

INFLUENCE OF COMPRESSIVE STRESSES ON THE PROPAGATION OF SURFACE SHEAR CRACKS IN RAILROAD RAILS

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UDC 539.375

We study the influence of longitudinal compressive residual stresses on the elastic state of a railroad head damaged by a surface crack under the conditions of rolling contact. For this purpose, we solve a two-dimensional contact problem of the theory of elasticity for a half plane weakened by an edge crack whose lips are in contact with friction under the action of a moving Hertz load on the edge of the half plane and uniform uniaxial compression at infinity. The stress intensity factors are found and the maps of contact of the crack lips are constructed for different values of the operating parameters typical of the wheel–rail system. We determine the most dangerous orientations of the crack susceptible to propagation in the zone of compression by the mechanism of transverse shear.

Keywords: crack, contact problem, friction, Hertz load, residual stresses, singular integral equations, stress intensity factors.

In the course operation of wheel–rail engineering couples, the rolling surfaces of railroad rails often fail. In this case, the longitudinal residual stresses (RS) arising in rails exert a substantial influence on this process. In the process of operation, in the subsurface layer of the rolling surface of a rail, we observe the formation of local plastic strain, which leads to mechanical hardening in this region of the railhead. As a result, the residual stresses, which were mainly tensile in the first stage (i.e., after manufacturing) become compressive and attain their maximum values near the surface [1, 2].

At present, on the European railroads, in about 30% of cases, the replacement of rails is caused by the presence of cracklike defects in their heads [3]. This is why surface cracks propagating in the rail head are especially dangerous for rails operating under the conditions of contact rolling fatigue and in the presence of residual stresses. However, the combined influence of the contact stresses and RS on the stressed state in the vicinity of these defects is poorly investigated [4–8]. In studies of this kind, the railroad rail was modeled by a half plane, the action of the wheel upon the rail was modeled by the Hertz contact load, and the influence of residual stresses was modeled by uniform tension/compression at infinity [4, 5], mass forces [6, 7], or by the presence of a plasticity zone [8].

Similar investigations were also performed in [9–12]. However, the case of contact of the crack lips with friction for a crack propagating under conditions of compression was not studied. In what follows, we continue these investigations and solved a contact problem of the theory of elasticity for the half plane weakened by an edge crack with regard for friction between its lips under the action of a moving Hertz load and compressive RS. We determine the orientations of a crack that are most favorable for its development according to the mechanism of transverse shear.

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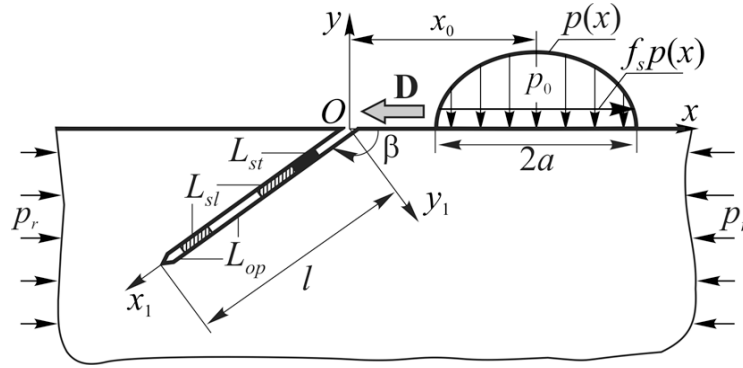


Fig. 1. Schematic diagram of a railhead weakened by an edge crack under the action of the model contact load and compressive residual stresses; $\mathbf{D}$ is the direction of motion of the contact load.

Statement of the Problem and Singular Integral Equations

Instead of a railroad rail damaged by a surface cracklike defect in the two-dimensional formulation, we consider an elastic half plane containing an edge rectilinear arbitrarily oriented crack (Fig. 1). We refer the half plane to the coordinate system xOy whose Ox -axis coincides with the edge of the half plane and the origin O coincides with the crack mouth. The crack is referred to a local coordinate system $x_1O_1y_1$ whose origin is also located at the crack mouth. The contact pressure exerted by the wheel upon the rail is modeled by a unidirectional repeated translation of Hertz contact forces along the edge of the half plane (from right to left). The tangential component of these forces is given by the formula

