

In designing and making sheet structural elements, it becomes necessary to evaluate the effect of reinforcing elements on the stress intensity factor at the tips of cracks. A survey of the studies done in this area can be found in [2, 7, 12], for example. Despite the practical importance of the problem, there has been almost no study of the growth of a crack around a hole in a reinforced plate.

Below, within the framework of the hypothesis of continuous contact between a rib and a plate along a line, we construct general representations of the solution of this problem for an anisotropic plate. The problem is reduced to a system of singular integral equations. We discuss the case of crack growth from the contour of a hole. The effect of the anisotropy of the material and the stiffness and location of the reinforcing elements on the stress intensity factors is examined. Corresponding results for isotropic plates are obtained by an asymptotic solution in anisotropy parameters in a numerical calculation.

1. We will examine an elastic anisotropic plate of constant thickness h weakened by an elliptical hole $\Lambda = \left\{ \left(\frac{x}{a} \right)^2 + \left(\frac{y}{b} \right)^2 = 1 \right\}$ and a system of smooth through nonintersecting curvilinear slits $L_j = \{z = x + iy = \tau^j(\beta) \mid |\beta| < 1\}$ ($j = \overline{1, k}$). The plate is reinforced along nonintersecting rectilinear segments $\Gamma_s = \{z = \tau^{k+s}(\beta) = z^s + \rho_s \beta \exp(i\vartheta_s) \mid |\beta| < 1\}$ ($0 \leq \vartheta_s < 2\pi$; $s = \overline{1, m}$) by stiffening ribs (stringers). We designate $L = \bigcup_{j=1}^k L_j$, $\Gamma = \bigcup_{s=1}^m \Gamma_s$. We direct the normal $\bar{n}$ to the right with the positive circumvention of L , Γ .

Let the plate be loaded by a prescribed system of external forces. The ribs are loaded at the ends $\tau^{k+s}(\mp 1)$ by forces $P_s^1 \exp[i(\vartheta_s + \pi)]$, $P_s^2 \exp[i\vartheta_s]$ ($s = \overline{1, m}$), respectively.

We will assume that the cross section of the ribs and the thickness of the plate h are small compared to the length of the reinforced sections; the plate is in a generalized stress state; the ribs are continuously attached to the plate, lie in its middle lane, and work only in tension-compression [6, 12]. The edges of the slits are free of external forces and are not in contact with one another. We will assume that all of the slits are internal. A generalization to the case of slits originating from the contour of the hole is given in Part 2.

We will use $r(t) = \{r_s(t) \mid t \in \Gamma_s; s = \overline{1, m}\}$ to designate the contact forces transmitted from the ribs to the plate, and we will examine $r(t)$ as body forces in the plate (the positive direction for $r_s(t)$ coincides with the direction of the vector $\exp[i(\vartheta_s + \pi)]$).

The stresses and displacements in the plate can be expressed through two analytic functions $\Phi_\nu(z_\nu)$ [8] satisfying the prescribed system of external forces. The boundary conditions on L and Γ can be assigned in the form [6]

$$\begin{aligned} a(t) \Phi_1^\pm(t_1) + b(t) \overline{\Phi_1^\pm(t_1)} + \Phi_2^\pm(t_2) &= 0 \quad (t \in L); \\ a(t) &= a_0 \frac{M_1(t)}{M_2(t)}; \quad b(t) = b_0 \frac{\overline{M_1(t)}}{\overline{M_2(t)}}; \quad M_\nu(t) = \mu_\nu \cos \psi - \sin \psi; \\ a_0 &= (\mu_1 - \mu_2) (\mu_2 - \mu_2)^{-1}; \quad b_0 = (\mu_1 - \mu_2) (\mu_2 - \mu_2)^{-1}; \\ \sigma_n^+ &= \sigma_n^-; \quad h(\tau_n^- - \tau_n^+) = r(t); \\ &(t \in \Gamma). \\ u^+ &= u^-; \quad v^+ = v^-; \quad \varepsilon^+ = \varepsilon^- = \varepsilon. \end{aligned} \tag{1.1}$$

