



THE CRACK PROPAGATION THEORY UNDER ROLLING CONTACT

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Abstract—A model to calculate the fatigue life (residual durability) of solids under rolling contact, which is based on the concepts of linear fracture mechanics, has been proposed. The construction of a crack growth path in one of the bodies of a contacting pair, using applicable criteria of local fracture, equations of crack growth rate and characteristics of cyclic crack growth resistance of the material, is a key element of the model. The rolling body is modelled by a half-plane and the contact interaction of the bodies in every rolling cycle is simulated by repeated translation motion of normal and tangential forces along the half-plane edge, which are distributed on the contact area by a certain law. The calculations of the path and stress intensity factors along it are carried out by the step-by-step process on the basis of the singular integral equations method. Edge crack growth paths have been investigated and the residual durability of the rolling surface of 75XTCT steel has been estimated.

1. INTRODUCTION

ROLLING PAIRS are widely spread elements of machines and devices. They are, for example, pairs in ball and roller bearings, rolls of rolling mills, different gearings, “rail–wheel” systems, etc. The determination of the working resource for such systems is an important engineering problem.

Since in many cases the working resource of a rolling pair is exhausted in the result of crack initiation and propagation at the contact zone of interacting bodies [1–3], this problem is to be solved with the help of fracture mechanics approaches. Some experimental investigations were carried out, on the basis of which damage analyses under rolling contact fatigue were made and contact fatigue durability of the rolling surfaces was determined in some cases [4–8]. Great attention is given to the determination of the stress–strain state in the vicinity of edge and subsurface stationary cracks. The majority of theoretical works are dedicated to two-dimensional problems, in particular to the determination of stress intensity factors (SIFs) at the tips of a crack in a half-plane under a given load on its boundary (concentrated force, Hertzian contact stresses) and different boundary conditions on the crack faces, which simulate their contact interaction, lubrication influence, etc. [9–15]. These results offered developing deterministic [10] and probabilistic [16, 17] models for the calculation of rolling surfaces durability. An attempt to use the experimentally obtained crack propagation path was undertaken in Keer and Bryant’s work [10]. Theoretical calculation of the crack path at such a complex stress state which appears under rolling is not a simple problem.

A two-dimensional model for the durability calculation under rolling has been suggested in this work, an algorithm of the crack propagation path construction in one of the bodies of a contacting pair being a key element of the model. Furthermore, local fracture criteria to be in accord with mechanical properties of the material and stress–strain state, crack growth rate equations, and characteristics of the cyclic crack growth resistance of the material are applied. The approach to the bodies durability evaluation from the point of view of fracture mechanics [18, 19] and the procedure of its realization under cyclic loading [20], as well as the step-by-step process of the crack propagation path construction [21, 22] on the basis of the singular integral equations method [21, 23], have been used in the model. The rolling body is modelled by a half-plane and the contact interaction in every cycle of rolling is simulated by a translation motion of normal and tangential forces along the edge of the half-plane, which are distributed on the contact area by a certain law (elliptic and parabolic load distribution, concentrated force, etc.). Two independent steps are introduced, making it possible to take into account the redistribution of the stresses related to both crack tip advance and contact area movement. It is pertinent to note

that data on the crack propagation path in tribosystems make it possible to predict break-away sizes of contacting bodies' surfaces, to reveal mechanisms and reasons of failures and damages, among them pittings and spallings.

Numerical results have been obtained for the case when the contact forces are modelled by a concentrated force, σ_θ -criterion is selected as the local fracture criterion and the crack growth rate is taken from Paris' equation. Edge crack propagation paths have been investigated and the fatigue life (residual durability) of the rolling surface has been evaluated for 75XГCT steel. Besides, some regularities of rolling surfaces' pittings have been established.

2. MODEL FOR ROLLING SURFACE DURABILITY CALCULATION

For convenience, we shall consider that there is a surface crack in one of the bodies of a rolling pair of cylindrical shape. Since the contact area width and crack length are small in comparison with the curvature radius of the body surface, a half-plane with an edge crack is to be considered instead of the real body (a two-dimensional statement of the problem). The contact action upon the body with the crack from the side of the other body (counter-body) of the pair is simulated by repeated translation motion of the contact forces along the half-plane boundary, which are distributed on the contact area as

$$N(x) + iT(x) = -(1 + if)p(x) = -p_0(1 + if)s(x, x_0), \quad |x - x_0| < c. \quad (1)$$

Here, the normal $N(x)$ and tangential $T(x)$ forces are related by the Coulomb law with the help of a coefficient of friction f , x_0 is the variable abscissa of the contact area centre (Fig. 1), $2c$ the width of this area, $s(x, x_0)$ a normalized contact pressure on the half-plane boundary and p_0 a force parameter (the maximum pressure on the contact area). It should be considered that frictional forces appearing at the bodies slip are taken into account by means of the tangential ones.

We shall use the apparatus of fracture linear mechanics to carry out an analysis of the phenomenon under consideration, as is customary [3, 9–15, 17].

For rolling bodies the loading is of a cyclic character. Supposing local fracture to be quasi-static, let us consider a relation

$$K = F[K_I(\lambda), K_{II}(\lambda), \theta^*(\lambda)], \quad (2)$$

defining the mechanism of local fracture at the complex stress state. Here, K is a parameter controlling this mechanism; K_I and K_{II} are the SIFs at the crack tip; $\lambda = x_0/l_1$ where l_1 is the length of the initial crack; the quantity

$$\theta^* = f[K_I(\lambda), K_{II}(\lambda)] \quad (3)$$

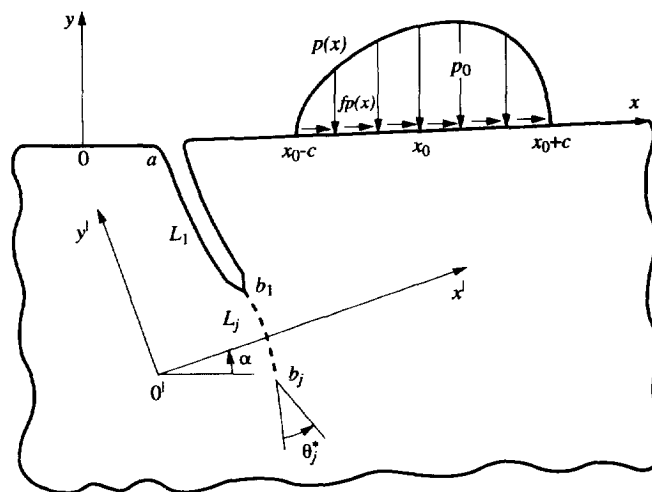


Fig. 1. General geometry-force scheme of the problem.

