

CRACK GROWTH IN ROLLING BODIES UNDER THE CONDITIONS OF DRY FRICTION AND WETTING

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Within the framework of a two-dimensional mathematical model, we study the development of a surface macrocrack in the course of unidirectional rolling under the conditions of dry friction and wetting. The damaged body in a rolling couple is modeled by an elastic half plane with an edge cut (crack) and the action of the counterbody is simulated by the translational motion of Hertzian contact forces along the boundary of the half plane with taking into account the friction forces. We compute the crack-growth paths depending on the coefficient of contact friction between the bodies and the orientation and length of the crack. The numerical calculations are performed by the step-by-step method based on the use of the singular integral equations of the theory of elasticity for bodies with curvilinear cracks and the criterion of generalized opening. It is shown that, for high coefficients of friction between the bodies of the rolling couple ($f = 0.20$ – 40 ; dry friction), a branch propagates from the original fatigue shear macrocrack into the bulk of the material of the driven body. In the case of low friction coefficients ($f < 0.20$; wetting), a branch mainly propagates along the boundary of the body in the direction of motion of the counterbody. We propose an explanation of the mechanism of development of burrow-type defects often encountered in railroad rails.

The phenomenon of fatigue fracture of structural materials subjected to rolling contact is often encountered in the engineering practice. Thus, it is observed in the process of operation of engineering wheel–rail couples as well as in various machine parts such as ball and roller bearings, mill rolls, gears, etc. Numerous experimental works were devoted to the investigation of fracture of rolling bodies under the conditions of dry friction and wetting (see, e.g., the surveys in [1–4]). At the same time, the number of theoretical studies of the process of crack propagation in rolling bodies is much smaller and, as a rule, the authors restrict themselves to the numerical analysis of the stress intensity factors (SIF) and the angles of deviation for initial cracks in two- [5–9] and three-dimensional [10] statements of the problem. As model contact loads, it is customary to use either concentrated [5, 6] or Hertzian contact forces [7–10].

In the present work, we use the two-dimensional model of fatigue fracture of rolling bodies proposed in [11]. The key element of this model is an algorithm of step-by-step calculation of the fatigue-crack growth path based on the use of singular integral equations of the theory of elasticity for bodies with curvilinear cracks and criteria of local fracture in the complex stressed state. The algorithm takes into account the changes in the stress-strain state at the crack tip caused not only by the process of crack growth in the damaged body but also by the motion of the counterbody. Unlike other approaches, the proposed model enables one to study the fracture processes in rolling bodies more efficiently.

It is known [12, 13] that, in the case of contact rolling fatigue, a surface macrocrack first propagates at an acute angle to the surface of the body according to the mechanism of shear and then according to the mechanism of tension. In the present work, we study the second stage of crack growth in detail. In our calculations, we use the criterion of generalized normal opening (σ_θ -criterion). We analyze the crack growth paths in the driven body for the case of unidirectional rolling depending on the coefficient of friction between the bodies and the initial length and orientation of the crack. We compare our results with the experimental data and crack growth paths constructed earlier by modeling the action of the counterbody by concentrated forces [11].

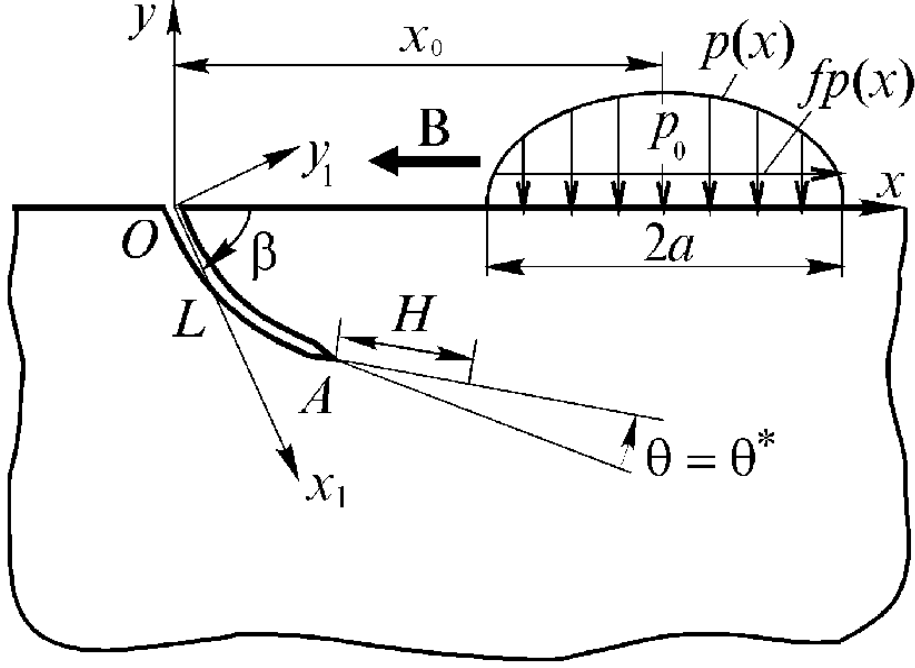


Fig. 1. Computational scheme of the model (B is the direction of motion of the counterbody).