$$\tag{1.2}$$

Here, the plus (minus) corresponds to the left (right) edge of L_j , Γ_s with movement from the beginning $a_j = \tau_j(-1)$ to the end; $b_j = \tau_j(1)$ ($j = 1, k + m$); $\psi = \psi(t)$ is the angle between the normal $\bar{n}$ to the left edge of the slit L at the point t and the axis x ; $u^\pm, v^\pm, \epsilon^\pm, \sigma^\pm, \tau_n^\pm$ are limiting values of the displacements and strains along Γ and the normal and shear stresses; μ_v are the roots of the corresponding characteristic equation [8].

We augment Eqs. (1.1) and (1.2) with conditions of strain compatibility between the rib and the plate ϵ along the contact line Γ_s ($s = 1, m$)

$$\epsilon = P(t) + \int_{\Gamma} k^*(t, \tau) r(\tau) ds;$$

$$P(t) = P_s^2 (E_s^0 F_s^0)^{-1}; \quad t = \tau^{k+s}(\beta); \quad \tau = \tau^{k+p}(\beta') \quad (p = \overline{1, m});$$

$$k^*(t, \tau) = 0 \quad (p \neq s); \quad k^*(t, \tau) = \begin{cases} (E_s^0 F_s^0)^{-1} & (\beta' > \beta); \\ 0 & (\beta' < \beta); \end{cases} \quad (p = s),$$
(1.3)

where E_s^0, F_s^0 are the Young's moduli and the cross-sectional area of the s -th rib.

We represent the function $\Phi_v(z_v)$ as the result of superposition in the form

$$\Phi_v(z_v) = \sum_{j=1}^3 \Phi_v^j(z_v). \quad (1.4)$$

Here $\Phi_v^1(z_v)$ is the solution for a crack-free plate with an elliptical hole which satisfies the prescribed system of external forces (including the boundary conditions on the contour of the hole and at infinity). We will assume that $\Phi_v^1(z_v)$ is known [8].

Proceeding on the basis of the solution of the problem of the action of a concentrated force at point τ of an infinite anisotropic plate with an elliptical hole free of external forces Λ [4, 8], we seek $\Phi_v^1(z_v)$ ($j = 2, 3$) in the form

$$\Phi_v^2(z_v) = \frac{1}{\omega'_v(\zeta_v)} \int_{\Gamma} \left\{ \frac{A_v(\tau)}{\zeta_v - \eta_v} + \frac{l_v \overline{A_1(\tau)}}{\zeta_v(\zeta_v \eta_1 - 1)} + \frac{n_v \overline{A_2(\tau)}}{\zeta_v(\zeta_v \eta_2 - 1)} \right\} r(\tau) ds; \quad (1.5)$$

$$\Phi_v^3(z_v) = \frac{1}{\omega'_v(\zeta_v)} \int_{\Gamma} \left\{ \frac{\Omega_v^*(\tau)}{\zeta_v - \eta_v} + \frac{l_v \overline{\Omega_1^*(\tau)}}{\zeta_v(\zeta_v \eta_1 - 1)} + \frac{n_v \overline{\Omega_2^*(\tau)}}{\zeta_v(\zeta_v \eta_2 - 1)} \right\} ds; \quad (1.6)$$

$$z_v = \omega_v(\zeta_v) = \frac{a - i\mu_v b}{2} \zeta_v + \frac{a + i\mu_v b}{2} \frac{1}{\zeta_v}; \quad \zeta_v = \zeta_v(z_v) = \frac{z_v + \sqrt{z_v^2 - a^2 - \mu_v^2 b^2}}{a - i\mu_v b};$$

