

Figures 3 and 4 show the stress distributions along the crack calculated by the method considered above.

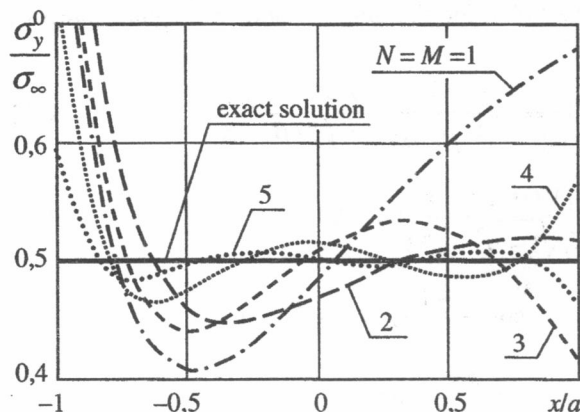


Fig. 3 Normal stress distribution along the crack

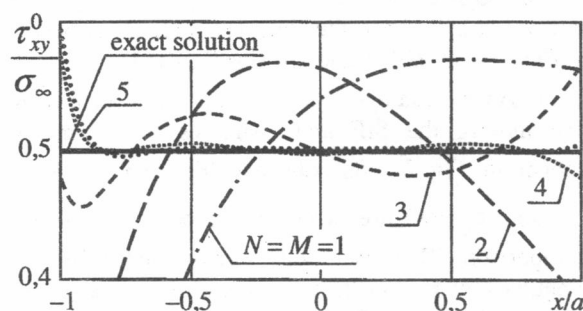


Fig. 4 Shear stress distribution along the crack

One can see from Figs. 3 and 4 that only three or five measurement points suffice to determine the SIF K_1 and K_2 and stresses σ_y and τ_{xy} with an accuracy of 5%–7%. This shows that the method possesses high convergence rate.

The present method is recommended to find: the stresses at the crack location, the residual strength reserve, the carrying capacity reserve in damaged structures under a static and oscillated loading in a field conditions and under a test.

References

- [1] Lekhnitskii S.G., *Anisotropic Plates*, Moscow: Gostechizdat, 1957 (in Russian).
- [2] Maksimenko V.N., Problem of a Crack in an Anisotropic Half Plane Stiffened with Elastic Cover Plates, *Continuous Media Dynamics*. 1990, n99, p.41–46 (in Russian).
- [3] Gakhov F.D., *Boundary-Value Problems*, Moscow: Physmatgiz, 1963 (in Russian).
- [4] *Damaging Mechanics and Material Strength*; Reference Manual in 4 vol.; edited by Panasyuk V.V., Kiev: Naukova Dumka, 1988 (in Russian).
- [5] Maksimenko V.N., Limit Equilibrium in an Anisotropic Plate with an Elliptical Hole and Compound Crack Pattern, *Uchenye Zapiski TSAGI* (Issues of Central Aerohydrodynamics Institute). 1987, v. 18, n3. p. 24 – 29 (in Russian).

[6] Berezin I.S., Zhidkov N.P., *Computational Methods*, v. 1, 2, Moscow: Nauka, 1966 (in Russian).

[7] Belotserkovskii S.M., Lifanov I.K., *Numerical Methods and Singular Integral Equations*, Moscow: Nauka, 1985 (in Russian).

CRACK PATH DETERMINATION

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Abstract

In responsible elements of the planes, bridges and other constructions from metal and composite materials the stress state in the tips of crack is mixed type. At that the conditions of rectilinear growth of a crack are violated as show numerous observations and experiments. Thus the problems about determination not only critical sizes of a crack and critical external loads but also path (trajectory) on which crack growth are arisen. The existing methods of mixed mode crack growth propagation are limited as a rule by consideration of isotropic materials and special kinds of geometry and loading where the crack has rectilinearly propagation. In this paper the method of crack growth trajectory determination in anisotropic plate are presented. This method based on well known solutions of the plane problem of the theory of elasticity for the anisotropic plates with cracks, any fracture criterion and spline theory.

Keywords: crack, stress intensity factors, crack propagation, crack trajectory, orthotropic plate.

This paper is concerned with predicting the direction of initial crack growth in the anisotropic (orthotropic) plate subjected arbitrary loading and determining the new crack contour after growth along this directions.