Basic Assumptions of the Model

We assume that one of cylindrical bodies in a rolling couple is damaged by a surface macrocrack. The typical linear size of the contact area and the crack length are small as compared with the radius of curvature of the contact surface of the bodies. Therefore, in the two-dimensional statement of the problem, instead of the actual body, we consider a half plane with edge cut (crack). We also assume that rolling is cyclic and unidirectional. The contact action of the second body (counterbody) upon the analyzed body is modeled by the repeated translational unidirectional motion of Hertzian contact forces along the boundary of the half plane

$$s(x) = \frac{-p_0(1+if)\sqrt{a^2 - (x - x_0)^2}}{a}, \quad |x - x_0| \leq a, \quad (1)$$

where p_0 is the maximum pressure in the middle of the contact segment of length $2a$ (Fig. 1) and f is the coefficient of Coulomb contact friction. The location of the contact segment relative to the crack mouth is specified by the parameter x_0 .

In the process of rolling, the bodies slide relative to each other with fairly high friction ($f = 0.1-0.4$) [9, 14]. This promotes the formation of zones of tension in the rolling bodies, especially in the driven body. Hence, the cracks may appear and grow in these bodies according to the mechanism of tension.

We now introduce a parameter $K_{I\theta}$ determining the intensity of normal hoop stresses at the crack tip (point A in Fig. 1) for a given configuration of the fatigue crack and loads. It can be expressed via the stress intensity factors (SIF) K_I and K_{II} by the formula [11]

$$K_{I\theta}(l, x_0, \theta) = \cos^3 \frac{\theta}{2} \left[K_I(l, x_0) - 3 \tan \frac{\theta}{2} K_{II}(l, x_0) \right], \quad (2)$$

where l is the length of the (rectilinear or curvilinear) crack and θ is the polar angle measured from the tangent to the crack at its tip A . In every cycle of motion of the contact load along the boundary of the half plane, the param-

ter $K_{I\theta}$ varies (as a function of x_0) and takes its maximum value $K_{I\theta\max} = K_{I\theta}(l, x_0^*, \theta^*)$ for some $x_0 = x_0^*$ and $\theta = \theta^*$. We assume that, in each loading cycle, the crack grows only if $x_0 = x_0^*$ and $K_{I\theta\max} > K_{th}$, where K_{th} is the threshold of fatigue crack growth in the material in tension. Thus, according to the σ_θ -criterion and the arguments presented above, the direction of crack propagation is specified by the angle [11]

$$\theta^* = 2 \arctan \frac{K_I(l, x_0^*) - \sqrt{K_I^2(l, x_0^*) + 8K_{II}^2(l, x_0^*)}}{4K_{II}(l, x_0^*)}. \quad (3)$$

The crack growth path is constructed step by step. For this purpose, we consider two types of steps: basic H connected with crack growth and auxiliary $\Delta\lambda$ ($\lambda = x_0/a$) caused by the motion of the contact load. In each stage of construction the crack path, its increment H is plotted from the crack tip A in the direction $\theta = \theta^*$, i.e., in the direction specified by relation (3). The auxiliary step (parameter) $\Delta\lambda$ is used to find $K_{I\theta\max}$ in a loading cycle, i.e., to determine x_0^* . In each step of construction of the path (for the corresponding number of loading cycles), we assume that the quantities θ^* , $\lambda^* = x_0^*/a$, and $K_{I\theta\max}$ are constant. Every increment of the path is approximated by a cubic polynomial whose coefficients are found from the conditions of smooth conjugation of the adjacent parts of the path [15]. Thus, the path turns into a smooth curve.

In each stage of construction of the crack growth path, we find the stress intensity factors K_I and K_{II} from the solution of the first basic problem of the theory of elasticity for a half plane with edge curvilinear cut (Fig. 1). The boundary conditions for this problem have the form

$$\sigma_y(x) - i \tau_{xy}(x) = s(x), \quad |x - x_0| \leq a, \quad \sigma_y(x) - i \tau_{xy}(x) = 0, \quad |x - x_0| > a, \quad (4)$$

on the boundary of the half plane and

$$N^\pm(t) + iT^\pm(t) = 0, \quad t \in L, \quad (5)$$

on the crack lips.¹ Here, L is the crack contour known from the construction of the increments of crack path in the previous stages, N and T are, respectively, the normal and tangential forces applied to the crack lips (the positive direction of tangential forces coincides with the direction of tracing of the crack contour when the body remains to the left), and the superscripts “+” and “−” denote the limiting values of the quantities on approaching the crack contour from the left and right, respectively. The problem is reduced to a singular integral equation. In the normalized form, it can be represented as [16]

$$\int_{-1}^1 [R(\xi, \eta)\varphi(\xi) + S(\xi, \eta)\bar{\varphi}(\xi)]d\xi = \pi P(\eta, \lambda), \quad |\eta| < 1. \quad (6)$$