$$\eta_v = \zeta_v(\tau_v); \quad \zeta_v(\infty) = \infty; \quad l_v = \frac{\mu_{3-v} - \bar{\mu}_1}{\mu_v - \mu_{3-v}}; \quad n_v = \frac{\mu_{3-v} - \bar{\mu}_2}{\mu_v - \mu_{3-v}} \quad (v=1, 2),$$

where $\Omega_v^*(t)$ are unknown complex functions on L ; $r(t)$ are the contact forces of interaction, while $A_v(\tau)$ with $\tau \in \Gamma_s$ ($s = \overline{1, m}$) are determined from the system of equations

$$\operatorname{Im} \left\{ \sum_{v=1}^2 \mu_v^{k-2} A_v \right\} = b_k (4\pi h)^{-1};$$

$$b_1 = -(a_{12} \cos \vartheta_s + a_{26} \sin \vartheta_s) / a_{22}; \quad b_2 = -\sin \vartheta_s;$$

$$b_3 = \cos \vartheta_s; \quad b_4 = (a_{16} \cos \vartheta_s + a_{12} \sin \vartheta_s) / a_{11}.$$

Using the relations $\Omega_v(t) = -2\pi i \Omega_v^*(t) / M_v(t)$, $d\tau_v = M_v(\tau) ds$ (ds is an element of an arc), we represent $\Phi_v^3(z_v)$ in the form

$$\Phi_v^3(z_v) = \frac{1}{2\pi i \omega'_v(\zeta_v)} \int_{\Gamma} \left\{ \frac{\Omega_v(\tau) d\tau_v}{\eta_v - \zeta_v} - \frac{l_v \overline{\Omega_1(\tau)} d\bar{\tau}_1}{\zeta_v(1 - \zeta_v \eta_1)} - \frac{n_v \overline{\Omega_2(\tau)} d\bar{\tau}_2}{\zeta_v(1 - \zeta_v \eta_2)} \right\}. \quad (1.7)$$

The functions $\phi_{\nu}^j(z_{\nu})$ ($j = 2, 3$) determined by Eqs. (1.5), (1.7) satisfy the boundary conditions $X_n = Y_n = 0$ on Λ and (1.2) on Γ and decays at infinity. Thus, the choice of the functions $\phi_{\nu}(z_{\nu})$ in the form (1.4) automatically ensures satisfaction of Eqs. (1.2) and the external loading conditions.