$$s(x) = -(1 + if_s)p(x) = -p_0(1 + if_s) \frac{\sqrt{a^2 - (x - x_0)^2}}{a}, \quad |x - x_0| \leq a, \quad (1)$$

where f_s is the coefficient of sliding friction between the rolling bodies, p_0 is the maximum value of contact pressure, and a is the half length of the contact region. The longitudinal RS are modeled by uniaxial uniform compressive forces p_r applied at infinity. The location of the contact region relative to the crack mouth is determined by the parameter $\lambda = x_0/a$, the relative crack length is given by the parameter $\varepsilon = l/a$, and its orientation is described by the angle β .

The boundary conditions of the problem on the edge of the half plane and at infinity have the form

$$\sigma_y(x, 0) - i\tau_{xy}(x, 0) = \begin{cases} s(x), & |x - x_0| \leq a, \\ 0, & |x - x_0| > a, \end{cases} \quad (2)$$

$$\sigma_x^\infty = -p_r. \quad (3)$$

For some locations of the Hertz load (some λ) and under the action of RS, the crack lips may be in contact. In what follows, we consider the general case of their contact, namely, the case of sliding with friction and with a possibility of restraining. Then the boundary conditions of the problem on the crack lips can be formulated as follows:

$$N^{\pm}(x_1) + iT^{\pm}(x_1) = 0, \quad x_1 \in L_{\text{op}}; \quad (4)$$

$$v^+(x_1) - v^-(x_1) = 0, \quad x_1 \in L_{\text{sl}}; \quad (5)$$

$$T^{\pm}(x_1) = f_c \operatorname{sign} [\tilde{T}^{\pm}(x_1)] |N^{\pm}(x_1)|, \quad x_1 \in L_{\text{sl}}; \quad (6)$$

$$u^+(x_1) - u^-(x_1) + i[v^+(x_1) - v^-(x_1)] = 0, \quad x_1 \in L_{\text{st}}; \quad (7)$$

$$N^+(x_1) - N^-(x_1) + i[T^+(x_1) - T^-(x_1)] = 0, \quad x_1 \in L_{\text{st}}. \quad (8)$$

In this case, the following condition is satisfied in the regions of restraining of the crack lips:

$$|T^{\pm}(x_1)| < f_c |N^{\pm}(x_1)|, \quad x_1 \in L_{\text{st}}. \quad (9)$$

Here, the contour of a rectilinear crack L consists of the collection of segments L_{op} on which the crack is open, segments L_{sl} with sliding of the crack lips, and segments L_{st} where the crack lips are restrained. We denote by f_c the coefficient of friction between the crack lips. By N , T and v , u we denote, respectively, the normal and tangential components of forces on the crack lips and of displacements of the crack lips in the local coordinate system $x_1 O_1 y_1$. The quantity $\operatorname{sign} [\tilde{T}(x_1)]$ specifies the sign of tangential contact stresses (discussed in what follows). The superscripts “+” or “−” denote the boundary values of the quantities on approaching the crack contour from above and from below, respectively.

We also consider the derivative of the jump of the unknown vector of displacements along the crack contour L in the form of a sum of two functions

$$g'(t) = g'_1(t) + g'_2(t), \quad t \in L, \quad (10)$$

where

$$g_1(t) = \frac{2G}{1+\kappa} [v^+(t) - v^-(t)]; \quad (11)$$

$$g_2(t) = -\frac{2iG}{1+\kappa} [u^+(t) - u^-(t)]. \quad (12)$$

In relations (11) and (12), G is the shear modulus and κ is the Muskhelishvili constant.

Satisfying the boundary conditions of problem (4)–(8) by using the integral transforms [13–15] of the Kolosov–Muskhelishvili complex potentials expressed in terms of the functions $g'_1(t)$ and $g'_2(t)$, we arrive at the following system of singular integral equations (SIE):

$$\operatorname{Re} D(\tau) = \pi \operatorname{Re} P(\tau), \quad \tau \in L_{\text{op}}; \quad (13)$$

$$\operatorname{Im} D(\tau) + n \operatorname{Re} D(\tau) = \pi [\operatorname{Im} P(\tau) + n \operatorname{Re} P(\tau)], \quad \tau \in L_{\text{op}} \cup L_{\text{sl}}, \quad (14)$$

where

$$D(\tau) = I\{L_{\text{op}}\} g'_1(t) + I\{L_{\text{op}} \cup L_{\text{sl}}\} g'_2(t), \quad (15)$$

and the operator $I\{L\}$ is given by the formula

$$I\{L\} \varphi(t) = \int_L \left[R(t, \tau) \varphi(\tau) + S(t, \tau) \overline{\varphi(\tau)} \right] d\tau. \quad (16)$$