defines the direction of crack growth (Fig. 1) and provides an extreme value of $F[K_I, K_{II}, \theta]$ if $K_I(\lambda)$ and $K_{II}(\lambda)$ are given.

In every cycle, as a result of contact area movement along the half-plane boundary (upon varying the parameter λ), the function $F[K_I, K_{II}, \theta^*]$ varies taking on its maximum and minimum values at corresponding λ values:

$$\begin{aligned} K_{\max} &= F[K_I(\lambda'), K_{II}(\lambda'), \theta^*(\lambda')] \\ K_{\min} &= F[K_I(\lambda''), K_{II}(\lambda''), \theta^*(\lambda'')]. \end{aligned} \quad (4)$$

The quantity

$$\Delta K = K_{\max} - K_{\min} \quad (5)$$

will be referred to as the range of the function $F(K_I, K_{II}, \theta^*)$ in the rolling cycle. Let us find the maximum absolute value of the function $F[K_I, K_{II}, \theta^*]$ as follows:

$$K_{\max}^* = \max(|K_{\max}|, |K_{\min}|) = |F[K_I(\lambda^*), K_{II}(\lambda^*), \theta^*(\lambda^*)]|, \quad (6)$$

where

$$\lambda^* = \begin{cases} \lambda' & \text{if } |K_{\max}| > |K_{\min}| \\ \lambda'' & \text{if } |K_{\max}| < |K_{\min}| \end{cases}. \quad (7)$$

Let us now count the fact that the crack grows in every rolling cycle in the direction defined by the angle θ^* in eq. (3) at $\lambda = \lambda^*$ if the criterion condition

$$\Delta K > \Delta K_{th} \quad (8)$$

is fulfilled. Here, ΔK_{th} is the threshold range of the function $F[K_I, K_{II}, \theta^*]$, which is a characteristic of the cyclic crack growth resistance of the material.

The crack path is constructed by stages. In the first stage, we solve the problem of the elasticity theory for a half-plane with an initial (in the general case, curvilinear) edge crack along the contour L_1 under the action of the forces given by eq. (1) on the half-plane boundary in view of the unknown abscissa x_0 of the contact area centre and boundary conditions on the contour L_1 taking into account the possibility of crack face contact. The values of the SIFs result from solving this problem. Next, with the help of relations (3), (4) and (6) we determine the value of the parameter $\lambda = \lambda^*$ (i.e. contact area displacement) and corresponding to it the crack propagation angle θ^* at which the value $K_{\max}^*$ of the function $F[K_I, K_{II}, \theta^*]$ is reached. The range ΔK_1 is found according to formula (5) as well. Furthermore, let us consider that condition (8) is fulfilled and the crack propagates for some time (number of cycles N_1) due to a small increment Δl_1 in the direction defined by the angle θ_1^* , i.e. the values of λ^* , θ^* and ΔK_1 are not varied from cycle to cycle in the stage under consideration. The newly constructed part Δl_1 of the path is approximated by the third degree polynomial. In every next stage of the path construction the above-described procedure is repeated (every time for the new extended contour of the crack). Thus, the crack propagation path is obtained as a smooth curve.

Evaluating the articles durability under cyclic loading they discern two periods [24], i.e. the general durability

$$N = N_0 + N_c = N_0 + \int_{l_1}^{l_c} \frac{dl}{v(\Delta K(l))}, \quad (9)$$

where N_0 is the cycles number of loading necessary for initiation of the initial macrocrack† of length l_1 ; N_c is the cycles number in the macrocrack growth phase when it grows along its path from the initial length l_1 to the critical one l_c , which defines the article residual durability. The function

$$v = v[\Delta K(l)] \quad (10)$$

†The phase of crack initiation is not under consideration in this work.

analytically describes a dependence of the crack growth rate vs the range of the parameter K controlling its propagation mechanism. Let us cite Paris' formula

$$v = C(\Delta K)^n, \quad (11)$$

which is frequently used[‡] and will be necessary subsequently.

With the range ΔK of the function $F[K_I, K_{II}, \theta^*]$ established in every stage (along the crack propagation path) we find the crack propagation rate from eq. (10) and, as a result, the residual durability of the rolling surface.

It is worth noting that the variation of a fracture mechanism, depending on the stress-strain state and mechanical properties of the material, may occur in the course of crack propagation. Then, a criterion equation of another mechanism and a condition of transition from one mechanism (criterion) to another should be introduced additionally to the algorithm of the path calculation. Also, crack branching is possible at the transition to the shear mechanism in two areas. In this case, the problem becomes more complicated since the branched contour should be considered and a stepping increment of every branch should be related to the rates of their growth. The step-by-step process of the path construction is acceptable to internal cracks as well. Then length increments at every tip are also related to the rates of their propagation.