2. With allowance for (1.4-1.7), boundary conditions (1.1) can be assigned in the form

$$\begin{aligned} \int_L \frac{\Omega(\tau) d\tau_1}{\eta_1 - \zeta_1} + \int_L \{K_1(t, \tau) \Omega(\tau) + K_2(t, \tau) \overline{\Omega(\tau)}\} ds + \int_{\Gamma} K_3(t, \tau) r(\tau) ds &= f_1(t); \\ \Omega_2(t) &= -a(t) \Omega(t) - b(t) \overline{\Omega(t)}; \quad \Omega_1(t) = \Omega(t); \\ K_1(t, \tau) ds &= \frac{\omega'_1(\zeta_1)}{2b(t)} \left\{ \frac{\overline{b(\tau)} - \overline{b(t)}}{\omega'_2(\zeta_2)(\eta_2 - \zeta_2)} d\tau_2 + \overline{b(\tau)} \left[\frac{\overline{\omega'_2(\eta_2)} - \overline{\omega'_2(\zeta_2)}}{\omega'_2(\zeta_2)(\eta_2 - \zeta_2)} d\eta_2 - \right. \right. \\ &\quad \left. \left. - \frac{\omega'_1(\eta_1) - \omega'_1(\zeta_1)}{\omega'_1(\zeta_1)(\eta_1 - \zeta_1)} d\eta_1 + d \ln \frac{\eta_2 - \zeta_2}{\eta_1 - \zeta_1} \right] + \frac{\overline{a(t)}}{\omega'_1(\zeta_1) \zeta_1} \left[\frac{\overline{l_1} d\tau_1}{1 - \zeta_1 \eta_1} - \right. \right. \\ &\quad \left. \left. - \frac{\overline{n_1} a(\tau) d\tau_2}{1 - \zeta_1 \eta_2} \right] - \frac{n_1 \overline{b(t)} \overline{b(\tau)} d\tau_2}{\omega'_1(\zeta_1) \zeta_1 (1 - \zeta_1 \eta_2)} - \frac{1}{\omega'_2(\zeta_2) \zeta_2} \left[\frac{\overline{l_2} d\tau_1}{1 - \zeta_2 \eta_1} - \frac{n_2 a(\tau) d\tau_2}{1 - \zeta_2 \eta_2} \right] \right\}; \\ K_2(t, \tau) ds &= -\frac{\omega'_1(\zeta_1)}{2b(t)} \left\{ \frac{\overline{a(t)} - \overline{a(\tau)}}{\omega'_2(\zeta_2)(\eta_2 - \zeta_2)} d\tau_2 + a(t) \left[\frac{\overline{\omega'_1(\eta_1)} - \overline{\omega'_1(\zeta_1)}}{\omega'_1(\zeta_1)(\eta_1 - \zeta_1)} d\eta_1 - \right. \right. \\ &\quad \left. \left. - \frac{\overline{\omega'_2(\eta_2)} - \overline{\omega'_2(\zeta_2)}}{\omega'_2(\zeta_2)(\eta_2 - \zeta_2)} d\eta_2 + d \ln \frac{\eta_1 - \zeta_1}{\eta_2 - \zeta_2} \right] + b(\tau) d\tau_2 \left[\frac{\overline{n_1} a(t)}{\omega'_1(\zeta_1) \zeta_1 (1 - \zeta_1 \eta_2)} + \right. \right. \\ &\quad \left. \left. + \frac{n_2}{\omega'_2(\zeta_2) \zeta_2 (1 - \zeta_2 \eta_2)} \right] + \frac{l_1 \overline{b(\tau)} d\tau_1}{\omega'_1(\zeta_1) \zeta_1 (1 - \zeta_1 \eta_1)} - \frac{n_1 \overline{b(t)} \overline{a(\tau)} d\tau_2}{\omega'_1(\zeta_1) \zeta_1 (1 - \zeta_1 \eta_2)} \right\}; \\ K_3(t, \tau) &= \frac{\pi i \omega'_1(\zeta_1)}{b(t)} \left\{ \frac{\overline{a(t)}}{\omega'_1(\zeta_1)} \overline{R_1(t, \tau)} + \frac{\overline{b(t)}}{\omega'_1(\zeta_1)} R_1(t, \tau) + \frac{1}{\omega'_2(\zeta_2)} \overline{R_2(t, \tau)} \right\}; \\ f_1(t) &= -\frac{\pi i \omega'_1(\zeta_1)}{b(t)} \{ \overline{a(t)} \overline{\Phi_1^1(t_1)} + \overline{b(t)} \Phi_1^1(t_1) + \overline{\Phi_2^1(t_2)} \}; \\ R_{\nu}(t, \tau) &= \frac{A_{\nu}(\tau)}{\zeta_{\nu} - \eta_{\nu}} + \frac{l_{\nu} \overline{A_1(\tau)}}{\zeta_{\nu}(\zeta_{\nu} \eta_1 - 1)} + \frac{n_{\nu} \overline{A_2(\tau)}}{\zeta_{\nu}(\zeta_{\nu} \eta_2 - 1)}. \end{aligned}$$