The coefficient n in relation (14) takes the following values:

$$n = \begin{cases} 0, & \tau \in L_{\text{op}}, \\ -f_c \operatorname{sign} [\operatorname{Im} (P(\tau))], & \tau \in L_{\text{sl}}. \end{cases} \quad (17)$$

The functions $R(t, \tau)$ and $S(t, \tau)$ are the known kernels of the SIE for the half plane weakened by an open edge crack [13]. The right-hand side of the system of equations (13), (14) is determined by the external loads (2), (3) applied to the half plane and expressed via the function [9, 15]

$$P(\tau) = p_0 \left\{ \operatorname{Re} [(1 + if_s)c(\tau)] - \left[(1 - if_s) \frac{\operatorname{Im} b(\tau)}{a(\tau)} - if_s \right] \overline{c(\tau)} e^{2i\beta} \right\} + \frac{p_r(1 - e^{2i\beta})}{2}, \quad (18)$$

where

$$a(\tau) = \sqrt{1 - b^2(\tau)}, \quad b(\tau) = e^{-i\beta} \tau / a - \lambda, \quad \text{and} \quad c(\tau) = a(\tau) - ib(\tau).$$

We solve the system of SIE (13), (14) by the numerical method of mechanical quadratures and then, on the basis of this solution, determine the stress intensity factors (SIF) from the well-known formula [13].

It should be emphasized that the boundaries of contact regions of the crack lips are not known in advance. We determine them in the process of solution of the integral equations of the problem from the auxiliary conditions of equality to zero of the normal contact stresses at these boundary points. At the same time, the zones of restraining and sliding of the contact regions are found from the auxiliary condition (9). In this case, we construct the solution of the contact problem by the method of successive approximations. In relation (6), the quantity $\tilde{T}(x_1) = T_k(x_1)$ is used in the iterative process for $k = 0, 1, 2, \dots$. As the zero-order approximation $T_0(x_1)$ ($k = 0$), we choose tangential stresses on the crack line in the solid half plane under the action of the external load. The subsequent approximations $T_k(x_1)$ ($k = 1, 2, \dots$) correspond to the values of tangential contact stresses on the crack lips.

Numerical Results and Discussion

We performed a numerical investigation of the mode II SIF K_{II} and their range $\Delta K_{\text{II}} = \max K_{\text{II}}(\lambda) - \min K_{\text{II}}(\lambda)$ in a rolling cycle (for a single passage of the contact load along the edge of the half plane) and constructed the maps of contact of the crack lips within a cycle.

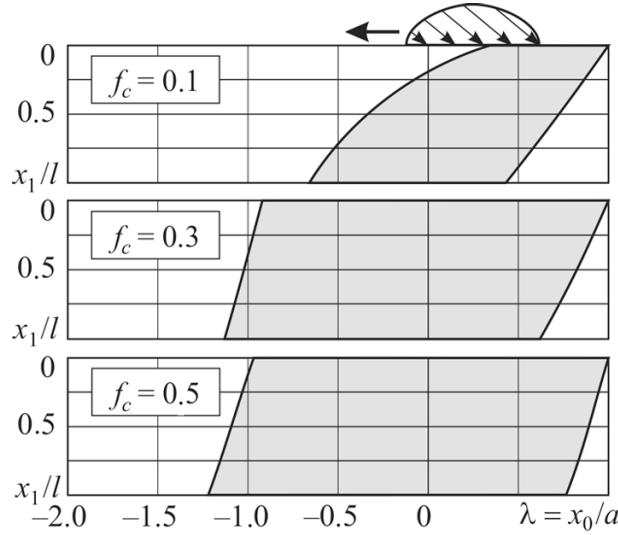


Fig. 2. Maps of contact of the crack lips in the course of motion of the Hertz load in the presence of residual stresses; $f_s = 0.1$; $\beta = 160^\circ$; $\varepsilon = l/a = 0.3$; ($\square$) sliding of the crack lips with friction; ($\blacksquare$) restraining.