3. ALGORITHM OF CRACK PROPAGATION PATH CONSTRUCTION

Let us assume that the crack grows in the opening mode.§ Then, according to the generalized normal opening criterion (σ_θ -criterion), crack propagation occurs in the direction for which the circumferential stresses σ_θ take on a maximum value $K_{I\theta}$ of the mixed-mode SIF. Furthermore, the relation (2) is rewritten as

$$K = K_{I\theta} = \cos^3(\theta^*/2)[K_I - 3 \tan(\theta^*/2)K_{II}] \quad (12)$$

and the angle θ^* is taken from the equality [23]

$$\theta^* = 2 \tan^{-1}[(K_I - \sqrt{K_I^2 + 8K_{II}^2})/(4K_{II})]. \quad (13)$$

Let us consider that in every cycle of rolling (consequently, in every stage of the path construction) there is such a range of variation of the parameter λ when the crack faces are not in contact ($K_I(\lambda) > 0$). It makes possible in every stage for the evaluation of $K_{I\theta, \max}^*$ values to use the solution of the first basic problem of the elasticity theory for a half-plane with a crack (more simple than the contact one). Besides, let us assume that $K_{I\theta, \min} = 0$. Then, the relation (6) becomes

$$K_{I\theta, \max}^* = K_{I\theta, \max} \quad (14)$$

and $\Delta K_{I\theta} = K_{I\theta, \max}$. The criterion condition of crack growth (8) is written, respectively, as

$$K_{I\theta, \max} > K_{Ith}. \quad (15)$$

So, let an elastic isotropic half-plane occupying the lower part of the complex variable plane $z = x + iy$ ($y \leq 0$) be weakened by a smooth edge curvilinear cut (crack) of length l_1 , whose initial contour L_1 related to a local coordinate system $x'O'y'$ has its origin at point a on the edge of the half-plane and the end at point b_1 (Fig. 1). The system $x'O'y'$ is connected with the main coordinate system xOy by the relation $z = z'e^{i\alpha} + z'_0$ in which a complex variable $z' = x' + iy'$, z'_0 is the affix of the point O' in the system xOy , α is the angle between abscissae of the main and local coordinate systems.

Now, according to the suggested model and the aforesaid assumptions the construction of the path increment in the next (j th) stage reduces to solving the first basic problem of the elasticity theory for the half-plane weakened by the edge curvilinear crack with the contour L_j when the load

[‡]Although obtained on the basis of the description of the fatigue fracture diagram of material by the normal opening mechanism in the symmetrical stress field, Paris' formula is applied by many investigators, also in the case of fracture by the generalized normal opening mechanism at the complex stress state [10, 20] or by the transverse shear one [10] by using corresponding characteristics of cyclic crack growth resistance.

§In general, fatigue crack growth under rolling can occur both in opening mode and in shear mode. The advantage of the opening mechanism, especially at the high levels of stresses, is established by experiment in refs [25, 26].

acts on the contact area with the centre at a point $x = x_0$. Thus, the condition (1) on the half-plane boundary together with the condition

$$N^\pm(\tau) + iT^\pm(\tau) = 0, \quad \tau \in L \quad (16)$$

on the faces of the open crack present the boundary conditions of the problem. Note that the indices $+$ and $-$ in eq. (16) correspond to the limiting values of the stresses on the crack faces if approaching its contour L from the left and from the right.

Let us represent the complex stress potentials of the problem as sums

$$\begin{aligned} \Phi(z) &= \Phi_0(z) + \Phi_1(z) \\ \Psi(z) &= \Psi_0(z) + \Psi_1(z). \end{aligned} \quad (17)$$

Here, the potentials Φ_0 and Ψ_0 , characterizing the stress state of a solid half-plane with the edge loaded according to the boundary condition (1), are determined by

$$\begin{aligned} \Phi_0(z) &= \frac{p_0(1+if)}{2\pi i} \int_{x_0-c}^{x_0+c} \frac{s(\zeta, l_1 \lambda)}{\zeta - z} d\zeta \\ \Psi_0(z) &= -\frac{p_0}{2\pi i} \int_{x_0-c}^{x_0+c} \frac{(1-if)z + 2if\zeta}{(\zeta - z)^2} s(\zeta, l_1 \lambda) d\zeta, \end{aligned} \quad (18)$$

and the stress state caused by the discontinuity of displacements $g(t)$ on the cut contour L is described by the functions Φ_1 and Ψ_1 as [21]:

$$\begin{aligned} \Phi_1(z) &= \frac{1}{2\pi} \int_L \left\{ \left[\frac{1}{T-z} - \frac{1}{\bar{T}-z} \right] e^{iz} g'(t) dt + \frac{T-\bar{T}}{(\bar{T}-z)^2} e^{-iz} \overline{g'(t)} d\bar{t} \right\} \\ \Psi_1(z) &= \frac{1}{2\pi} \int_L \left\{ \left[\frac{1}{(\bar{T}-z)^2} - \frac{1}{(T-z)^2} \right] \bar{T} e^{iz} g'(t) dt \right. \\ &\quad \left. + \left[\frac{1}{T-z} - \frac{1}{\bar{T}-z} + \frac{(\bar{T}-T)(\bar{T}+z)}{(\bar{T}-z)^3} \right] e^{-iz} \overline{g'(t)} d\bar{t} \right\} \end{aligned} \quad (19)$$

where $T = re^{ix} + z'_0$.

Satisfying the boundary condition (16) by the potentials (17) we obtain the singular integral equation in the derivative of discontinuity of the displacements $g'(\tau)$, which in the normalized form can be written as [27]

$$\int_{-1}^1 [R(\xi, \eta) \phi(\xi) + S(\xi, \eta) \bar{\phi}(\xi)] d\xi = \pi P(\eta, \lambda), \quad |\eta| < 1 \quad (20)$$

