

ESTIMATING CORROSION-LAYER GROWTH NEAR THE VERTEX OF A CRACK UNDER MECHANICAL LOADING

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The worst corrosion cracking concentrators are [1-4] surface defects such as pitting corrosion, cracks, fatigue cracks, and so on, whose nucleation and growth are often controlled by the medium and the applied internal stresses (e.g., residual ones) or external mechanical forces. When a corrosive medium enters a crack, a corrosion layer is formed at the edges, and as it thickens, the contact between the medium and the metal is reduced. The greatest zone of stable penetration for the corrosive medium to the metal lies at the crack vertex, which is a stress concentrator. It is therefore important to estimate the extent to which reagents penetrate into the corrosion defect around the vertex and the thickness of the corrosion layer in the region of maximum stress concentration.

Formulation. Consider an elastically deformed body that satisfies the conditions of the planar theory of elasticity [5], in which there is a rectilinear crack (Fig. 1), whose edge is γ_0 , and where there is a constant velocity vector $\vec{v}(v_x, v_y)$. The corrosion layer extends at the same speed as the crack. Line γ_1 is the boundary of the contact between the corrosion layer and the metal. We assume that there are constant concentrations c_0 and c_1 of the reagent at γ_0 and γ_1 , namely

$$c|_{\gamma_0}=c_0, \quad c|_{\gamma_1}=c_1, \quad (1)$$

with $c_0 > c_1$. At infinitely remote points or in the inner part of the elastic body, where the conditions of the planar theory of elasticity apply, one has mechanical forces distributed in any fashion. Also, at γ_1 , the activity of the medium is proportional to δc and is subject to the balance condition [6] for the solution:

$$D\partial c/\partial n|_{\gamma_1}=-\delta c \langle \vec{\gamma}, \vec{n} \rangle, \quad (2)$$

in which D is the diffusion coefficient for the reagent, while $\partial c/\partial n$ is the derivative of the concentration c along the normal $\vec{n}$ at the edge of the corrosion layer, while the vector is $\vec{\gamma}_1(\gamma_{1x}, \gamma_{1y})$; then $\dot{\gamma}_{1x}=\partial \gamma_{1x}/\partial t$, $\dot{\gamma}_{1y}=\partial \gamma_{1y}/\partial t$; in which $\langle \vec{\gamma}, \vec{n} \rangle$ is the scalar product of the velocity vector for γ_1 and the normal $\vec{n}(n_x, n_y)$.

One has to determine c around the crack vertex, which is stimulated by the stresses, and to estimate the thinning in the corrosion layer in the maximally stressed zone.

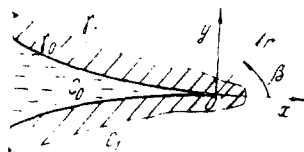


Fig. 1. General formulation scheme.

Initial Equations and Solution. We specify the flux of corrosive medium caused by the mechanical stress pattern as [7]

$$\vec{J}_c = -D \left(\vec{\nabla} c + \frac{cB}{D} V_{ef} \vec{\nabla} p \right), \quad (3)$$

in which B is the mobility of the chemically active medium; V_{ef} is the effective interaction volume between the medium and the metal; p is the hydrostatic pressure, which is related to the mechanical stress tensor; and $\vec{\nabla}$ the Hamilton operator. If we take the diffusion of the reagent into the body as quasistationary, the concentration c satisfies [7, 8].

$$\vec{\nabla}^2 c + k \vec{\nabla} p \vec{\nabla} c = 0, \quad (4)$$

in which $k = BV_{ef}/D$.

We convert to polar coordinates (r, β) and take the concentration function c(r, β) as

$$c(r, \beta) = u(r, \beta) e^{f(r, \beta)}, \quad (5)$$

with [7]

$$\frac{\partial f}{\partial r} = -\frac{k}{2} \frac{\partial p}{\partial r}; \quad \frac{\partial f}{\partial \beta} = -\frac{k}{2} \frac{\partial p}{\partial \beta}, \quad (6)$$

to get the solution to the vector differential equation (4) as equivalent to deriving the unknown function u(r, β):

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \beta^2} - \frac{k^2}{4} \left[\left(\frac{\partial p}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial p}{\partial \beta} \right)^2 \right] u = 0. \quad (7)$$