Main attention was given to the study of the combined influence of the Hertz load and RS on the parameter ΔK_{II} that controls the process crack growth in the contact (compression) zone by the mechanism of transverse shear. The dependence of this parameter on the orientation of the crack was investigated.

The numerical calculations were carried out for an edge arbitrarily oriented crack with relative length $\varepsilon = l/a = 0.3$. The orientation angle β of this crack varied from 20° to 165° . We choose the following operating parameters of the wheel–rail system: the maximum Hertz pressure $p_0 = 1100$ MPa [1, 16], the length of the contact region $2a = 14$ mm, the compressive RS $p_r = 175$ MPa [2], the coefficient of sliding friction in rolling contact $f_s = 0.1$, and the coefficients of friction between the crack lips in the rail $f_c = 0.1, 0.3$, and 0.5 .

We investigated the case where the contact load is applied in the immediate vicinity of the crack mouth ($-2.0 \leq \lambda \leq 1.0$). Then it was assumed that, under the action of the Hertz load and compressive RS, the crack lips are in contact along the entire crack length. In this case, to find the zones of sliding and restraining of the crack lips for a crack whose lips are in contact, we use the following algorithm. First, we assume that the crack lips contact without restraining and solve the SIE (14) in which $\tau \in L$, i.e., τ changes along the entire crack contour. The solution of this problem is used to find contact stresses on the contour L and determine the zones in which the auxiliary condition (9) is satisfied. In the first approximation, the endpoints of these segments specify the boundaries of the zones of restraining of crack lips. If these zones exist, then we take them into account in constructing the second approximation. Thus, we get the iterative process, which can be terminated as soon as we get the required accuracy.

In practice, the surface cracks propagate in railheads mainly at angles of 10° – 30° in the direction opposite to the direction of action of the tangential contact forces [1, 16, 17]. To illustrate what happens with a flat crack in the zone of compression under the combined action of the Hertz load and RS, we choose, as an example, the mean value of the angle equal to 20° , which corresponds to the orientation $\beta = 180^\circ - 20^\circ = 160^\circ$. For this crack, we constructed the maps of contact of its lips and computed the SIF K_{II} in the course of motion of the contact load in the vicinity of its mouth (Figs. 2 and 3).

The results of calculations confirm the appearance of sliding and restraining zones for the chosen direction of displacements of the Hertz load from the right to the left (Fig. 2). The larger the friction between the crack lips, the longer the period of restraining of the lips. In this case, the complete restraining of the lips occurs faster.

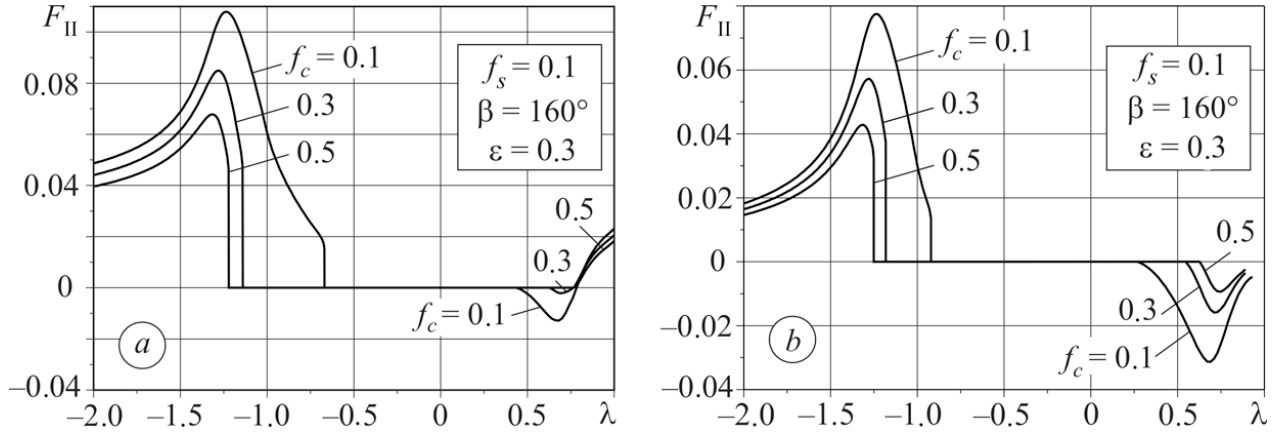


Fig. 3. Dependences of the normalized SIF F_{II} on the location of the Hertz load in the presence (a) and in the absence (b) of residual stresses for different friction coefficients between the crack lips f_c .