where

$$\begin{aligned} R(\xi, \eta) &= \operatorname{Re} \left[\frac{\omega'(\eta)}{\omega(\xi) - \omega(\eta)} + \frac{\bar{\omega}'(\eta)}{w(\xi, \eta)} \right] + \frac{1}{2} \frac{w(\xi, \xi)}{w^2(\xi, \eta)} \left[\bar{\omega}'(\eta) + \omega'(\eta) e^{2iz} + 2e^{2iz} \bar{\omega}'(\eta) \frac{\omega(\xi) - \omega(\eta)}{w(\xi, \eta)} \right] \\ S(\xi, \eta) &= -\frac{1}{2} \frac{\partial}{\partial \eta} \frac{\omega(\xi) - \omega(\eta)}{\bar{\omega}(\xi) - \bar{\omega}(\eta)} + \frac{\omega'(\eta)}{2} \left[w^{-1}(\xi, \eta) + \frac{\omega(\xi) - \omega(\eta)}{w^2(\xi, \eta)} \frac{\bar{\omega}'(\eta)}{\omega'(\eta)} + e^{-2iz} \frac{\bar{w}'(\xi, \xi)}{\bar{w}^2(\xi, \eta)} \right] \\ w(\xi, \eta) &= \bar{\omega}(\eta) - \omega(\xi) e^{2iz} - 2ie^{iz} \operatorname{Im} z'_0 \\ \phi(\xi) &= g'(t) \omega'(\xi) \end{aligned} \quad (21)$$

$$\begin{aligned} P(\eta, \lambda) &= -2\omega'(\eta) \operatorname{Re} \Phi_0(H) - e^{-2iz} \bar{\omega}'(\eta) [H \bar{\Phi}_0'(H) + \bar{\Psi}_0(H)] \\ H &= \omega(\eta) e^{iz} + z'_0. \end{aligned} \quad (22)$$

¹Subsequently, the indices indicating the sequence number of some stage are, as a rule, omitted. So, e.g. in the first stage the initial crack is under consideration and $L = L_1$.

The parametric equation for the crack contour L_j , which is known from the constructions of the path increments in the $j - 1$ preceding stages, is given by

$$t = \omega(\xi) = x'(\xi) + iy'[x'(\xi)], \quad |\xi| \leq 1. \quad (23)$$

Here, $x'(\xi) = \text{Re}[(b - a)\xi + b + a]/2$ where b is the affix of the running crack tip; $y'(x')$ is the function describing the contour L_j in the local coordinate system $x'O'y'$ placed in such a way as to provide the uniqueness of the function $y'(x')$ through all the stages of the path construction.

The integral equation (20) is solved numerically by the method of mechanical quadratures [21, 23]. Let us represent the unknown function $\phi(\eta)$ in the form $\phi(\eta) = u(\eta)/\sqrt{1 - \eta^2}$. Here, $u(\eta)$ is the continuous function in the closed interval $[-1, 1]$ which satisfies the additional condition

$$u(-1) = 0, \quad (24)$$

providing the solution limitation at the point $\eta = -1$ on the half-plane boundary. Applying Gaussian quadratures to the integral equation (20) and representing the function $u(\eta)$ in the condition (24) as the Lagrange interpolation polynomial by points ξ_k we come to the set of M linear algebraic equations in M unknowns $u(\xi_k)$ ($k = 1, \dots, M$)

$$\begin{aligned} \sum_{k=1}^M [R(\xi_k, \eta_m)u(\xi_k) + S(\xi_k, \eta_m)\bar{u}(\xi_k)] &= MP(\eta_m, \lambda), \quad m = 1, \dots, M-1 \\ \sum_{k=1}^M \left[(-1)^k \tan\left(\frac{2k-1}{4M}\pi\right) u(\xi_k) \right] &= 0, \end{aligned} \quad (25)$$

where $\xi_k = \cos[(2k-1)\pi/(2M)]$ and $\eta_m = \cos(\pi m/M)$ are the zeros of the Chebyshev polynomials of the first $T_M(\xi)$ and the second $U_{M-1}(\eta)$ kinds, respectively.

On the basis of the set solution by the formula [21]

$$K_I - iK_{II} = \frac{1}{M} \frac{\sqrt{\pi|\omega'(1)|}}{\omega'(1)} \sum_{k=1}^M \left[(-1)^k \cot\left(\frac{2k-1}{4M}\pi\right) u(\xi_k) \right], \quad (26)$$

we seek the SIFs and according to formulae (12) and (13), the values of $K_{I\theta}$ and θ^* .

Now, the value of $\lambda = \lambda^*$ can be found when $K_{I\theta}$ reaches its maximum $K_{I\theta, \max}^*$. The search for λ^* is carried out numerically and step-by-step. In the first stage of the path construction all the ranges of values of λ^* are tested in which the crack is open ($K_I(\lambda) > 0$), and in the next stages the ranges are tested in the vicinity of the previous values of λ^* , i.e.

$$\lambda_j^* = \lambda_{j-1}^* \pm mH_1, \quad j = 2, 3, \dots, \quad m = 0, 1, 2, \dots \quad (27)$$

where $H_1 > 0$ is a constant step of the increment of the parameter λ and m is the steps number. In every step eq. (20) is solved, but within one stage of the path construction the procedure of the search for the maximum $K_{I\theta, \max}^*$ is simplified since only the right side of eq. (20) depends on λ .

When λ_j^* is known, from eq. (13) we can find the angle θ_j^* to be the basis for the construction of the $j + 1$ st part of the path. The abscissa of the crack tip is determined from step to step by the relation (Fig. 2) [28]

$$x'_{j+1} = \text{Re } b_1 + \sum_{k=1}^j h_k, \quad (28)$$

where $h_k = H_2 \cos \gamma_k$ is the abscissa increment step in every stage of the path construction, $H_2 > 0$ a constant step of the path increment laid off in the direction of the crack propagation, and γ_k the angle between the local axis $O'x'$ and the path increment direction. In the first stage, the initial crack is under consideration and the next parts of the path increment are approximated by the third degree polynomials for their smoothing in the vicinity of angle points (see refs [21, 22] for details):

$$\begin{aligned} y'_{j+1}(x') &= a_{j+1}(x' - q_j)^3 + b_{j+1}(x' - q_j)^2 + c_{j+1}(x' - q_j) \\ &+ d_{j+1}(q_j \leq x' \leq x'_{j+1}, q_j = x'_j - h_j/2). \end{aligned} \quad (29)$$

The parameters of these polynomials are determined from the conditions of joint evenness of the neighbouring parts of the path increment and by the direction of the next crack growth.