Its right-hand side is given by the formula [17]

$$P(\eta, \lambda) = p_0 \left\{ \operatorname{Re}[(1 + if)c(\eta, \lambda)]\omega'(\eta) - e^{2i\beta}\bar{\omega}'(\eta) \left[\frac{(1 - if)\bar{c}(\eta, \lambda)}{l_1\bar{a}(\eta, \lambda)} \operatorname{Im}(\varepsilon\omega(\eta)e^{-i\beta}) - if\bar{c}(\eta, \lambda) \right] \right\}, \quad (7)$$

$$c(\eta, \lambda) = a(\eta, \lambda) - ib(\eta, \lambda),$$

¹ We omit the subscripts indicating the number of the stage of construction of the path. Actually, in the first stage, for the initial crack, we must write $L = L_1$, in the second stage, $L = L_2$, etc.

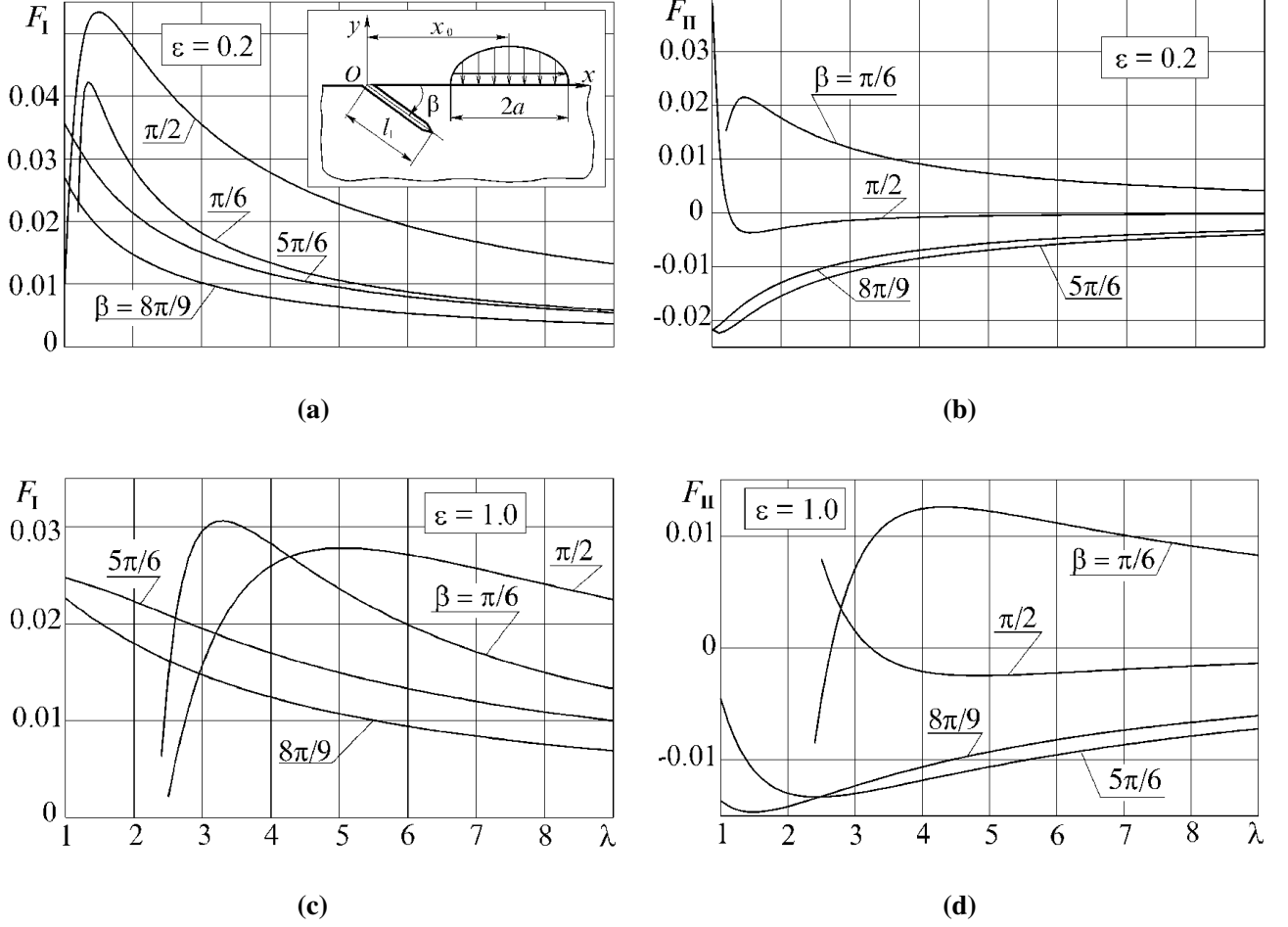


Fig. 2. Dependences of the coefficients $F_I = K_I / (p_0 \sqrt{\pi a})$ and $F_{II} = K_{II} / (p_0 \sqrt{\pi a})$ for an edge rectilinear crack of length $l_1 = \varepsilon a$ on the relative distance $\lambda = x_0/a$ between the crack mouth and the load (for the coefficient of contact friction between the rolling bodies $f = 0.25$).