After certain transformations, the strain compatibility condition (1.3) can be written in the form

$$\begin{aligned} \operatorname{Re} \left\{ \int_{\Gamma} \frac{f^*(t, \tau) r(\tau) ds}{\tau - t} + \int_L K_4(t, \tau) \Omega(\tau) ds \right\} + \int_{\Gamma} K_5(t, \tau) r(\tau) ds &= f_2(t); \\ f^*(t, \tau) &= 2(\tau - t) \sum_{\nu=1}^2 \frac{S_{\nu}(t) A_{\nu}(\tau)}{\omega'_{\nu}(\zeta_{\nu})(\zeta_{\nu} - \eta_{\nu})}; \\ S_{\nu}(t) &= p_{\nu} \cos^2 \vartheta_s + r_{\nu} \sin \vartheta_s \cos \vartheta_s + q_{\nu} \mu_{\nu} \sin^2 \vartheta_s \quad (t \in \Gamma_s); \\ K_4(t, \tau) ds &= \frac{1}{\pi i} \left\{ \frac{\overline{S_1(t)}}{\omega'_1(\zeta_1)} \frac{d\tau_1}{\eta_1 - \zeta_1} - \frac{S_2(t)}{\omega'_2(\zeta_2)} \frac{a(\tau) d\tau_2}{\eta_2 - \zeta_2} + \frac{\overline{S_2(t)}}{\omega'_2(\zeta_2)} \times \right. \\ &\quad \times \frac{\overline{b(\tau)} d\tau_2}{\eta_2 - \zeta_2} + \sum_{\nu=1}^2 \frac{S_{\nu}(t)}{\omega'_{\nu}(\zeta_{\nu}) \zeta_{\nu}} \frac{n_{\nu} \overline{b(\tau)} d\tau_2}{1 - \zeta_{\nu} \eta_2} - \sum_{\nu=1}^2 \frac{\overline{S_{\nu}(t)}}{\omega'_{\nu}(\zeta_{\nu}) \zeta_{\nu}} \left[\frac{a(\tau) \overline{n_{\nu}} d\tau_2}{1 - \zeta_{\nu} \eta_2} - \frac{\overline{l_{\nu}} d\tau_1}{1 - \zeta_{\nu} \eta_1} \right] \Big\}; \\ K_5(t, \tau) &= 2 \operatorname{Re} \left\{ \sum_{\nu=1}^2 \frac{S_{\nu}(t)}{\omega'_{\nu}(\zeta_{\nu})} \left[\frac{l_{\nu} \overline{A_1(\tau)}}{\zeta_{\nu}(\zeta_{\nu} \eta_1 - 1)} + \frac{n_{\nu} \overline{A_2(\tau)}}{\zeta_{\nu}(\zeta_{\nu} \eta_2 - 1)} \right] \right\} - k^*(t, \tau); \\ f_2(t) &= P(t) - 2 \operatorname{Re} \left\{ \sum_{\nu=1}^2 S_{\nu}(t) \Phi_{\nu}^1(t_{\nu}) \right\}. \end{aligned} \tag{2.2}$$

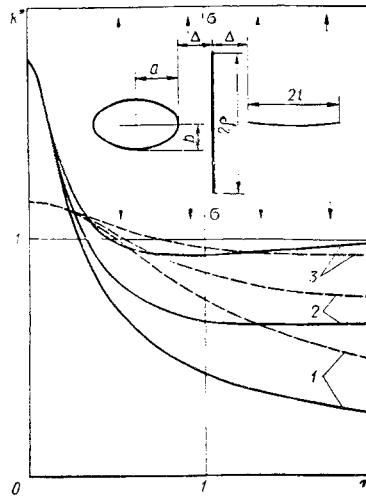


Fig. 1

TABLE 1

i	ρ	N		
		8	16	24
1	0,2	1,272	1,262	1,262
	1	0,687	0,677	0,671
	2	0,632	0,654	0,654
3	0,2	1,340	1,233	1,231
	1	0,731	0,701	0,694
	2	0,662	0,661	0,661

In conformity with the assumptions made regarding L , the kernels $K_j(t, \tau)$ ($j = \overline{1, 5}$), $f^*(t, \tau)$ are continuous.