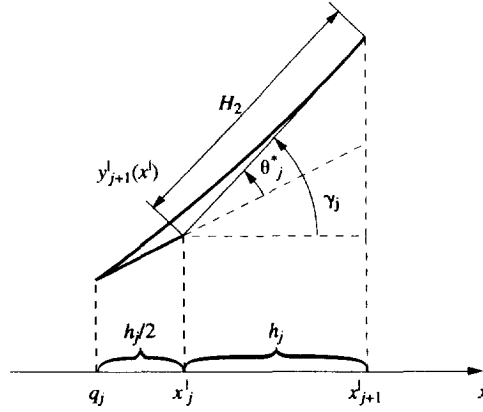


Fig. 2. Construction diagram of next ($j + 1$ st) part of crack path increment.

Thus, the equation of the whole path is as follows:

$$y'(x') = \begin{cases} y'_1(x'), & \text{Re } a \leq x' \leq q_1 = \text{Re } b_1 - h_1/2 \\ y'_k(x'), & q_{k-1} \leq x' \leq q_k, \quad k = 2, \dots, j \\ y'_{j+1}(x'), & q'_j \leq x' \leq x'_{j+1}, \end{cases} \quad (30)$$

where the contour of the initial crack is described by the function $y'_1(x')$ in the interval $[\text{Re } a, \text{Re } b_1]$.

Finally, let us write the expression for durability N_c in eq. (9) in the form

$$N_c \approx \sum_{k=1}^{j_c} \frac{\Delta l_k}{v_k(K_{l\theta, \max})} = H_2 \sum_{k=1}^{j_c} v_k^{-1}(K_{l\theta, \max}), \quad (31)$$

being convenient in the step-by-step construction of the crack propagation path. Here, j_c is the sum number of the path increment steps necessary for growth of the crack from its initial length to the critical one.

4. NUMERICAL ANALYSIS (PATHS AND SIFS)

Let us construct paths of an edge originally rectilinear crack of length l_1 forming an angle β with the half-plane boundary $[z'_0 = 0, a = 0, b_1 = l_1 e^{-i\beta}, \beta = -\alpha; \text{ see scheme in Fig. 3(a)}]$ on the assumption that fracture occurs by the generalized normal opening mechanism. The contact load under rolling is modelled by a concentrated force, which has a normal component P and a tangential component fP and acts at a point $x = x_0$ on the half-plane boundary. The parametric equation of the contour of the initial crack is

$$t = \omega(\xi) = l_1(\xi + 1)/2, \quad |\xi| \leq 1, \quad (32)$$

and the right side (22) of the integral equation (20) can be written as [29]

$$P(\eta, \lambda) = \frac{P}{\pi} \left\{ \text{Re} \left[\frac{(f - i)e^{-i\alpha}}{\varsigma(\eta, \lambda)} \right] \omega'(\eta) - \frac{\text{Re}[\varsigma(\eta, \lambda)(f + i)e^{i\alpha}]}{\bar{\varsigma}^2(\eta, \lambda)} \bar{\omega}'(\eta) \right\},$$

$$\varsigma(\eta, \lambda) = \omega(\eta) + (z'_0 - \lambda l_1)e^{-i\alpha}. \quad (33)$$

The coefficient of friction f is equal to 0.25 and 0.30, which corresponds, e.g. to friction between the wheel and rail in dry weather [30].

The variations of the stress state characteristics in the vicinity of the crack tip during the initial rolling cycles (in the first stage of the path construction) are shown in Fig. 4 [29].

The ranges of the variations of the parameter λ for which the initial crack is fully open ($K_I(\lambda) > 0$) are established on the basis of the numerical data analysis. For the chosen direction of the action of the tangential force fP [Fig. 3(a)] the corresponding ranges of λ are situated on the right from the crack mouth (λ is positive). Full or partial contact of the crack faces takes place with other values of λ . The more detailed analysis can be made on the basis of Fig. 4(a) where

the values of the SIF K_I are presented ($\lambda > 0$). At equal angles between the crack and half-plane boundary the range of λ where $K_I(\lambda) > 0$ is broader for the obtuse angles, which is attributed to the action direction of the tangential force fP . As the angles between the crack and the boundary increase, the corresponding ranges of λ become narrower on the whole. As the tangential force fP (coefficient f) increases, the range of λ where the crack is open becomes broader at the fixed angle β .

The values of the SIFs (for the open crack) vary intensively during concentrated force movement near the crack [Fig. 4(a, b)], which reflects in the behaviour of the quantities θ^* and $K_{I\theta}$ depending on them [Fig. 4(c, d)]. Here the SIFs reach their maximum absolute values. With increasing distance between the point of contact and the crack, the values of the SIFs, including also $K_{I\theta}$, decrease to zero in a gradual manner (hence $K_{I\theta, \min} = 0$).

