

OPTIMIZATION OF THE SHAPES OF HOLES IN A QUASIORTHOTROPIC PLATE UNDER BIAXIAL TENSION

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UDC 539.3

On the basis of solutions of the plane problems of the elasticity theory for a quasiorthotropic body with curvilinear hole, we propose an analytic-numerical algorithm for the determination of the optimal shape of the hole (with minimum stress concentration) under the conditions of biaxial tension of the plate. The direct problems of the theory of elasticity for quasiorthotropic plates with smooth holes are solved by the method of singular integral equations. The solution of inverse problems with unknown shapes of the holes is reduced to the minimization of a multiparameter functional of standard deviations of the tensile stresses along the contours of the holes from their specified values. We determine the optimal shapes of the holes in quasiorthotropic plates for different tensile stresses applied at infinity in the directions of the orthotropy axes.

Keywords: quasiorthotropic material, plane stress state, optimal holes, stress concentration, singular integral equations.

Introduction

The problems of optimization of the shapes of various structural elements belong to the inverse problems of the theory of elasticity with unknown boundaries. They can be reduced to the minimization of the so-called quality functional whose construction is based on the solutions of the corresponding direct problems. Numerous problems for the elastic isotropic planes with holes were solved by using various methods [1–5]. The corresponding problems of optimization of the shapes of holes in the anisotropic bodies (various composite materials) are much more complicated because the stressed state of these bodies strongly depends on the loading mode and on the orientation of the holes relative to the anisotropy axes of the material. Some solutions of problems of this kind were obtained for anisotropic plates in [4, 6].

In what follows, we consider the problem of optimization of the shape of a hole in a quasiorthotropic plate under the conditions of biaxial tension at infinity. For the direct problems of the two-dimensional theory of elasticity for quasiorthotropic bodies with holes, the effective solutions were obtained by the method of singular integral equations (SIE) [7–10]. The solutions of the inverse problems for the regions with unknown boundaries were constructed on the basis of the well-known procedure from [5]. For the minimization of the multiparameter functional of mean-square deviations of tensile stresses acting on the contour of the hole in the tangential direction from the specified values, we applied the quasi-Newton method based on the use of the first finite-difference derivatives [11].

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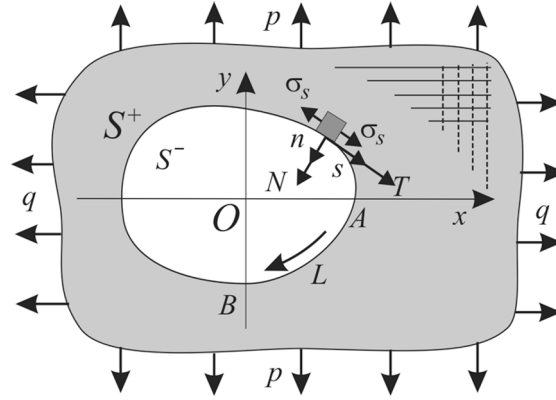


Fig. 1. Elastic quasiorthotropic plate with smooth curvilinear hole.

Statement of the Inverse Problem with Unknown Boundary of the Elastic Body

We consider a problem of the theory of elasticity for an infinite quasiorthotropic plate containing a smooth hole. Its contour L is subjected to the action of given stresses

$$N^*(t) + iT^*(t) = p^*(t), \quad t \in L, \quad (1)$$

where $N^*(t)$ and $T^*(t)$ are the normal and tangential components of the vector of stresses acting on the contour of hole with the outer normal n (Fig. 1). The direction of traversing of the contour L for which the region of the body S^+ is located to the left is regarded as positive. At infinity, the plate is stretched by the stresses $\sigma_x^\infty = q$ and $\sigma_y^\infty = p$. The directions of the axes x and y of a Cartesian coordinate system are chosen along the principal axes of orthotropy. In our case, the analyzed quasiorthotropic material belongs to a special class of degenerate orthotropic materials for which the roots of the characteristic equation are multiple and pure imaginary: $\mu_1 = \mu_2 = i\gamma$ [7, 10]. For the plane stressed state, $\gamma = 4\sqrt{E_x/E_y}$ is the principal parameter of orthotropy and the shear modulus can be represented via the other constants as follows: $G = E_x/[2(\sqrt{E_x/E_y} + \nu_{xy})]$, where E_x , E_y are the elasticity moduli along the axes x and y , respectively, and ν_{xy} is Poisson's ratio. For isotropic materials, the parameter $\gamma = 1$.