In addition to (2.3), (2.4), conditions expressing the nonambiguity of the displacements in the circumvention of the crack L_j and the equilibrium of the rib located along Γ_s must be satisfied [6]:

$$\int_{L_j} \Omega(\tau) d\tau_1 = 0 \quad (j = \overline{1, k}); \quad (2.3)$$

$$\int_{\Gamma_s} r(\tau) ds = P_s^1 - P_s^2 \quad (s = \overline{1, m}). \quad (2.4)$$

In accordance with the assumptions made regarding L and Γ , Eqs. (2.1) and (2.2) constitute a system of singular integral equations having an index equal to "1." With the additional restrictions (2.3), (2.4), in the class of functions

$$\chi_j(t) = \chi_j^0(\beta) (1 - \beta^2)^{-1/2}, \quad t = \tau^j(\beta) \quad (j = \overline{1, k+m}), \quad (2.5)$$

where $\chi_j^0(\beta)$ is a bounded function continuous on $[-1, 1]$ in accordance with the Holder condition, a solution of the system exists and is unique [3].

The limiting cases of the problem being examined: a reinforced half-plane ($x > 0$) with a system of curvilinear slits ($b/a \rightarrow \infty$); a reinforced infinite plate with a rectilinear slit $\Lambda = \{|x| < a; y = 0\}$ and a system of curvilinear slits L_j ($b/a = 0$).

In the customary manner, we reduce the solution of system (2.1-2.4) to the solution of a system of linear algebraic equations relative to approximate values of sought functions of the form (2.5) at Chebyshev nodes $\beta_p = \cos\left(\frac{2p-1}{2N} \cdot \pi\right)$ ($p = \overline{1, N}$). Theoretical estimates of the convergence of this numerical method are given in [1], for example.

If the end $a_p = \tau P(-1)$ of the slit L_p reaches the contour of the hole Λ , then condition (2.3) is not satisfied with $j = p$ because the connectedness of the region changes. In this case, the condition should be discarded, while potentials (1.7) and the integral equations are refined by a familiar method [9]. To numerically solve the equations which result in this case, below we use a simplified procedure [9] based on representation of the sought after function in the form (2.5).

3. Representations (1.4) turned out to be an effective tool for determining the stresses in the neighborhood of the tips of internal and edge discontinuities in reinforced and unreinforced anisotropic plates with an elliptical hole Λ .

Below, for the case of uniform tension by the forces $\sigma_y^\infty = \sigma$, we present certain results of calculations of the stress intensity factor for tensile stresses $k_1 = \lim_{t \rightarrow c} \sigma_n \sqrt{2\pi r}$ at the tips

c of a crack (t is a point lying on the continuation of the crack beyond the end c along a tangent, $r = |t - c|$) in a plate made of a glass-epoxy composite ($E_1 = 53.84$ GPa; $E_2 = 17.95$ GPa; $G_{12} = 8.63$ GPa; $\nu_1 = 0.25$) and an isotropic material ($\nu = 0.33$). The values of k_1 were calculated by the formulas presented in [6]. Data for the isotropic material was obtained from an asymptotic numerical solution in anisotropy parameters.