$$a(\eta, \lambda) = \sqrt{1 - b^2(\eta, \lambda)}, \quad b(\eta, \lambda) = \frac{\varepsilon \omega(\eta) e^{-i\beta}}{l_1} - \lambda,$$

where

$$t = \omega(\eta) \quad (t \in L, \quad |\eta| < 1) \quad (8)$$

is a parametric equation of the crack contour L and $\varepsilon = l_1/a$ is the relative length of the initial macrocrack. The integral equation (6) is solved numerically by the method of mechanical quadratures. As a result, we arrive at a system of N linear algebraic equations in N unknowns whose solution enables us to find the stress intensity factors K_I and K_{II} (2) at the crack tip.

In each consecutive stage of construction of the path, the outlined procedure is repeated (for a new value of crack length).

Numerical Results (Analysis of the SIF and Crack Paths)

We performed numerical calculations for various friction coefficients ($f = 0.1-0.4$). These coefficients, in particular, correspond to different conditions of operation of actual wheel-rail couples (dry or wet weather). It is as-

sumed that, under the conditions of wet weather, only the coefficient of contact friction decreases but the amount of water penetrating into the crack is insignificant for its wedging in the process of rolling, i.e., condition (5) is satisfied. We studied initial rectilinear cracks of different relative lengths ($\varepsilon = l_1/a = 0.2-2.0$) and angles of orientation β (see the diagram in Fig. 2a). By analyzing the accumulated numerical data on the SIF, we established the ranges of the parameter λ for which the initial crack is open ($K_I > 0$). For the specified direction of action of the tangential forces, the corresponding ranges of λ are located to the right of the crack mouth ($\lambda \geq 1$). For all other λ , we observe either complete or partial contact of the crack lips. In particular, this follows from the plots presented in Figs. 2a and c, where the values of the factor K_I are given for $f = 0.25$.

As can be expected, near the crack mouth, the quantities K_I and K_{II} for initial open cracks strongly depend on the distance between the contact area and the crack mouth (Fig. 2). In this region, the SIF attain their maximum absolute values and, moreover, the smaller the crack length, the higher these values. As the distance between the contact area and the crack mouth increases, the SIF gradually decrease to zero.

Table 1. Starting Values of the Quantities λ^* , θ^* , and $F_{I\theta_{\max}}$ for Initial Cracks in the Process of Rolling under the Conditions of Dry Friction Depending on the Coefficient of Contact Friction f between the Bodies (a), Angle of Orientation β (b), and Relative Crack Length $\varepsilon = l_1/a$ (c)

		λ^*	F_I	F_{II}	θ^* , rad	$F_{I\theta_{\max}}$
<i>a</i>		$\beta = 5\pi/6, \quad \varepsilon = 1.0$				
<i>f</i>	0.10	1.3	0.0003	0.0135	-1.222	0.0158
	0.15	1.0	0.0076	0.0126	-1.041	0.0191
	0.20	2.5	0.0151	-0.0083	0.735	0.0201
	0.25	1.9	0.0226	-0.0128	0.746	0.0304
	0.30	1.5	0.0311	-0.0180	0.754	0.0422
	0.40	1.1	0.0497	-0.0311	0.783	0.0697
<i>b</i>		$f = 0.25, \quad \varepsilon = 1.0$				
β	$\pi/6$	3.6	0.0300	0.0116	-0.604	0.0355
	$\pi/2$	5.0	0.0278	-0.0025	0.174	0.0282
	$5\pi/6$	1.9	0.0226	-0.0128	0.746	0.0304
	$8\pi/9$	1.0	0.0227	-0.0135	0.766	0.0312
<i>c</i>		$f = 0.25, \quad \beta = 5\pi/6$				
ε	0.2	1.0	0.0357	-0.0217	0.773	0.0495
	0.5	1.3	0.0292	-0.0174	0.765	0.0401
	1.0	1.9	0.0226	-0.0128	0.746	0.0304

Table 2. Starting Values of the Quantities λ^* , θ^* , and $F_{I\theta_{\max}}$ for an Initial Crack in the Process of Rolling with Wetting Depending on the Angle of Its Orientation β (a) and Relative Length $\varepsilon = l_1/a$ (b)

		λ^*	F_I	F_{II}	θ^* , rad	$F_{I\theta_{\max}}$
<i>a</i>		$f = 0.10, \quad \varepsilon = 1.0$				
β	$8\pi/9$	1.0	0.0046	0.0062	− 0.996	0.0100
	$5\pi/6$	1.3	0.0003	0.0135	− 1.223	0.0158
	$3\pi/4$	3.1	0.0003	0.0048	− 1.208	0.0057
	$2\pi/3$	10.4	0.0044	− 0.0012	0.477	0.0048
<i>b</i>		$f = 0.10, \quad \beta = 5\pi/6$				
ε	0.2	1.5	0.0082	− 0.0043	0.722	0.0107
	0.5	1.0	0.0026	0.0129	− 1.165	0.0164
	1.0	1.3	0.0003	0.0135	− 1.222	0.0158

