the row of internodes with a reduced (by 20%) static shear resistance.

Results of Numerical Analysis. For the calculations we used steel with the following parameters: $\sigma_Y = 300 \cdot 10^6$ Pa, $E = 2.1 \cdot 10^{11}$ Pa, $G = 8.26 \cdot 10^{10}$ Pa, $K = 1.52 \cdot 10^{11}$ Pa, $n = 0.3$, $A = 0.3$, $\nu = 0.27$, $\mu = 2 \cdot 10^4$ Pa · s [8], $T_c = 750^\circ\text{C}$, and $\rho = 7800$ kg/m³. The displacement rate at the boundaries of the layer $v_0 = 45$ m/s.

After the initial wave stage of the deformation process, the temperature rise causes a more intensive plastic axisymmetrical deformation in the central area of the layer of metal. The values of stresses and strains on the x and y axes are approximately one order of magnitude lower than those of shear stresses, and they do not affect the qualitative nature of the strain localization. The propagation of the localized plastic strain began in the center of the layer at a strain value of 4.5.

The plastic strain distribution in the layer of metal is illustrated in Fig. 4. The influence of a linear defect prior to the beginning of localization is presented in Fig. 4a. It is obvious from the figure that the defect have no significant influence on the general plastic strain distribution. An increase in the strain level in the defect region accounts for less than 2% as contrasted to the strain in the adjacent internodes. The situation changes as the localization begins. The plastic strain level in the defect increases dramatically. Then the strain propagates symmetrically along the central

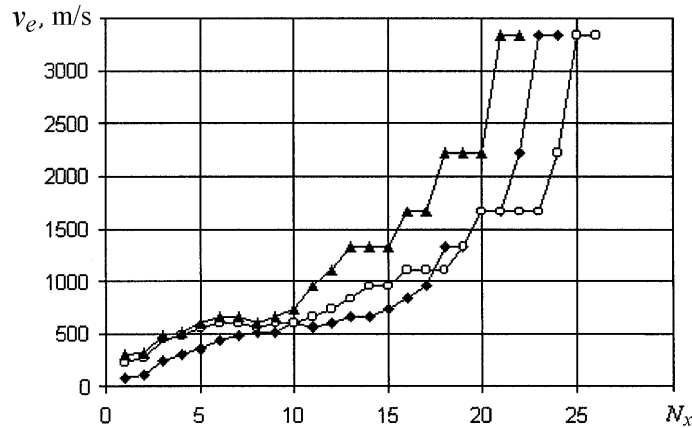


Fig. 5. Propagation rates of a given plastic strain (dot) moving away from the defect in the row of internodes containing the above defect [(◆) $\varepsilon_i^P = 4.72$; (○) $\varepsilon_i^P = 5.59$; (▲) $\varepsilon_i^P = 6.78$].

area of the layer moving away from the defect. The plastic strain distribution in the layer within $24.75 \mu\text{s}$ after the instant of initiation of the localization is given in Fig. 4b. One can see that the propagation of the plastic strain localization is the key process occurring in the layer of metal. Meanwhile, the mean level of the plastic strain (without taking into account the localization) in the layer grows insignificantly ($< 7\%$).

Figure 5 presents the curves of the propagation rates of plastic strains of a given level moving away from the defect in the row of internodes containing the above defect. For either strain level, the propagation rate of the localized shear v_e increases when moving away from the defect. In addition, the curves can be divided into two portions. Within the first portion we observe only a small increase and equalization of the strain propagation rate whereas within the second one it grows sharply up to the value close to the shear-wave velocity in metal. We note that the propagation rate of the intermediate plastic strain $\varepsilon_i^P = 5.59$ reaches the shear-wave velocity later than that of the strain $\varepsilon_i^P = 6.78$, which calls for further investigation.

Conclusions. The propagation rate of the localized plastic strain of a given level increases when moving away from the site of its initiation. At the sufficiently large distance from the linear defect, it is close to the velocity of the shear wave in metal.

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