Let $L = \{\tau^1(\beta) = a + 2\Delta + l(1 + \beta) \mid |\beta| < 1\}$ be an internal crack on the axis x in an anisotropic plate. The principal directions of anisotropy E_1, E_2 coincide with the axes x, y , respectively. The plate is reinforced by a stiffening rib along the segment $\Gamma = \{\tau^2(\beta) = a + \Delta + i\rho\beta \mid |\beta| < 1\}$ passing through the middle of the bridge of material between the tip of the crack $\tau^1(-1)$ and the hole Λ . Figure 1 shows the dependence of the correction factor $k^* = k_1(-1)/k_0$ ($k_0 = \sigma\sqrt{\pi\ell}$) at the left tip of L on $\eta = \rho/a$ (2ρ is the length of the rib) with $\ell/a = 1$, $m = a - b/a + b = 0$ (circular hole), $u = E_1 h a / E^0 F^0 = 0$ (curves 1-3). The solid (dashed) lines pertain to the case $\Delta/a = 0.1$ (0.5). The length of the rib ρ and its relative stiffness u depend appreciably on the value of k^* . The limiting case $\eta = 0$ gives the solution of the problem of an internal crack in an unreinforced plate with a hole. At $m = 1$ (two cracks symmetrically located relative to the rib), the results of calculations agree with the data in [6]. Our calculations demonstrated the good convergence of the algorithm. With $a = 1$, $m = 0$, $\Delta = 0.1$, and $u = 0$ (see Fig. 1), Table 1 shows the values of k^* for different ℓ and ρ . The crack and the rib were subdivided by $N = 8, 16$, and 24 Chebyshev nodes. The accuracy decreases somewhat with a decrease in Δ and an increase in u .

Let $L_{1,2} = \{\tau^{1,2}(\beta) = \pm[a + l(1 + \beta)] \mid |\beta| < 1\}$ be boundary cracks originating from the contour of the hole at the vertices of the ellipse $(\pm a, 0)$. The plate is reinforced by stiffening ribs along the symmetrical segments $\Gamma_{1,2} = \{\tau^{3,4}(\beta) = \pm W + i\rho\beta \mid |\beta| < 1\}$ ($W > a + 2l$).

Figure 2 shows the dependence of $K = k_1/k_*$ ($k_* = \sigma\sqrt{\pi(a + 2l)}$) - the stress intensity factor at the tips of an "equivalent" crack $L^* = \{|x| < a + 2\ell, y = 0\}$ in an infinite unreinforced plate from the action of the stresses $\sigma_y^\infty = \sigma$ on $\eta = \rho/a$ at $\delta = 2\ell/a = 0.1, 0.5$, and 0.9 (curves 1-3), $w = W/a = 2$, $u = 1$, and $m = 0$. At $\eta > 3$, the values of K are relatively constant. At $\eta = 0$, we obtain the solution of the problem of two boundary cracks in an unreinforced plate. The solid (dashed) lines here and below pertain to the case when the principal direction of anisotropy E_1 of the plate material forms the angle $\varphi = 0$ ($\pi/2$) with the axis x .

Figure 3 shows the dependence of K on δ with $w = 2$, $u = 1$, $\eta = 2$, and different values of $m = 1; 0.5; 0; -0.5; -0.9$ (curves 1-5). At $m = 1$, the calculated results agree with the data in [6].

The calculations show that the value of k_1 depends appreciably on the anisotropy of the material, the stiffness, length, and location of the reinforcing elements, and the form of the hole. With an increase in the degree of anisotropy E_1/E_2 , the difference between the values of k_1 at $\varphi = 0$ and $\varphi = \pi/2$ increases. It follows from analysis of the curves in Figs. 2 and 3 that it is not always possible in calculations to examine the given effect (hole - boundary cracks) as an "equivalent" crack, as is usually done in practice.

Many investigators (see the survey in [5]) have examined the Bowie problem concerning radial cracks reaching the edge of a circular hole in an isotropic plate. Table 2 shows values of $k_1/(\sigma\sqrt{2\pi\ell})$ calculated at the tips of one ($k = 1$) or two ($k = 2$) boundary cracks $L_{1,2}$ originating from points $(\pm a, 0)$ of the contour of a hole in an unreinforced isotropic plate. The values were obtained for the case of $N = 8.16$ Chebyshev nodes and different $\delta = 2\ell/a$ by