We consider the vertex of a sharp-ended crack-type defect. If we locate the origin of the (r, β) polar coordinate system at that vertex (Fig. 1), the hydrostatic pressure near it has the asymptote

$$p = \frac{2(1+\mu)}{3\sqrt{2\pi r}} \left(K_I \cos \frac{\beta}{2} - K_{II} \sin \frac{\beta}{2} \right), \quad (8)$$

in which μ is Poisson's ratio and K_I and K_{II} are the stress intensity coefficients related to the stress pattern. The equation (7) in partial derivatives with respect to that asymptote takes the form [7, 8]

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \beta^2} - \frac{\lambda^2}{r^3} u = 0, \quad (9)$$

in which

$$\lambda^2 = s^2 (K_I^2 + K_{II}^2)/16, \quad s = -\frac{2(1+\mu)B V_{ef}}{3\sqrt{2\pi D}}. \quad (10)$$

We neglect the λ^2 term and represent the solution to (9) as

$$u(r, \beta) = r^a \cos \frac{\beta}{2}. \quad (11)$$

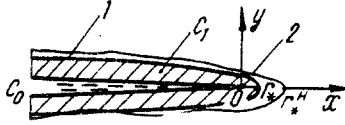


Fig. 2. Scheme for growth of a corrosion layer under mechanical stress: 1) parabolic layer boundary; 2) disrupted layer boundary.

We calculate the partial derivatives and substitute them into (9) to get

$$\alpha = \pm 1/2, (\lambda^2 \approx 0) \quad (12)$$

Then the general solution to (9) is

$$u(r, \beta) = \left(A\sqrt{r} + \frac{B}{\sqrt{r}} \right) \cos \frac{\beta}{2}. \quad (13)$$

From (5), the solution to (4) is

$$c(r, \beta) = \left(A\sqrt{r} + \frac{B}{\sqrt{r}} \right) \cos \frac{\beta}{2} \times \\ \times e^{-\frac{s}{2\sqrt{r}} \left(K_I \cos \frac{\beta}{2} - K_{II} \sin \frac{\beta}{2} \right)} + c_0. \quad (14)$$

As the concentration c of the reagent is c_0 at γ_0 and therefore at the vertex, $c(r, \beta)$ satisfies this condition with $B = 0$. From the condition that the concentration is constant $c = c_1$ at γ_1 , we get that $c(r, \beta)$ satisfies this for $A = c_1 - c_0$, and then the solution becomes

$$c(r, \beta) = c_0 - (c_0 - c_1) \chi(r, \beta) / \chi_1, \quad (15)$$

in which

$$\chi(r, \beta) = \sqrt{2r} e^{-\frac{s}{2\sqrt{r}} \left(K_I \cos \frac{\beta}{2} - K_{II} \sin \frac{\beta}{2} \right)} \cos \frac{\beta}{2}, \quad (16)$$

where χ_1 is the constant value of $\chi(r, \beta)$ on γ_1 . In fact, the equation

$$\chi(r, \beta) = \chi_1 \quad (17)$$

is the equation of γ_1 , on which the reagent concentration is constant and takes the value c_1 . Then this equation enables one to introduce the equations for the edge representing the corrosion front near the crack vertex, at which the mechanical stress concentration is represented in terms of the known stress intensity coefficients.

Corrosion Layer Thickness Estimation. As the growth rate of the corrosion layer in this formulation is dependent on the crack extension rate ($\dot{\gamma}_{1x} = v_x$, $\dot{\gamma}_{1y} = v_y$) and the vertex is taken as rectilinear ($v_y = 0$), the (2) balance equation in this case takes the form

$$\frac{\partial c}{\partial n} \Big|_{\gamma_1} = -\delta c v_x n_x. \quad (18)$$