The process of restraining begins from the crack mouth and gradually propagates onto the entire crack. As the Hertz load passes through the mouth, the zone of restraining gradually shrinks from the mouth and then completely disappears.

By using the determined contact regions of the crack lips, we obtain the numerical values of the normalized SIF

$$F_{II} = \frac{K_{II}}{p_0 \sqrt{\pi a}}$$

at the crack tip (Fig. 3a). As the Hertz load moves, they change and take, for the corresponding locations of the load (for the values of λ), its minimum (negative) and maximum (positive) values. Moreover, as the coefficient of friction f_c between the crack lips increases, both the maximum and minimum absolute values of the SIF decrease. The larger the friction between the crack lips, the longer the horizontal segments of the curves corresponding to the event of restraining near the crack tip.

For comparison, to reveal the influence of RS, we also present a plot of the SIF for the case where these stresses are absent (Fig. 3b). We see that the RS lead to an increase in the maximum SIF F_{II} and to decrease in the absolute value of the minimum SIF.

Similar calculations were also performed for other orientation angles of the crack β . On the basis of the accumulated results, we plotted the dependence of the normalized SIF range

$$\Delta F_{II} = \frac{\Delta K_{II}}{p_0 \sqrt{\pi a}}$$

on the angle β for different friction coefficients f_c between the crack lips. For comparison, the same calculations were performed for the case of action solely of the Hertz load. It is easy to see (Fig. 4) that, depending on the orientation of the crack, there exist two local maxima of the range of SIF K_{II} : one maximum is attained for $\beta = \beta_1^*$ within the range of acute angles of orientation of the crack and the second maximum is attained for $\beta = \beta_2^*$ within the range of obtuse angles (see Table 1, where H is the Hertz load and $H + R$ is the combined action of the Hertz load and residual stresses).

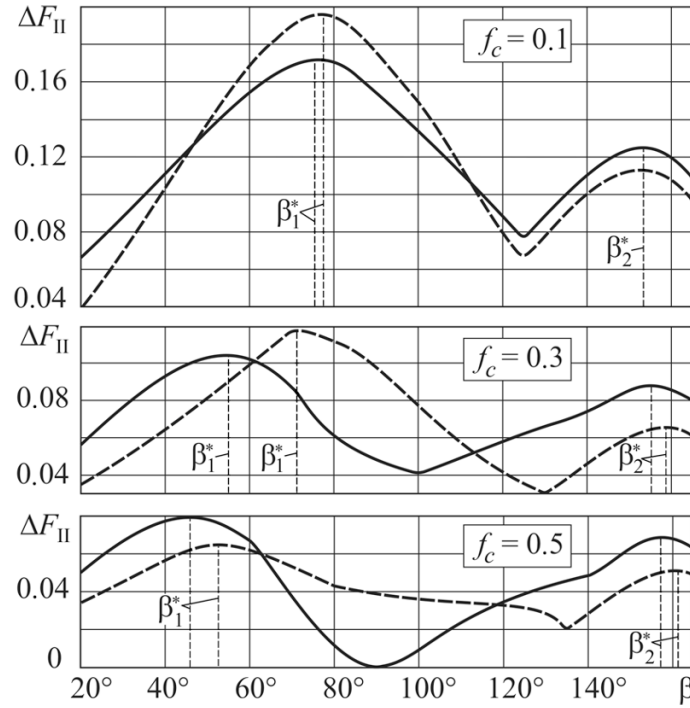


Fig. 4. Dependences of the normalized range ΔF_{II} on the angle of orientation of the crack β for different coefficients of friction f_c between the crack lips; $f_s = 0.1$; $\varepsilon = l/a = 0.3$; the solid curve corresponds to the combined action of the Hertz load and residual stresses and the dashed line corresponds to the action solely of the Hertz load.