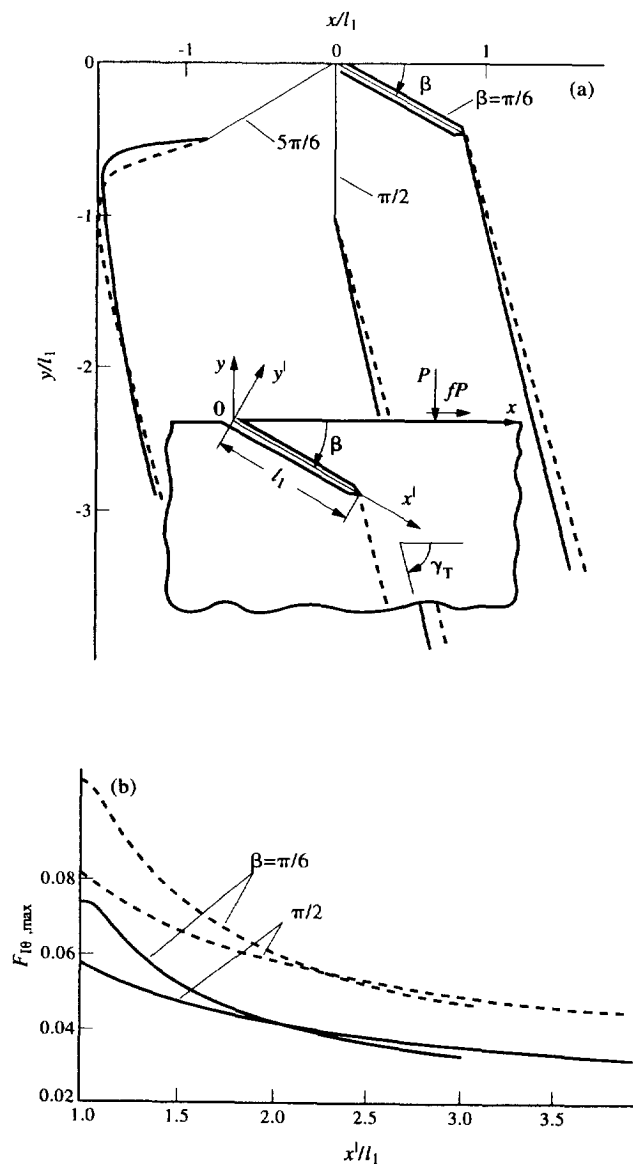


Fig. 3. The growth paths due to the opening-mode mechanism (a) and normalized SIF $F_{I0, \max} = \sqrt{\pi l_1} K_{I0, \max} / P$ along the paths (b) for an initially rectilinear edge crack with length l_1 oriented at an angle β to the half-plane boundary under the action of concentrated force at the boundary, which simulates rolling contact interaction. Solid lines are for the coefficient of friction $f = 0.25$ in contact, while dashed ones are for $f = 0.30$.

It is to be noted that obtained numerical data for K_I and K_{II} are in good agreement with analytical results for the crack perpendicular to the half-plane boundary [31].

The moment of incipient growth of the initial crack (when the coefficient $K_{I\theta}$ reaches the maximum value during loading cycle for the open crack) and the initial direction of its growth (the angle θ_1^* corresponding to λ_1^*) as well can be established on the basis of the analysis of Fig. 4. The numerical values of these quantities for the different angles β of crack orientation and coefficients of friction f are presented in Table 1.

On the basis of crack propagation path analysis [Fig. 3(a)] such a feature should be noted: the direction of further crack propagation is little dependent on initial crack orientation and after some initial period the crack grows into the depth of the material along a straight line at a certain acute angle γ_T to the half-plane boundary towards the tangential contact force action. Furthermore, the higher the friction in contact, the more the crack "presses" to the boundary: the angle $\gamma_T \approx 78^\circ$ for the coefficient of friction $f = 0.25$ and $\gamma_T \approx 76^\circ$ for $f = 0.30$. It should be noted also that in the vicinity of the orientation angle $\beta = \gamma_T$ the behaviour of the curve $K_{II}(\lambda)$ varies [Fig. 4(b)].

Clearly, there exists such an orientation angle of the initial rectilinear crack ($\beta = \gamma_T$) for which $\theta_1^* \approx 0$ and the crack will propagate along its extension. Having found the value of this angle, one can predict the growth direction of the arbitrarily oriented crack propagating by the normal opening mechanism for the specific coefficient of friction f .¶ Crack path predictions on the basis of the initial angles of crack growth, as made by some authors, may cause error [see the path for $\beta = 5\pi/6$ in Fig. 3(a)].

Since this occurred in the depth of the plate, crack propagation is accompanied by the fall of $K_{I\theta, \max}$ [Fig. 3(b)], the main characteristic in the calculation of crack propagation rate and rolling surface durability, which may in some time cause the change of its propagation mechanism.

Let us compare the obtained paths with paths of a crack whose faces are under the cyclic action of a uniformly distributed pressure [Fig. 5(a)] [32]. As an approximation, such a load makes it possible to simulate lubrication action on the faces of the surface crack under rolling contact. The arbitrarily oriented crack (obviously, except symmetry when $\beta = \pi/2$) propagates to the half-plane boundary under pressure on its faces. Thus, the pressure of lubrication or other filling materials penetrating into the interior of the crack is one of the main reasons for surface pitting of bodies under rolling, which is confirmed experimentally [4]; pits of pitting were not observed on

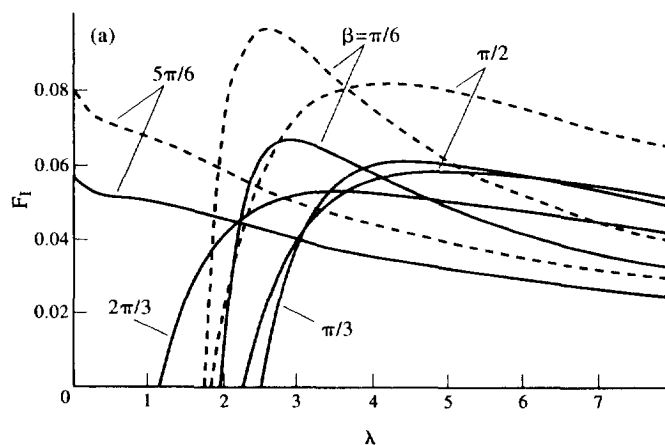


Fig. 4(a). Caption on page 188

¶It is to be noted that edge crack propagation due to transverse shear in a driven body of a rolling pair occurs at an angle of 20° – 30° to the surface in the direction opposite to the tangential forces' action. This regularity is established by experiment [5, 26] when contact fatigue was tested at small coefficients of friction in contact.