The numerical analysis of the available data enables us to establish the time of start of the initial crack, i.e., the location of the load (the value of λ^*) for which the parameter $K_{I\theta}$ for an open crack attains its maximum value $K_{I\theta_{\max}}$ in a loading cycle and determine the initial direction of crack propagation (the angle θ^* corresponding to λ^*). The values of these quantities depending on the orientation and length of the crack and the coefficient of contact friction are presented in Tables 1 and 2. Here, the angle of orientation $\beta = 5\pi/6$ should be regarded as preferable as the most typical angle of formation and propagation of a shear macrocrack in the driven body of a rolling couple in the first stage of its development [12, 13].

In what follows, we analyze some specific features of the paths of edge cracks (Figs. 3 and 4). The major part of numerical calculations was carried out for $N = 90$, $H = 0.02$, and $\Delta\lambda = 0.05$. In finding the starting values of θ^* and $K_{I\theta_{\max}}$, we, as a rule, used the values of K_I and K_{II} with four correct significant digits.

For large friction coefficients ($f = 0.20$ – 0.40), the direction of propagation of a tensile crack is practically independent of its initial orientation and length (Fig. 3). Moreover, after a short initial period, the crack grows rectilinearly into the bulk of the material at the same, for any given f , acute angle γ to the boundary toward the place of action of the tangential contact forces. The higher the contact friction, the stronger the “attraction” between the boundary and the crack (Fig. 3a): Thus, for the friction coefficient $f = 0.15$, we have $\gamma \approx 86^\circ$, for $f = 0.20$, we have $\gamma \approx 81^\circ$, for $f = 0.25$, we have $\gamma \approx 79^\circ$, for $f = 0.30$, we have $\gamma \approx 77^\circ$, and, finally, for $f = 0.40$, we have $\gamma \approx 72^\circ$. It is obvious that if the friction coefficient f is fixed, then there exists an angle of orientation of the initial crack $\beta = \gamma$ such that $\theta^* = 0$ and the crack propagates in the plane of its initial location. If we know this angle, then it is possible to predict the direction of growth of an arbitrarily oriented crack propagating according to the mechanism of tension.

If the friction coefficients are small, then the process of crack growth exhibits a somewhat different tendency. Thus, for $f = 0.10$ and broad ranges of orientations ($\beta > \pi/2$) and lengths of the initial crack, the starting values of θ^* are negative (see Table 3), i.e., the crack initially grows toward the boundary. Nevertheless, the analysis of the paths (Fig. 4) shows that, after its start, the crack grows not to the boundary but along it (the curves for $\beta = 5\pi/6$

and $\beta = 3\pi/4$ in Fig. 4a and the curve for $\varepsilon = 1.0$ in Fig. 4b). Note that this phenomenon takes place for large starting values of the angle θ^* ($\theta^* \approx -1.22$ rad). If the angles of orientation satisfy the inequality $\beta < \pi/2$, then the crack, as a rule, starts from the boundary and propagates into the bulk of the material (Table 3).

At the same time, one may also observe intermediate paths (the curve for $f = 0.15$ in Fig. 3a, the curve for $\beta = 8\pi/9$ in Fig. 4a, and the curve for $\varepsilon = 0.5$ in Fig. 4b): For a certain period of time, the crack propagates along the boundary and then turns into the bulk of the material. This phenomenon is observed if the values of the parameters f , β , and ε are such that the angles θ^* are negative and relatively small.

The analysis of crack paths demonstrates that the prediction of the behavior of cracks based only on the starting values of θ^* , often encountered in the literature, can be erroneous. Indeed, to give an adequate description of the process of crack propagation under the conditions of complex stressed state typical of rolling contact, it is necessary to construct the crack path.