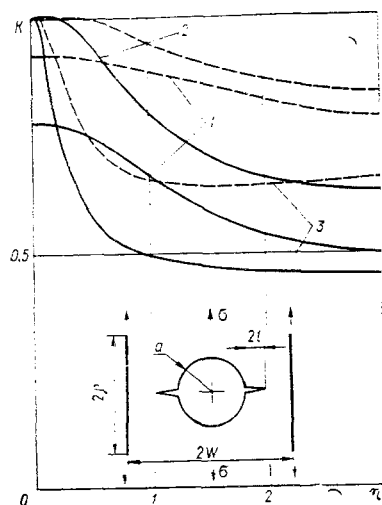


Fig. 2

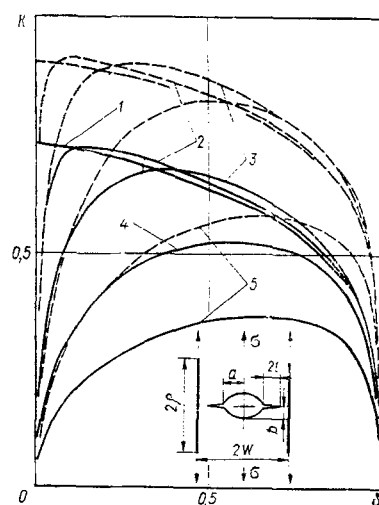


Fig. 3

TABLE 2

δ	$k=1$			$k=2$		
	$N=8$	$N=16$	[13]	$N=8$	$N=16$	[11]
0.01	3,288	3,292	3,291	3,289	3,293	—
0.1	2,769	2,772	2,771	2,782	2,785	2,780
1.0	1,305	1,306	1,306	1,470	1,471	1,471
10	0,779	0,780	—	1,048	1,049	1,049
∞	0,707	0,707	—	1,000	1,000	1,000

finding the asymptote for an isotropic medium in a numerical solution. The results are compared with data from [10, 12] for different crack lengths. The data in Table 2 shows the high degree of efficiency of the simplified algorithm for numerically solving the integral equations of the problem in the case of boundary cracks: there is no loss of accuracy for short cracks; to obtain a result that is accurate to within 0.1%, it is necessary to have approximately 16 Chebyshev nodes in the solution of the corresponding equations.

LITERATURE CITED

1. S. M. Belotserkovskii and I. K. Lifanov, Numerical Methods in Singular Integral Equations [in Russian], Nauka, Moscow (1985).
2. L. T. Berezhnitskii, V. V. Panasyuk, and N. G. Stashchuk, Interaction of Stiff Linear Inclusions and Cracks in a Deformable Body [in Russian], Naukova Dumka, Kiev (1983).
3. N. P. Vekua, Systems of Singular Integral Equations and Certain Boundary-Value Problems [in Russian], Nauka, Moscow (1970).
4. D. V. Grilits'kii, Prikl. Mekh., 2, No. 2, 159-166 (1956).
5. A. A. Kaminskii, Brittle Fracture near Holes [in Russian], Naukova Dumka, Kiev (1982).
6. V. N. Maksimenko, "Problem of an anisotropic plate weakened by curvilinear cracks and reinforced by stiffening ribs," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 163-169 (1982).
7. V. Z. Parton and E. M. Morozov, Mechanics of Elastoplastic Fracture [in Russian], Nauka, Moscow (1985).
8. G. N. Savin, Stress Distribution around Holes [in Russian], Naukova Dumka, Kiev (1968).
9. M. P. Savruk, Two-Dimensional Problems of the Theory of Elasticity for Bodies with Cracks [in Russian], Naukova Dumka, Kiev (1981).
10. Y. C. Hsu, "The infinite sheet with two radial cracks from cylindrical hole under inclined tension or in-plane shear," Int. J. Fract. Mech., 13, No. 6, 839-845 (1977).
11. P. S. Theocaris and D. Bardzokas, "The influence of a finite stringer on the stress intensities around cracks in plates," Eng. Fract. Mech., 14, No. 3, 493-506 (1981).
12. J. Tweed and D. P. Rooke, "The distribution of stress near the tip of radial crack at the edge of circular hole," Int. J. Eng. Sci., 11, No. 11, 1185-1195 (1973).