It is necessary to determine the shape of a curvilinear hole L_0 at all points of which the normal stresses in the tangential direction $\sigma_s^*(t)$ take constant values:

$$\sigma_s^*(t) = c = \text{const}, \quad t \in L_0. \quad (2)$$

These holes are called optimal. For any load applied to the contour of the hole (1), the constant c should be found from the solution of the problem. In the case of free holes ($p^*(t) = 0$) in isotropic plates subjected to the action of the indicated biaxial load, this constant is known [2, 5]: $c = p + q$.

For the uniqueness of the solution of the inverse problem with unknown boundary of the body, it is necessary to impose additional conditions. For a plate with a single hole, it is sufficient to specify one parameter, e.g., its diameter or area. Moreover, the solutions of these problems exist not always. In what follows, we consider solely the loads for which the optimal shapes of holes exist.

Direct Problem

The first basic problem of the two-dimensional theory of elasticity for an unbounded quasiorthotropic body with a smooth hole is solved by the SIE method [10]. We use the method of superposition and seek the complex stress potentials for the quasiorthotropic body in the form

$$\Phi_1^*(z_1) = \Phi_1^0(z_1) + \Phi_1(z_1) \quad \text{and} \quad \Psi_1^*(z_1) = \Psi_1^0(z_1) + \Psi_1(z_1),$$

where

$$\Phi_1^0(z_1) = (p + \gamma^{-2}q)/4 \quad \text{and} \quad \Psi_1^0(z_1) = (p - \gamma^{-2}q)/2$$

are given functions that describe the homogeneous stress-strain state in the continuous quasiorthotropic plate without holes. The potentials $\Phi_1(z_1)$ and $\Psi_1(z_1)$ determine the perturbed stressed state caused by the holes [9, 10]. The complex stresses potentials $\Phi_1(z_1)$ and $\Psi_1(z_1)$ and the integral equations of this problem have the same form as in the case of isotropic material [5]. However, they are written in the mathematical plane $z_1 = x + i\gamma y$ (here and in what follows, the subscript “1” means that the corresponding quantities belong to the plane z_1 ; in particular, the point $t_1 \in L_1$ corresponds to a point $t \in L$ in the plane $z = x + iy$ [8–10]). We represent the boundary condition (1) for the perturbed stressed state in the plane z_1 in the following form:

$$\left[\left(\frac{i}{\gamma} \right) X_n - Y_n \right] \frac{ds}{dt_1} = \tilde{P}(t) = \tilde{P}_1(t_1) = \frac{1}{2\gamma} \left[(1 + \gamma)p(t) - (1 - \gamma)\overline{p(t)} \frac{d\bar{t}}{dt} \right] \frac{dt}{dt_1}, \quad t \in L, \quad (3)$$

where X_n and Y_n are Cartesian components of the stress vector act from the side of the outer normal n on the curvilinear contour L and

$$p(t) = p^*(t) - \frac{p + q + (p - q)\overline{dt}/dt}{2}.$$

Finding the limit values of the complex potentials as $z_1 \rightarrow t_1 \in L_1$ and satisfying, with the use of these values, the boundary condition (3), we arrive at the following SIE:

$$\int_{L_1} [K(\tau_1, t_1)g'_1(\tau_1)d\tau_1 + L(\tau_1, t_1)\overline{g'_1(\tau_1)}d\bar{\tau}_1] - \frac{M_1}{2i\bar{t}_1^2} \frac{d\bar{t}_1}{dt_1} + \frac{a_1}{l} \frac{ds_1}{dt_1} = \pi\tilde{P}_1(t_1), \quad (4)$$