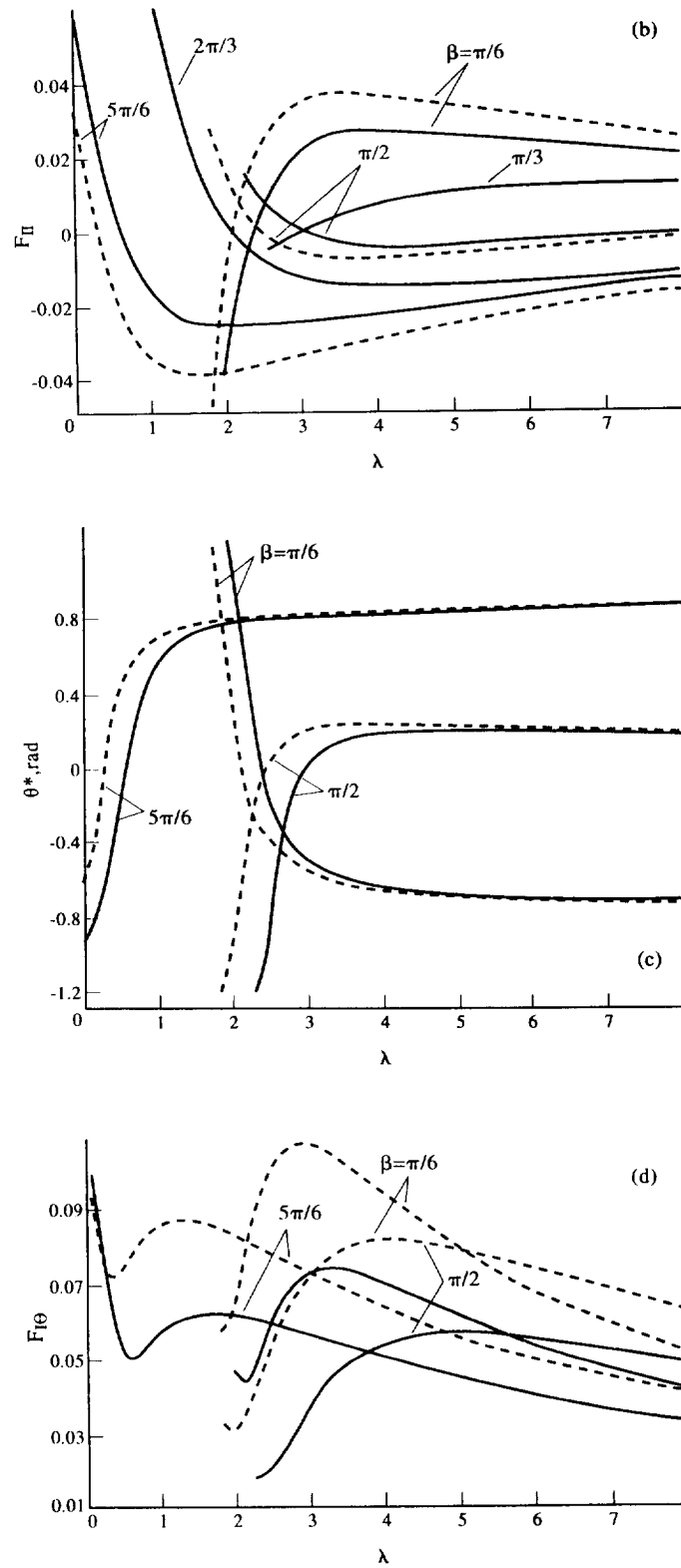


Fig. 4(b), (c) and (d)

Fig. 4. Variation of normalized SIFs $F_I = \sqrt{\pi l_1} K_I / P$ (a) and $F_{II} = \sqrt{\pi l_1} K_{II} / P$ (b) early angles of crack growth (c) and normalized SIF $F_{I\theta} = \sqrt{\pi l_1} K_{I\theta} / P$ (d) for an initially rectilinear edge crack of length l_1 oriented at an angle β to the half-plane boundary vs distance of concentrated force application point on the boundary from crack mouth $x_0 = l_1 \lambda$. Solid lines are for coefficient of friction $f = 0.25$ in contact, while dashed ones are for $f = 0.30$.

Table 1. Stress state characteristics for initial crack under rolling

β (rad)	λ_1^*	θ_1^* (rad)	$F_{I0, \max}$
$\pi/6$	3.3	-0.580	0.0739
	2.9	-0.562	0.1071
$\pi/2$	4.9	0.177	0.0569
	4.1	0.210	0.0815
$5\pi/6$	0.1	-0.904	0.94
	0.1	-0.551	0.88

Data for $f = 0.25$ are above the line, while data for $f = 0.30$ are under the line.

non-lubricated surfaces.†† The pit size depends on the length and orientation of the initial crack: the shorter the crack and the more it is inclined to the body boundary, the less the pit of pitting. It is also to be noticed that the intensive growth of K_{I0} [Fig. 5(b)] takes place when the crack tip approaches the boundary, which causes its spontaneous propagation in some time.