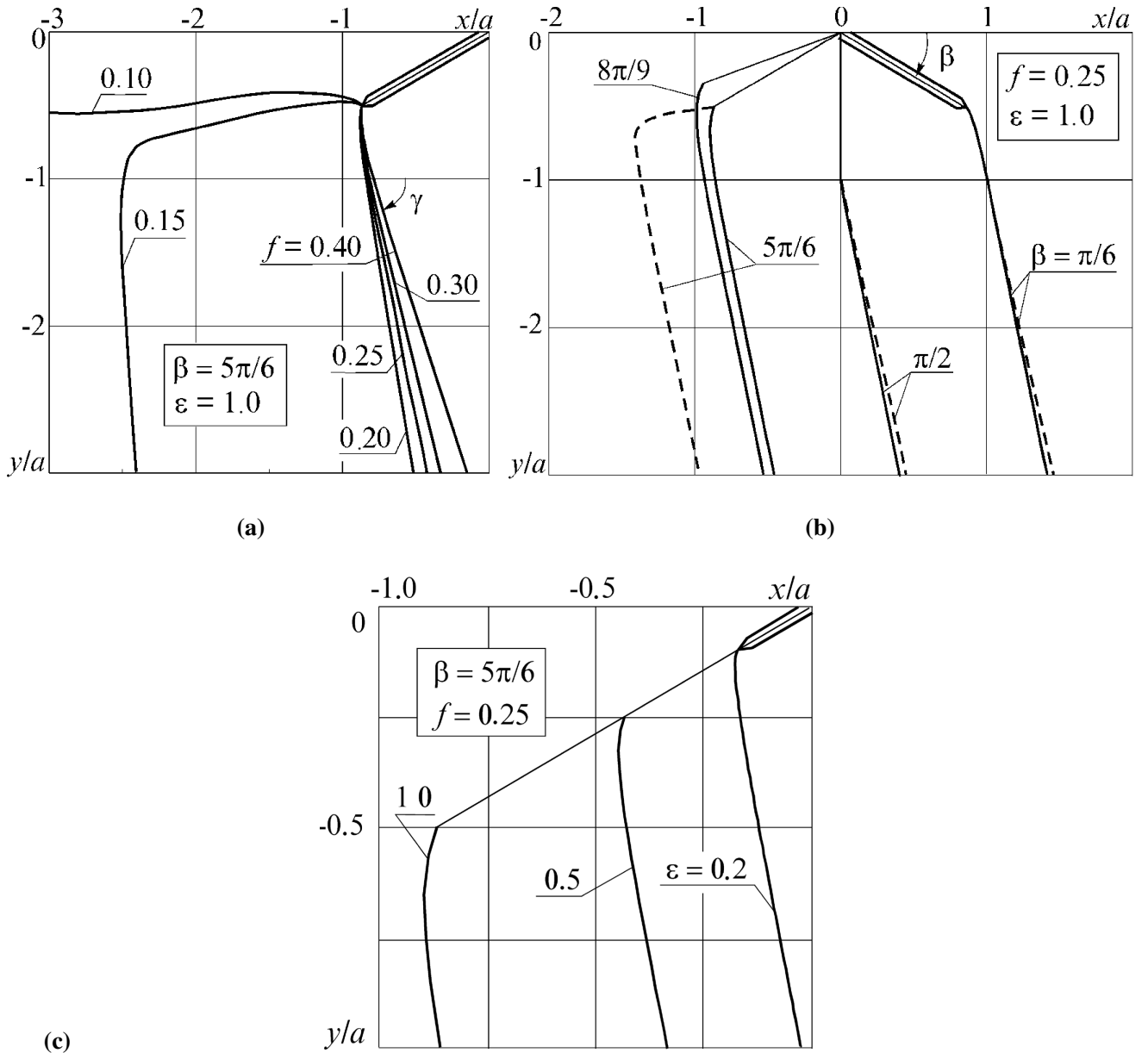


Fig. 3. Crack growth paths in the process of rolling under the conditions of dry friction depending on the friction coefficient f (a), angle of initial orientation of the crack β (b), and relative crack length ε (c). The solid and dashed lines correspond to $\varepsilon = 1.0$ and $\varepsilon = \infty$ [11], respectively.

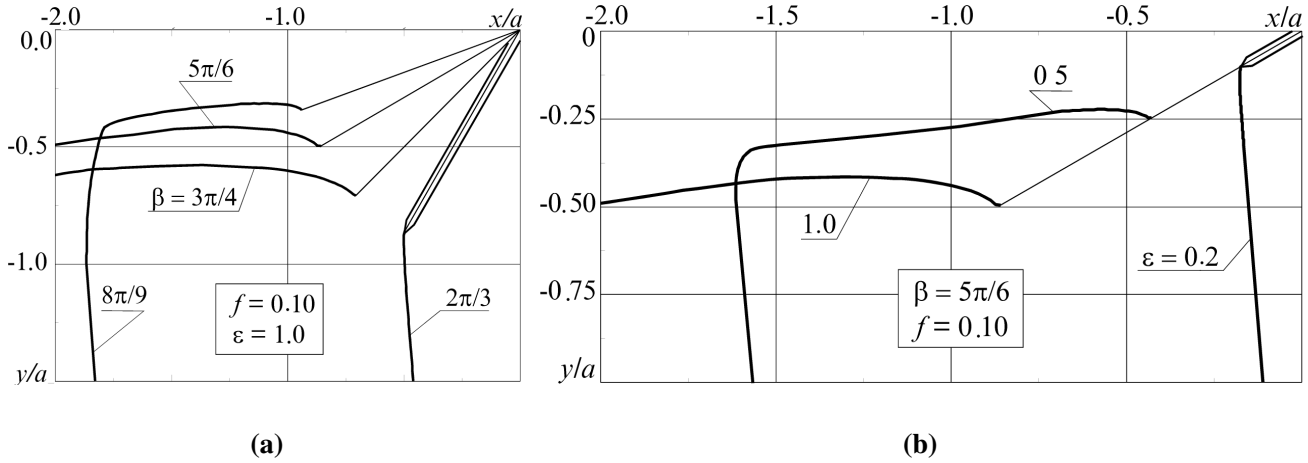


Fig. 4. Crack growth paths in the process of rolling with wetting depending on the initial orientation (a) and length (b) of the crack for the friction coefficient $f = 0.10$.