where

$$K(\tau_1, t_1) = \frac{1}{2} \left[\frac{1}{\tau_1 - t_1} + \frac{1}{\bar{\tau}_1 - \bar{t}_1} \frac{d\bar{t}_1}{dt_1} \right], \quad L(\tau_1, t_1) = \frac{1}{2} \left[\frac{1}{\bar{\tau}_1 - \bar{t}_1} - \frac{\tau_1 - t_1}{(\bar{\tau}_1 - \bar{t}_1)^2} \frac{d\bar{t}_1}{dt_1} \right],$$

$g'_1(t_1)$ is the required complex continuous function on the contour L_1 ; s_1 is the arc abscissa corresponding to the point t_1 , l is an arbitrary parameter with the dimension of length, and

$$M_1 = i \int_{L_1} [g'_1(t_1)\bar{t}_1 dt_1 - \overline{g'_1(t_1)}t_1 d\bar{t}_1] \quad \text{and} \quad a_1 = \int_{L_1} g'_1(t_1) dt_1$$

are zero additional terms guaranteeing the uniqueness of a solution of SIE for any right-hand side $\tilde{P}_1(t_1)$ (3) [9, 10]. The numerical solution of SIE is found by the method of quadratures.

If the contour of the hole is free of loads ($p^*(t) = 0$), then the contour stresses are given by the formula

$$\begin{aligned}\sigma_s &= \operatorname{Re} \left\{ 2(1 + \gamma^2) i g'_1(t_1) + (1 - \gamma^2) i [\overline{g'_1(t_1)} - g'_1(t_1)] \frac{d\bar{t}_1}{dt_1} \right\} \\ &= -2 \operatorname{Im} \{ g'_1(t_1) \} \left[(1 + \gamma^2) - (1 - \gamma^2) \operatorname{Re} \left\{ \frac{d\bar{t}_1}{dt_1} \right\} \right].\end{aligned}\quad (5)$$

Here and in what follows, $\sigma_s \equiv \sigma_s^*(t)$, $t \in L$, are the normal stresses on the contour of the hole in the tangential direction (Fig. 1).

Analytic Solution for an Elliptic Hole in the Quasiorthotropic Plate

In order to determine the analytic solution for the stresses σ_s under the conditions of biaxial tension at infinity $\sigma_x^\infty = q$, $\sigma_y^\infty = p$ in the quasiorthotropic plate containing a free ($p^*(t) = 0$) elliptic hole

$$t = \omega(\xi) = a\{\cos(\xi) - i\varepsilon \sin(\xi)\}, \quad \varepsilon = \frac{b}{a}, \quad \xi \in [0; 2\pi] \quad (6)$$

(see the schematic diagram in Fig. 2a), we use the following known [10, 12] solution obtained for the vertical tension of this plate:

$$\sigma_s(p, \xi) = \frac{\gamma^2(\sin^2 \xi + \varepsilon^2 \cos^2 \xi)(\varepsilon^2 \gamma^2 \cos^2 \xi + 2\varepsilon \gamma \cos^2 \xi - \sin^2 \xi)}{(\sin^2 \xi + \varepsilon^2 \gamma^2 \cos^2 \xi)^2} p. \quad (7)$$

By the change of variables

$$p \rightarrow q, \quad \gamma \rightarrow 1/\gamma, \quad \varepsilon \rightarrow 1/\varepsilon, \quad \xi \rightarrow \xi + \pi/2$$

in relation (7), for the case of horizontal tension $\sigma_x^\infty = q$, we find

$$\sigma_s(q, \xi) = \frac{(\cos^2 \xi + (1/\varepsilon)^2 \sin^2 \xi)((1/\varepsilon \gamma)^2 \sin^2 \xi + (2/\varepsilon \gamma) \sin^2 \xi - \cos^2 \xi)/\gamma^2}{(\cos^2 \xi + (1/\varepsilon \gamma)^2 \sin^2 \xi)^2} q. \quad (8)$$