Table 1. Maximum Values of the Ranges ΔK_{II} for Acute β_1^* and Obtuse β_2^* Angles of Orientation of a Crack of Length $l = 2.1$ mm; $p_0 = 1100$ MPa, $p_r = 175$ MPa

f_c	0.1		0.3		0.5	
External load	H	$H + R$	H	$H + R$	H	$H + R$
β_1^*	79°	76°	74°	56°	52°	43°
$\Delta K_{II}(\beta_1^*), \text{ MPa}\sqrt{\text{m}}$	32.14	28.06	19.41	16.96	10.77	12.89
β_2^*	153°	153°	158°	154°	161°	157°
$\Delta K_{II}(\beta_2^*), \text{ MPa}\sqrt{\text{m}}$	18.76	20.55	10.77	14.35	8.32	11.26

It was established that the friction coefficient f_c strongly affects the range ΔK_{II} . The increase in the coefficient of friction between the crack lips leads to a decrease both in the range ΔK_{II} and, especially, in its maximum values $\Delta K_{II}(\beta_1^*)$ and $\Delta K_{II}(\beta_2^*)$ both under the combined action of the Hertz load and RS and under the action solely of the Hertz load. The acute angle β_1^* (at which the maximum of $\Delta K_{II}(\beta_1^*)$ is realized) also undergoes significant changes. Thus, as f_c increases, this angle becomes much smaller. Indeed, in the presence of RS, for $f_c = 0.1$, the angle $\beta_1^* = 76^\circ$, whereas for $f_c = 0.5$, we have $\beta_1^* = 43^\circ$ (see Table 1), i.e., it becomes almost twice smaller.

The obtuse angle β_2^* is more stable. It remains almost constant both with variations of f_c and with the appearance of RS. Moreover, for the obtuse angles of orientation of the crack in the presence of RS, the maxima $\Delta K_{II}(\beta_1^*)$ and $\Delta K_{II}(\beta_2^*)$ are predominant independently of f_c , whereas for the acute angles, this regularity is observed for only large $f_c = 0.5$.

It is worth noting that, as the coefficient f_c increases, the difference between the values of $\Delta K_{II}(\beta_1^*)$ and $\Delta K_{II}(\beta_2^*)$ strongly decreases both in the absence and in the presence of RS. For large f_c , both angles become equally favorable for the growth of shear cracks in these directions. Thus, it is possible to assume that this fact facilitates the formation of defects of the “dark-spot” type [18].

The obtained numerical values of the ranges ΔK_{II} (see Table 1) corresponding to the most favorable directions of crack propagation (angles β_1^* and β_2^*) were compared with the threshold value [19]

$$\Delta K_{II,th} = 13.01 \text{ MPa} \sqrt{\text{m}}$$

obtained for RSB12 rail steel. As could be expected, the intensification of friction between the crack lips inhibits its growth by the shear mechanism and, for the large friction coefficient $f_c = 0.5$, the propagation of the crack in both directions under given operating conditions is completely impossible, both in the presence and in the absence of RS.

CONCLUSIONS

We have established two most favorable directions for the propagation of surface shear cracks in railroad railheads under the conditions of unidirectional rolling. One of this directions is the direction of action of tangential contact forces (angle β_1^* ; Fig. 4) and the other direction is opposite to the first direction (angle β_2^* ; Fig. 4). The angle β_2^* weakly varies under the influence of compressive residual stresses and friction between the crack lips. At the same time, the variations of the angle β_1^* are significant. In this case, as the coefficient of friction between the crack lips f_c increases, the angle β_1^* changes from an almost right angle to an acute angle. For large f_c , this angle becomes even smaller under the influence of residual compressive stresses.

It is shown that the friction between the crack lips has a significant influence on the maximum value of the range ΔK_{II} responsible for the propagation of shear cracks. Thus, the increase in the friction coefficient f_c leads to a substantial decrease in the maximum value of ΔK_{II} as a result of which the difference between the values of $\Delta K_{II}(\beta_1^*)$ and $\Delta K_{II}(\beta_2^*)$ noticeably decreases.