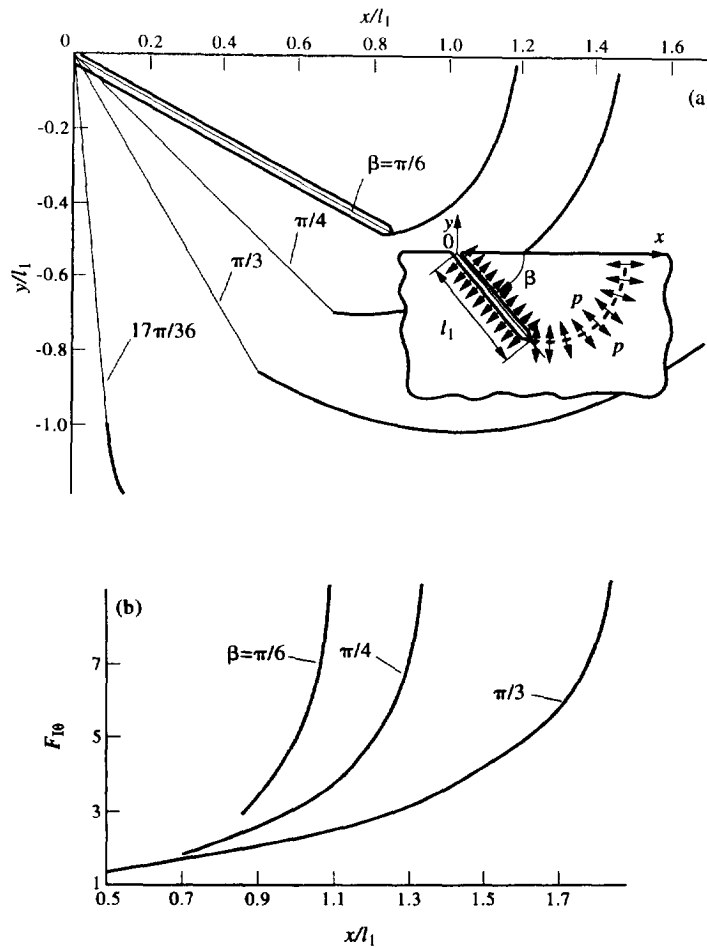


Fig. 5. The growth paths due to the opening-mode mechanism (a) and normalized SIF $F_{I0} = K_{I0}/(p\sqrt{\pi l_1})$ along the paths (b) for an initially rectilinear edge crack with length l_1 oriented at an angle β to the half-plane boundary under pulsating pressure on its faces.

††Pittings may occur on non-lubricated surfaces if surface cracks propagating under the action of contact forces into the depth of the material cross with other cracks or branch towards the boundary. The question of rolling surface spalling being usually connected with the process of subsurface crack propagation is not under consideration in the present work.

Table 2. Residual durability evaluation of rolling surface of 75XГCT steels

β (rad)	B	H_1	H_2	λ_1^*	λ_e^*	Δl (mm)	N_c (cycle)
Concentrated force at the half-plane boundary							
$f = 0.25$							
$\pi/6$	10	$0.1l_1$	$0.05l_1$	3.3	18.0	3.0	2×10^5
$\pi/2$	10	$0.1l_1$	$0.05l_1$	4.9	19.5	3.0	3×10^5
$5\pi/6$	10	$0.1l_1$	$0.01l_1$	0.1	12.8	3.0	1×10^5
$f = 0.30$							
$\pi/6$	10	$0.1l_1$	$0.05l_1$	2.9	15.2	3.0	6×10^4
$\pi/2$	10	$0.1l_1$	$0.05l_1$	4.1	16.4	3.0	8×10^4
$5\pi/6$	10	$0.1l_1$	$0.01l_1$	0.1	10.6	3.0	5×10^4
Uniformly distributed pressure along the crack faces							
$\pi/6$	0.4	—	$0.01l_1$	—	—	0.40	6×10^4
$\pi/4$	0.6	—	$0.01l_1$	—	—	0.76	1×10^5
$\pi/3$	0.8	—	$0.01l_1$	—	—	1.42	2×10^5

5. RESULTS AND CONCLUSIONS

Fatigue life (residual durability) of the rolling surface damaged by an initially rectilinear edge crack of length $l_1 = 1$ mm has been evaluated. The surface is made of 75XГCT steel with lamellar pearlite structure applied in railway rails manufacture. The calculation was carried out on the basis of relation (31) using the numerical data for the SIF $K_{I0, \max}$ [Figs 3(b), 5(b)]. Furthermore, the crack propagation rate was determined by Paris' formula (11). The characteristics of cyclic crack growth resistance of steel are as follows [33]: $K_{I-2} = 5.2 \text{ MPa} \sqrt{\text{m}}$, $K_{2-3} = 29.1 \text{ MPa} \sqrt{\text{m}}$, $C = 3.09 \times 10^{-12} \text{ MPa}^{-n} \text{ m}^{1-n/2}$, $n = 3.48$.

Two ways of plate loading were under consideration: the cyclic translation motion of a concentrated force along the half-plane boundary (to simulate the contact interaction of bodies under rolling) and the cyclic action of a uniformly distributed pressure on the crack faces (to take account of the lubrication influence under rolling). In the first case, in every rolling cycle the normal contact force $P = BK_{2-3} \sqrt{\pi l_1}$; in the second one, the maximum pressure on the crack faces $p_{\max} = BK_{I-2} \sqrt{\pi l_1}$. Furthermore, a constant B was chosen in such a way that the condition

$$K_{I-2} \leq K_{I0, \max} \leq K_{2-3} \quad (34)$$

should be fulfilled in the moment of the crack start and in the process of its propagation. Here, as it is known, K_{I-2} and K_{2-3} are Paris part bounds in the fatigue fracture diagram.

Numerical results of the calculation are presented in Table 2 where the values of the parameter λ^* in the beginning ($\lambda^* = \lambda_1^*$) and in the end ($\lambda^* = \lambda_e^*$) of computations, the number of cycles N_c during which the initial crack grows up to length l_e , and full crack increment $\Delta l = l_e - l_1$ are also given. It is to be noticed that for the first way of loading the initial crack was given the possibility to grow up by 3 mm and for another case the calculation was carried out until the coefficient $K_{I0, \max}$ was not more than K_{2-3} .

As one would expect, the friction increase in contact results in a durability decrease of the rolling surface. It has been established that with the increase of the coefficient f from 0.25 to 0.30 the number of cycles necessary for increasing the crack length by 3 mm is decreased (Table 2). For another way of loading one can come to recognize that the greater the angle between the initial crack and the half-plane boundary, the greater the distance its tip travels to the boundary, which, in turn, causes the surface durability increase in the growth of the crack to the permissible length. It is to be noted that the obtained durability values for the case of the uniform pressure on the crack faces are basically of the same order of magnitude as already known ones [4, 5, 10]. This fact, in our opinion, corroborates Tellian's conclusion [16], that the stage of edge crack propagation under rolling contact (residual durability of the rolling surface) is the main part of the general durability.

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