Adding relations (7) and (8), we get the contour stresses $\sigma_s(t)$ for the elliptic hole in the quasiorthotropic plate subjected to biaxial tension by the stresses q and p along the coordinate axes x and y :

$$\sigma_s(\xi) = \sigma_s(p, \xi) + \sigma_s(q, \xi), \quad \xi \in [0; 2\pi]. \quad (9)$$

We now determine the ratio $\varepsilon_0 = b_0/a$ of the semiaxes of ellipse (6) for which the stresses (9) at the axial points $A = \omega(0)$ and $B = \omega(\pi/2)$ are identical: $\sigma_s(A) = \sigma_s(B)$. In view of relations (7)–(9), this condition

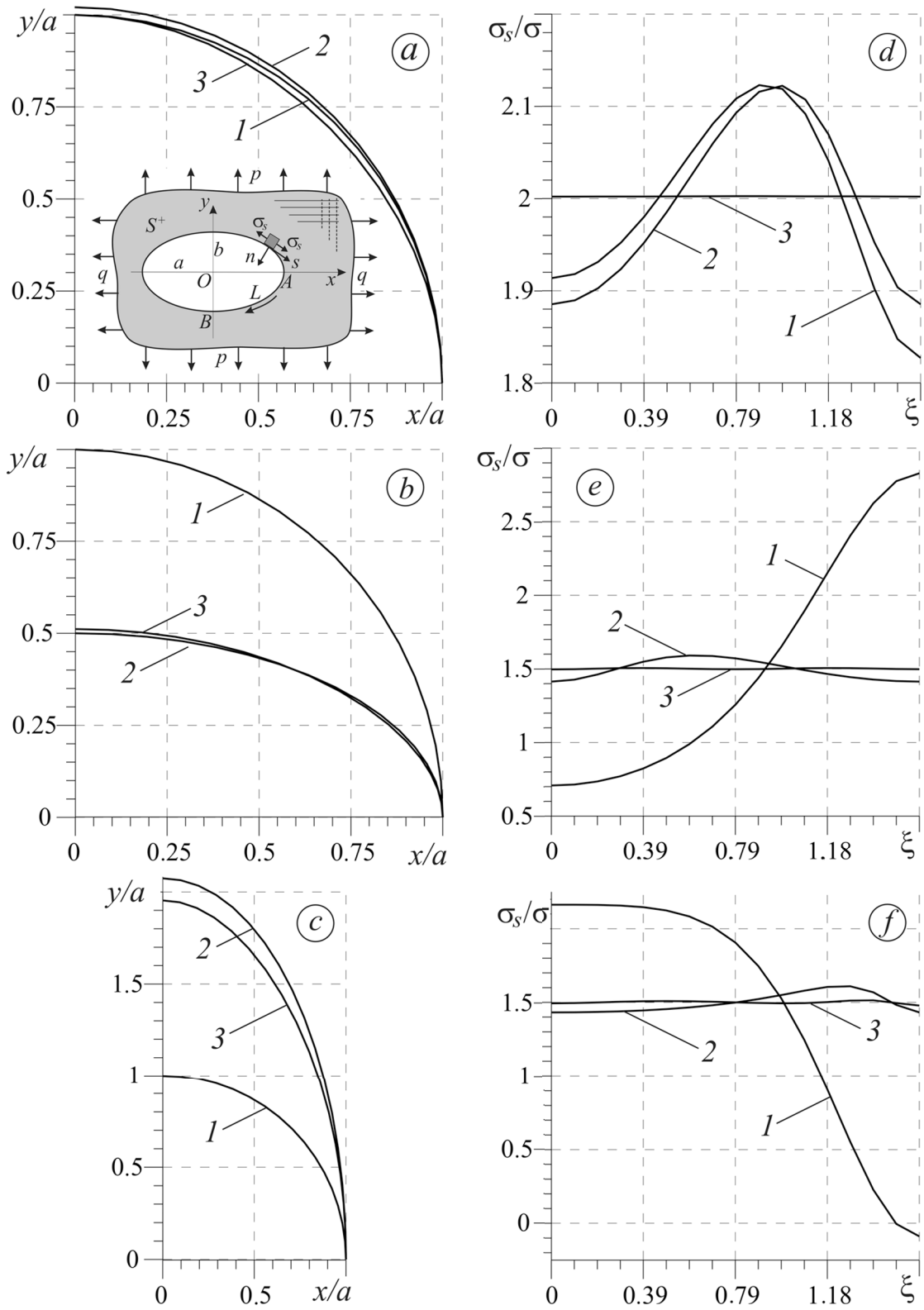


Fig. 2. Shapes of the holes in the first quadrant (a–c) and the corresponding distributions of stresses σ_s/σ (d–f) for the parameter of orthotropy $\gamma = 1.4$ under the loads $p = q = \sigma$ (a, d); $p = 0.5\sigma$, $q = \sigma$ (b, e); $p = \sigma$, $q = 0.5\sigma$ (c, f): (1) circle; (2) optimal ellipse from the set of ellipses; (3) optimal hole.

implies that:

$$1 + \frac{2}{\gamma \epsilon_0} - \frac{\tilde{\epsilon}}{\gamma^2} = (1 + 2\gamma \epsilon_0) \tilde{\epsilon} - \gamma^2, \quad \epsilon_0 = \frac{\sqrt{c_2^2 + 8c_1/\gamma} - c_2}{2c_1}, \quad (10)$$

where

$$\tilde{\epsilon} = \frac{q}{p}, \quad c_1 = 2\gamma \tilde{\epsilon}, \quad \text{and} \quad c_2 = (\gamma^2 + 1) \left(\frac{\tilde{\epsilon}}{\gamma^2} - 1 \right).$$

These ellipses (the optimal ellipses from the set of ellipses) may serve as the first approximation in finding the optimal holes in the quasiorthotropic plate. For the isotropic plate ($\gamma = 1$), we get the well-known result $\epsilon_0 = b_0/a = p/q$ [1–3, 5] with identical stresses $\sigma_s(\xi) = p + q$ along the entire contour of the elliptic hole.