We use the explicit expressions

$$\frac{\partial \chi}{\partial n} = \frac{\chi(r, \beta)}{2r} \sqrt{1 + \operatorname{tg}^2 \frac{\beta}{2} + \frac{s}{\sqrt{r}} K_I \left(\cos \frac{\beta}{2} - \operatorname{tg} \frac{\beta}{2} \sin \frac{\beta}{2} \right)},$$

$$n_x = \left[1 + \frac{s}{\sqrt{r}} \left(K_I \cos \frac{\beta}{2} - K_{II} \sin \frac{\beta}{2} \right) \right] / \left(\frac{2r}{\chi(r, \beta)} \frac{\partial \chi}{\partial n} \right), \quad (19)$$

and (15) for $c(r, \beta)$ to represent the balance equation (18) on γ_1 as follows:

$$\frac{D(c_0 - c_1)}{2r} \left\{ 1 + \operatorname{tg}^2 \frac{\beta}{2} + \frac{s}{\sqrt{r}} K_I \left(\cos \frac{\beta}{2} - \operatorname{tg} \frac{\beta}{2} \sin \frac{\beta}{2} \right) \right\} = \delta c v_x \left[1 + \frac{s}{\sqrt{r}} \left(K_I \cos \frac{\beta}{2} - K_{II} \sin \frac{\beta}{2} \right) \right]. \quad (20)$$

We note that the other formulas with the above replacement of (9) by Laplace equations represent approximate equations for reagent balance at the edge of the corrosion layer. However, for low K_I and K_{II} , this enables one to derive an estimate for the motion of the layer relative to that for $K_I = K_{II} = 0$. We use (20) with known physicochemical characteristics for the medium and the corroding material and the given values for K_I and K_{II} to estimate the thickness of the corrosion layer around the vertex along the straight lines $\beta = \text{const}$.

We consider the case where $\beta = 0$. Then (20) simplifies to a cubic equation:

$$x^3 - a\sqrt{1 + 4\lambda^2 x} - 2a\lambda + O(\lambda^2) = 0, \quad (21)$$

in which

$$x = \sqrt{r}, \quad a = 0,5D(c_0 - c_1)/\delta c v_x.$$

If we put $\lambda = 0$ in (21), which implies the absence of mechanical stresses ($K_I = K_{II} = 0$), the thickness of the corrosion layer in the crack filled with corrosive medium is

$$r_* = 0,5 \frac{D(c_0 - c_1)}{\delta c v_x}. \quad (22)$$

This value of r_* for this particular case completely coincides with the analogous final result from [9], where the effects from mechanical stresses on the corrosion were neglected. We see from (22) that the thickness of the corrosion layer at the end of the crack is inversely proportional to δc and also to v_x .

If there are no mechanical stresses in the body and we take $\lambda \equiv 0$ in (16), we get the contour of the corrosion front from (17) as

$$\sqrt{2r} \cos \frac{\beta}{2} = \chi_1. \quad (23)$$

We transfer to a Cartesian coordinate system xOy with its center at the vertex of the crack and write the equation for the parabola [10]

$$x = 0,25 r_*^{-1} y^2 + r_*, \quad (24)$$

whose focus lies at the vertex of the crack, $r_* = 0,5\chi_1^2$ (Fig. 2). In this particular case (absence of stresses) this is the equation of the parabola derived in [9] by conformal mapping. This agreement over the shape of the corrosion front confirms that this approach to estimating the corrosion layer thickness is reliable for the case of stresses around the corrosion crack vertex.

If we assume that the corrosion front is perturbed by the stresses, and that the growth rate v_x is constant and is provided by low stresses, namely with $\lambda \ll 1$, we get the equation for the increase in the thickness r_x^1 of the corrosion layer in the presence of mechanical loading relative to r_* in the absence of loading from (21) without incorporating terms of order λ^2 :

$$x^3 - ax - 2a\lambda = 0. \quad (25)$$

The solution for small λ is

$$x = \sqrt{a} + \lambda x_1 + O(\lambda^2), \quad (26)$$

and then substitution into (25) gives $|x_1| = 1$. Then the deviation in the thickness of the corrosion layer r^i stimulated by the medium and the mechanical stresses at the end of the crack relative to r_* is

$$|r^i - r_*| < \left| \frac{2(1+\mu)BV_{ef}}{3} \right| \sqrt{\frac{c_0 - c_1}{\pi D \delta c v_x}}. \quad (27)$$

That difference around the crack vertex is inversely proportional to the square root of the diffusion coefficient D and δc , as well as to that of the crack extension rate. The analogous estimate for the maximal radius of curvature at the vertex of the corrosion layer is found to be finite, so the vertex of the corrosion crack in the presence of mechanical stresses is blunted.

SUMMARY

An analytic determination has been given for the concentration of the reagent near a crack vertex as stimulated by mechanical stresses, and an estimate is made of the displacement of the layer thickness in the maximum stress zone.

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