REFERENCES

1. U. Zerbst, R. Lunden, K.-O. Edel, and R. A. Smith, “Introduction to the damage tolerance behavior of railway rails—a review,” *Eng. Fract. Mech.*, **76**, 2563–2601 (2009).
2. G. Budnitzki and K.-O. Edel, “Railway rails can fracture under service conditions,” *Int. Polymer. Proc.*, Nos. 11–12, 674–681 (2005).
3. M. Heinsch, “Improving rail durability and life,” *Int. Railway J.*, No. 2, 13–15 (2004).
4. I. I. Kudish, “Contact problem of the theory of elasticity for prestressed bodies with cracks,” *Zh. Prikl. Mat. Tekh. Fiz.*, No. 2, 144–152 (1987).
5. N.-A. Noda, M. Yagishita, and T. Kihara, “Effect of crack shape, inclination angle, and friction coefficient in crack surface contact problems,” *Int. J. Fract.*, **105**, No. 13, 367–389 (2000).
6. A. Kapoor and D. I. Fletcher, *Post Hatfield Rolling Contact Fatigue. The Effect of Residual Stress on Contact Stress Driven Crack Growth in Rail, Part 1: The Model, Technical Report WR061106-2*, Newcastle University (2006).

7. D. I. Fletcher, A. Kapoor, F. J. Franklin, et al., *Post Hatfield Rolling Contact Fatigue. The Effect of Residual Stress on Contact Stress Driven Crack Growth in Rail. Comparison of the Hatfield and Alternative UK Rails Using Models to Assess the Effect of Residual Stress on Crack Growth from Rolling Contact Fatigue. Research Report No. WR061106-6*, Newcastle University (2006).
8. B. Trollé, M.-C. Baietto, A. Gravouil, et al., “2D fatigue crack propagation in rails taking into account actual plastic stresses,” *Eng. Fract. Mech.*, **123**, 163–181 (2014).
9. H. P. Marchenko, “Influence of residual stress on the stress intensity factors for a surface crack in the rail head,” *Fiz-Khim. Mekh. Mater.*, No. 1, 57–62 (2010); **English translation: Mater. Sci.**, **46**, No. 1, 64–69 (2010).
10. O. P. Datsyshyn and H. P. Marchenko, “On the influence of residual stresses on the stressed state near a surface crack in the rail-head,” *Mashynoznavstvo*, Nos. 3–4, 16–20 (2011).
11. O. P. Datsyshyn and H. P. Marchenko, “Evaluation of the influence of residual stresses on the surface fracture of railroad rails,” *Zalozn. Transport Ukr.*, No. 6, 38–41 (2012).
12. O. Datsyshyn and H. Marchenko, “On shear surface cracks in railroad rails,” in: *Proc. of the 5th Internat. Conf. “Fracture Mechanics of Materials and Strength of Structures”* [in Ukrainian], Lviv (2014), pp. 379–384.
13. V. V. Panasyuk, M. P. Savruk, and A. P. Datsyshyn, *Stress Distributions near Cracks in Plates and Shells* [in Russian]. Naukova Dumka, Kiev (1976).
14. M. P. Savruk, *Two-Dimensional Problems of Elasticity for Bodies with Cracks* [in Russian], Naukova Dumka, Kiev (1981).
15. O. P. Datsyshyn, R. E. Pryshlyak, S. V. Prykhods'ka, et al., “Influence of the form of a model contact load on the stress intensity factors for edge cracks,” *Probl. Tribol.*, No. 3, 3–16 (1998).
16. J. W. Ringsberg and A. Bergkvist, “On propagation of short rolling fatigue cracks,” *Fatigue Fract. Eng. Mater. Struct.*, **26**, No. 10, 969–983 (2003).
17. A. F. Bower, “The influence of crack face friction and trapped fluid on surface initiated rolling contact fatigue cracks,” *Trans. ASME: J. Tribol.*, **110**, No. 4, 704–711 (1988).
18. M. Kaneta, K. Matsuda, K. Murakami, and H. Nishikawa, “A possible mechanism for rail dark spot defects,” *Trans. ASME*, **120**, 304–309 (1998).
19. Bold P. E., Brown M. W., and Allen R. J. “Shear mode crack growth and rolling contact fatigue,” *Wear*, **144**, No. 2, 307–317 (1991).