Numerical Optimization of the Shapes of Holes

As a result of the numerical solution of the SIE (4) for some initially given [elliptic (6) or circular] contour L , we find stresses (9) at the n nodal points $\omega(\xi_j)$, $\xi_j = 2\pi(j-1)/n$, $j = \overline{1, n}$. Consider the functional of mean-square relative deviations of these stresses from their mean values [5]:

$$F[\mathbf{P}] = \frac{1}{n} \sum_{j=1}^n \left\{ \frac{\sigma_s[\omega(\mathbf{P}, \xi_j)] - \tilde{\sigma}_s}{\tilde{\sigma}_s} \right\}^2, \quad \tilde{\sigma}_s = \frac{1}{n} \sum_{j=1}^n \sigma_s(\xi_j), \quad (11)$$

where $\mathbf{P} = (P_1, P_2, \dots, P_M)$ is the collection of parameters of the optimization problem. By using these parameters, we construct the M -parameter equation of the contour L :

$$t = \omega(\mathbf{P}, \xi) = x(\mathbf{P}, \xi) - iy(\mathbf{P}, \xi), \quad \xi \in [0; 2\pi]. \quad (12)$$

Minimizing functional (11) for P_m ($m = \overline{1, M}$), we find the values of these parameters P_m^0 (parameters of the optimal shape of the contour) for which the stresses $\sigma_s(\xi)$ become identical over the entire contour. In other words, the functional condition $F[\mathbf{P}^0] < \Delta$ is satisfied; here, Δ is a given admissible deviation of the relative contour stresses from the stresses averaged over the required contour. To minimize the multiparameter functional (11), we apply the quasi-Newtonian method based on the use of the first finite-difference derivatives [11].