Table 3. Dependences of the Initial Angle of Crack Propagation θ^* (rad) on the Relative Crack Length $\varepsilon = l_1/a$ and the Angle of Orientation β for $f = 0.10$

ε	$\beta = 8\pi/9$	$\beta = 5\pi/6$	$\beta = 3\pi/4$	$\beta = 2\pi/3$	$\beta = \pi/4$	$\beta = \pi/6$	$\beta = \pi/8$
0.2	0.782	0.722	-1.184	0.485	-0.549	-0.686	-0.754
0.3	0.765	-0.974	-1.229	0.478	-0.539	-0.661	-0.734
0.5	0.750	-1.166	-1.212	0.478	-0.531	-0.644	-0.707
0.7	-0.827	-1.219	-1.227	0.476	-0.529	-0.642	1.225
1.0	-0.996	-1.223	-1.221	0.468	-0.528	-0.639	1.227
1.2	-1.045	-1.225	-1.226	0.401	-0.526	-0.637	1.221
1.4	-1.076	-1.227	-1.230	0.278	-0.527	-0.637	1.222
1.6	-1.096	-1.228	-1.225	0.294	-0.527	-0.636	1.225
1.8	-1.111	-1.230	-1.229	-1.213	-0.526	-0.636	1.229
2.0	-1.122	-1.224	-1.226	-1.229	-0.526	1.230	1.218

The results presented above agree with the experimental data obtained for large [13, 18] and small [13, 19, 20] friction coefficients and well agree with our own results obtained in [11] where the contact load is modeled by concentrated forces (see Fig. 3b for $\varepsilon = \infty$).

Discussion

Cracklike burrow-type defects (Figs. 5a and b) represent one of the most typical types of damage to railroad rails [3, 4, 12]. Kaneta et al. experimentally reproduced this defect with the help of a two-roller machine [13]. In Fig. 5c, a defect of this sort is depicted schematically as in [4].