The algorithm of determination of the optimal shapes of holes in a quasiorthotropic plate is illustrated for two values of the principal parameter of orthotropy $\gamma = \sqrt[4]{E_x/E_y} = 1.4$ (Fig. 2) and $\gamma = 2.6$ (Fig. 3) for which the ratios E_x/E_y are chosen according to the characteristics of the following orthotropic materials: E-Glass/Epoxy ($E_x = 41.4$ GPa, $E_y = 10.3$ GPa, $G_{xy} = 4.27$ GPa, and $\nu_{xy} = 0.28$) and Graphite/Epoxy ($E_x = 294$ GPa, $E_y = 6.34$ GPa, $G_{xy} = 4.9$ GPa, and $\nu_{xy} = 0.23$) [13]. We considered three cases of biaxial tension of the plate at infinity by the stresses $\sigma_x^\infty = q$ and $\sigma_y^\infty = p$: $p = q = \sigma$ (Figs. 2a, d and 3a, d); $p = 0.5\sigma$, $q = \sigma$ (Figs. 2b, e and 3b, e), and $p = \sigma$, $q = 0.5\sigma$ (Figs. 2c, f and 3c, f), where σ is the general parameter of loading. For the parameter of orthotropy $\gamma = 1.4$, the optimal contour is sought in the form (12) with the help of trigonometric

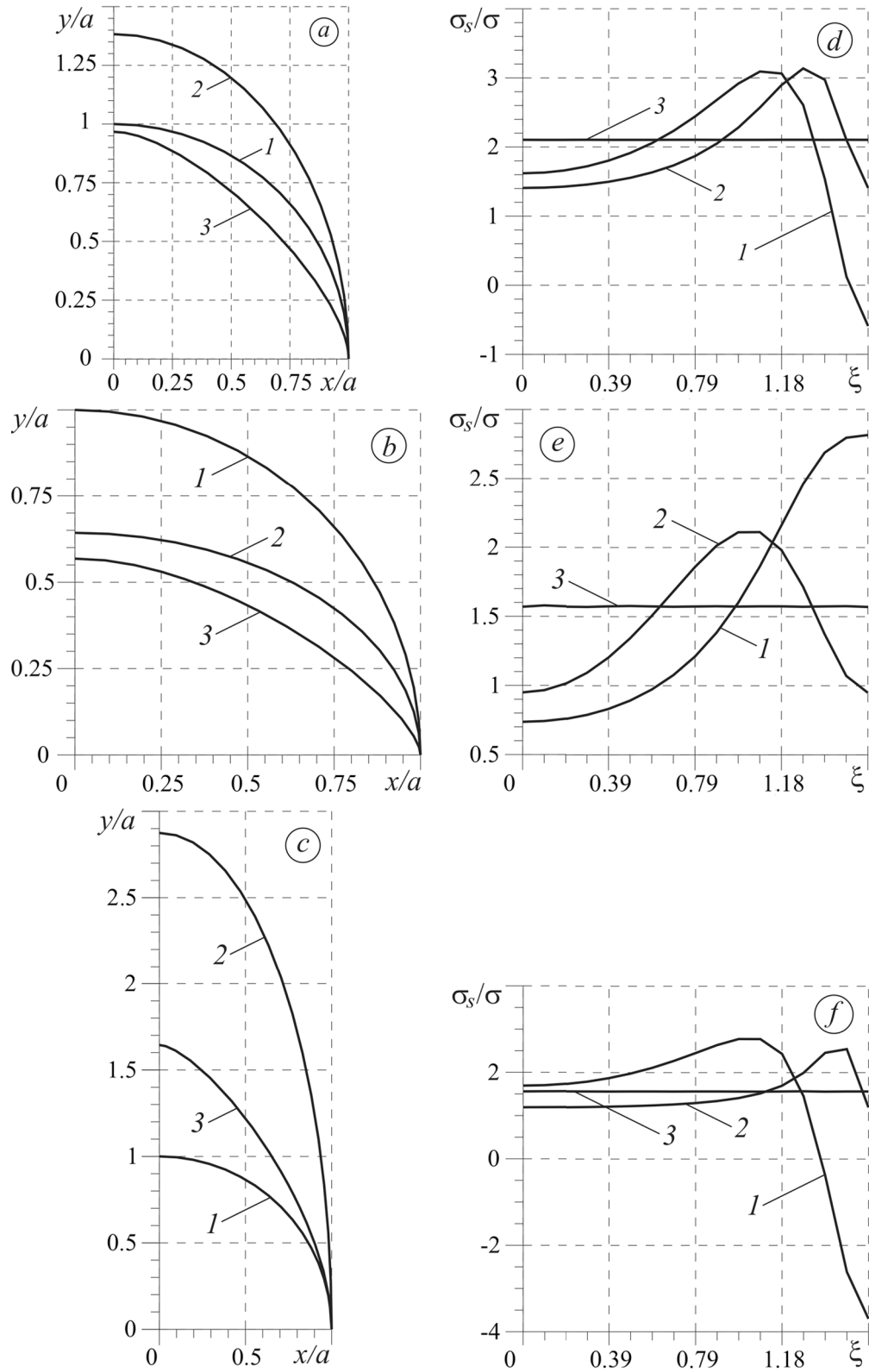


Fig. 3. Shapes of the holes in the first quadrant (a–c) and the corresponding distributions of stresses σ_s/σ (d–f) for the parameter of orthotropy $\gamma = 2.6$ under loads $p = q = \sigma$ (a, d); $p = 0.5\sigma, q = \sigma$ (b, e); $p = \sigma, q = 0.5\sigma$ (c, f): (1) circle; (2) optimal ellipse from the set of ellipses; (3) optimal hole.

Table 1
Parameters of the Optimal Holes ($M = 5$; $\gamma = 1.4$) and the Maximum Relative Deviations
of the Contour Stresses from the Averaged Values

$\gamma = 1.4$	P_1^0	P_2^0	P_3^0	P_4^0	P_5^0	$\delta, \%$	$\delta_0, \%$	$\varepsilon_0 = b_0/a$
$p = q = \sigma$	0.989	0.016	0.991	-0.014	-0.005	0.12	6.12	1.021
$p = \sigma/2, q = \sigma$	0.993	0.023	0.506	-0.015	-0.010	0.52	6.07	0.500
$p = \sigma, q = \sigma/2$	0.986	0.021	1.952	-0.011	-0.007	0.82	7.30	2.074

series with $M = 5$ parameters:

$$\begin{aligned} \frac{x(\mathbf{P}, \xi)}{a} &= P_1 \cos(\xi) + P_2 \cos(3\xi) + (1 - P_1 - P_2) \cos(5\xi), \\ \frac{y(\mathbf{P}, \xi)}{a} &= P_3 \sin(\xi) + P_4 \sin(3\xi) + P_5 \sin(5\xi), \quad \xi \in [0; 2\pi], \end{aligned} \quad (13)$$

where the x, y -symmetry of the problem is taken into account and the contour is fixed at the point A ($x(\xi = 0) = a$).

We determined the distributions of stresses (9) over the initial circular holes (Figs. 2d–f and 3d–f, curves 1), for the optimal ellipses from the set of ellipses (whose parameters are such that $\sigma_s(A) = \sigma_s(B)$) (curves 2), and for the obtained optimal holes (12), (13) (curves 3). We computed the parameters of the optimal holes and the maximum relative deviations δ (%) of stresses $\sigma_s(\xi_j)$ from their averaged values on these holes $\tilde{\sigma}_s$

$$(\delta = \max |(\sigma_s(\xi_j) - \tilde{\sigma}_s) / \tilde{\sigma}_s| 100\%),$$

as well as the corresponding values of δ_0 (%) on the optimal ellipses (Table 1).