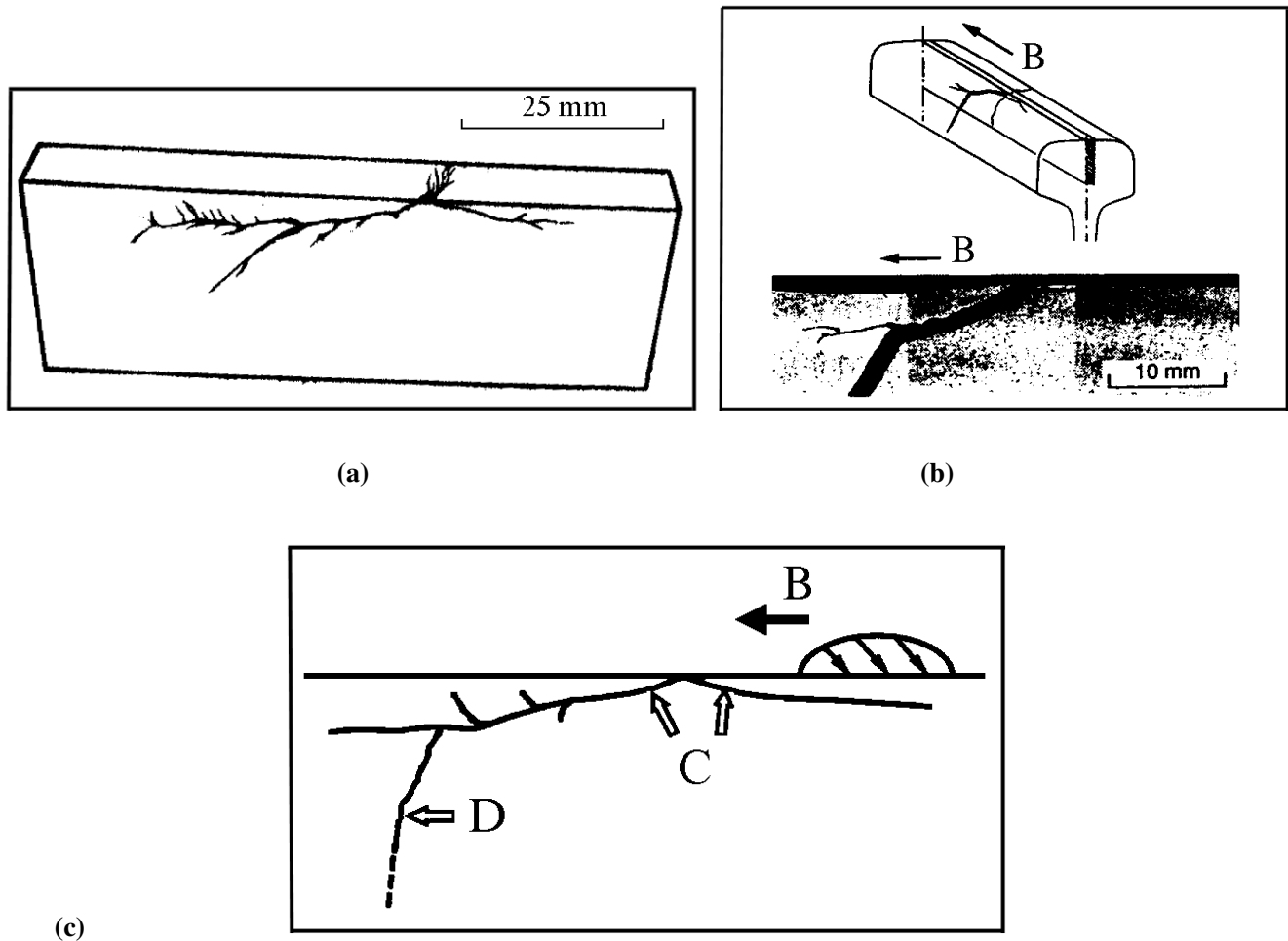


Fig. 5. Cracklike burrow-type defect in a rail: (a) cross section of the defect [12], (b) the same defect on passing of a weight of 1.5×10^{11} kg [13], (c) possible mechanisms of the development of a burrow [4]; (B) direction of motion, (C) shear mechanism of crack growth, (D) possible causes of crack growth into the bulk of the rail (bending? friction forces? thermal stresses? residual stresses? hydraulic effect?).

In [12, 13], it was shown that incipient macrocracks are formed on the surface of the driven element of a rolling couple. First, they propagate according to the mechanism of transverse shear at an acute angle to the rolling surface not only in the direction of motion of the load but also in the opposite direction (Fig. 5). The development of the left branch of the defect, i.e., in the direction of motion of the counterbody is accompanied, unlike the case of the right branch, by the appearance of new branches directed toward the surface of the body and in the opposite direction. After a certain period of time, the left branch splits into two main cracks. One of these cracks propagates along the surface of the body but the second one turns into the bulk of the rail. The mechanism of development of defects of this sort is studied insufficiently, especially concerning the formation of branches propagating into the bulk of the material (see Fig. 5c).

The results obtained in the present work enable us to make the following conclusions: In the process of its propagation, a crack initiated in the subsurface layer of a rolling body turns from the surface into the bulk of the material if the friction forces between the rail and the wheel are sufficiently large, i.e., under the conditions of dry friction (dry weather). This crack grows by the mechanism of tension into the bulk of the material with subsequent deviation (from the normal to the surface) in the direction of rolling which coincides with the direction of tangential forces (see Fig. 3). These observations answer the question posed by Murakami et al. [4] about the main branch of the crack propagating from the boundary (Fig. 5c) and confirm the experimental data concerning the direction of crack growth [18].

In the case of wetting, the coefficient of contact friction between the rolling bodies is small [14] and the crack does not go into the bulk of the material but may propagate (also according to the mechanism of tension) along the boundary of the body (see Fig. 4).

In our opinion, the phenomenon of crumbling of the surfaces of railroad rails may be caused by wetting of these surfaces. This becomes possible if the horizontal branch of the initial crack crosses another edge crack [19] or water penetrates into the crack and promotes its wedging. As indicated in [21], in the second case, the crack grows to the surface of the body.

CONCLUSIONS

In the present work, we study the development of an edge fatigue macrocrack in the driven body of a rolling couple under the conditions of dry friction and wetting when the rolling load is modeled by the unidirectional cyclic motion of Hertzian contact forces. The crack growth paths are computed under the assumption that the crack propagates according to the mechanism of tension.

For large coefficients of contact friction between the bodies ($f = 0.20\text{--}0.40$; dry friction), the direction of subsequent crack propagation insignificantly depends on the initial orientation and length of the crack. After a certain starting period, the crack grows into the bulk of the material at a practically constant (for any given f) acute angle to the boundary of the body toward the place of action of the tangential contact forces (see Fig. 3).

In the case where the friction coefficients in the contact zone are small ($f < 0.20$; wetting), the cracks start to the boundary of the body ($\theta^* < 0$; Table 3) within fairly broad ranges of their initial lengths and orientations ($\beta \geq 2\pi/3$). If the starting values of θ^* are large, then these cracks propagate along (parallel to) the boundary. At the same time, if the angles θ^* are negative and relatively small, then the cracks first grow along the boundary and then turn into the bulk of the material.

If the crack makes an acute angle with the direction of tangential contact forces ($\beta < \pi/2$), then it mainly grows into the bulk of the material.

The results obtained in the present work explain the mechanism of the formation of cracklike burrow-type defects typical of railroad rails.

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