For the quasiorthotropic plate with a larger parameter γ , the range of the contour stresses σ_s is much wider. Therefore, for the satisfactory minimization of functional (11), the optimal contour should be sought for a larger number of parameters P_m . For $\gamma = 2.6$ (Fig. 3), we performed our search by using similar expansions with $M = 9$ parameters P_m (Table 2):

$$\begin{aligned} \frac{x(\mathbf{P}, \xi)}{a} &= P_1 \cos(\xi) + P_2 \cos(3\xi) + P_3 \cos(5\xi) + P_4 \cos(7\xi) + (1 - P_1 - P_2 - P_3 - P_4) \cos(9\xi), \\ \frac{y(\mathbf{P}, \xi)}{a} &= P_5 \sin(\xi) + P_6 \sin(3\xi) + P_7 \sin(5\xi) + P_8 \sin(7\xi) + P_9 \sin(9\xi), \quad \xi \in [0; 2\pi]. \end{aligned}$$

For the parameter of orthotropy $\gamma = 1$ (isotropic material), we obtained well-known elliptic contours of the holes with the ratio of semiaxes $b/a = p/q$ [1–3, 5].

The distributions of stresses σ_s (9) over the circular and elliptic holes in quasiorthotropic plates subjected to biaxial tension qualitatively differ from the analogous distributions obtained for orthotropic plates [6, 14].

Table 2
Parameters of the Optimal Holes $P_m = P_m^0$ ($m = \overline{1,9}$) and the Maximum Relative Deviations
of the Contour Stresses from the Averaged Values ($\gamma = 2.6$)

$\gamma = 2.6$	P_1/P_5	P_2/P_6	P_3/P_7	P_4/P_8	P_9	$\delta, \%$	$\delta_0, \%$	$\varepsilon_0 = b_0/a$
$p = q = \sigma$	$\frac{1.030}{0.820}$	$\frac{0.037}{-0.205}$	$\frac{-0.079}{-0.040}$	$\frac{0.011}{0.018}$	-0.001	5.22	56.9	1.383
$p = \sigma/2, q = \sigma$	$\frac{0.908}{0.557}$	$\frac{0.084}{-0.014}$	$\frac{0.039}{-0.024}$	$\frac{-0.021}{-0.024}$	-0.003	5.23	40.8	0.643
$p = \sigma, q = \sigma/2$	$\frac{0.951}{1.524}$	$\frac{0.028}{-0.082}$	$\frac{-0.027}{0.093}$	$\frac{0.051}{0.033}$	-0.022	4.49	69.6	2.875

In particular, in the case of uniform tension ($p = q$) of the quasiorthotropic plate with circular hole, the maximum stresses σ_s are attained within the range of the parameter $\xi \in (\pi/4; \pi/3)$ (Figs. 2d and 3d). At the same time, for the orthotropic plates, the stresses are maximum at the axial points ($\xi = 0, \xi = \pi/2$) [6, 14]. In view of the different distributions of stresses σ_s in the quasiorthotropic and orthotropic plates, the shapes of the optimal holes are also different (for the identical biaxial loads). As the parameter γ increases, the optimal holes shrink within the interaxial range ($\pi/8 < \xi < 3\pi/8$) for quasiorthotropic plates (Fig. 3d, e, f, curves 3) but, on the contrary, stretch for orthotropic plates [6].

CONCLUSIONS

We propose a method for the determination of the optimal contours of holes in quasiorthotropic plates based on the numerical solution of singular integral equations. The inverse problems of the theory of elasticity for quasiorthotropic bodies with unknown boundary are reduced to the minimization of a multiparameter functional of mean-square deviations of normal stresses from the given constant values. For this purpose, we use the effective solutions of the corresponding direct problems for plane quasiorthotropic bodies with smooth holes. We determined the optimal shapes of the holes for three cases of biaxial tension of the plates with various parameters of orthotropy. Unlike the isotropic bodies, insignificant changes in the shape of a hole in the quasiorthotropic plate may lead to significant changes in the distributions of contour stresses.

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