As show supervision and experiments, growth of a crack in isotropic and anisotropic materials considerably differs from each other. The solving of a crack trajectory determination problem (path of the crack grows) can make essential amendments to an estimation of a resource of structures and strength calculations. On the other hand, the solution of this problem enables comparisons of various criteria of fracture.

In the solution three basic steps are precisely looked through: 1) determining the stress state in a plane element with crack; 2) determining the direction of initial crack growth and increments its tips; 3) modeling the crack growth trajectory.

In papers [1,2,3] by author the effective methods to determine the stress state of infinite or finite anisotropic plates with a system of cuts and cracks of the complicated form (fig.1) are constructed. The values of stress intensity factors (SIF) are obtained from numerical solution of the singular integral equations of the problem.

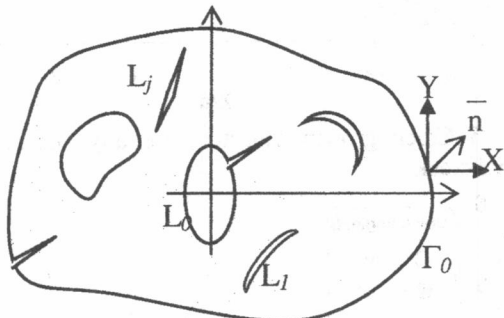


Fig. 1 The plate with a system of cuts and cracks.

There are many various fracture criteria for determination of an initial fracture angle of a crack growth. A number of the criteria which have received the greatest distribution are considered in works [4,5,6]. In the given work for definition of a crack growth direction θ^* the criterion proposed in work [4] is used.

It assumes that the direction of initial crack growth is controlled by the ratio of normal stress to tensile strength on a given plane. The normal stress ratio we defined here as:

$$R(\theta) = \frac{\sigma_{\theta\theta}(r_0, \theta)}{\sigma_{E1} \sin^2(\gamma) + \sigma_{E2} \cos^2(\gamma)},$$

where σ_{θ} – normal stress acting on the radial plane defined by θ , γ – angle from the plane of interest to the direction of the main principle elastic axis (fig. 2), σ_{E1} , σ_{E2} – longitudinal and transverse tensile strength respectively. According these criteria we have:

$$\frac{dR(\theta)}{d\theta} = 0; \quad \frac{d^2R(\theta)}{d\theta^2} < 0 \text{ for } \theta = \theta^*; \quad -\pi < \theta^* < \pi$$

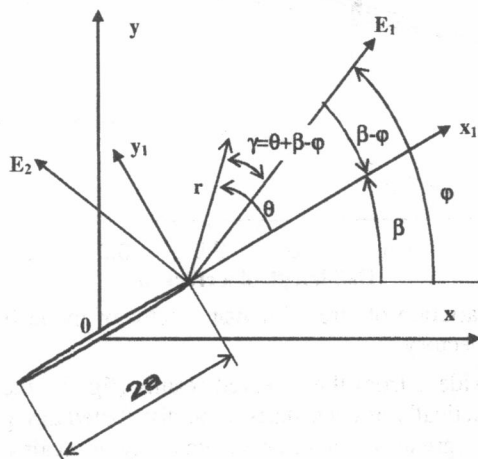


Fig. 2 Crossbar systems at the tip of crack.

Then the formulas for the definition crack configuration in process of it growth will be presented.

For simplicity we shall limit by reviewing of a first step on which the crack is propagated from the initial L_0 up to some new L_1 configuration [7,8,9].

Let outline of an initial crack is described by the arrays of points $z_j^{(0)}$ $j = 1, \dots, n_0$ (Fig. 3). Then the crack initial contour will outline with the help of cubic spline. Let's enter a new array $\{S_0\}$ in form:

$$s_j^{(0)} = s_{j-1}^{(0)} + |z_j^{(0)} - z_{j-1}^{(0)}| = \sum_{i=2}^j |z_i^{(0)} - z_{i-1}^{(0)}|; \quad s_1^{(0)} = 0; \quad j = 2, \dots, n_0 \quad (1)$$

It is not difficult to understand that:

$$\lim_{n_0 \rightarrow \infty} s_{n_0}^{(0)} = \lim_{n_0 \rightarrow \infty} \sum_{i=2}^{n_0} |z_i^{(0)} - z_{i-1}^{(0)}| = 2a_0,$$

where $2a_0$ – crack length.

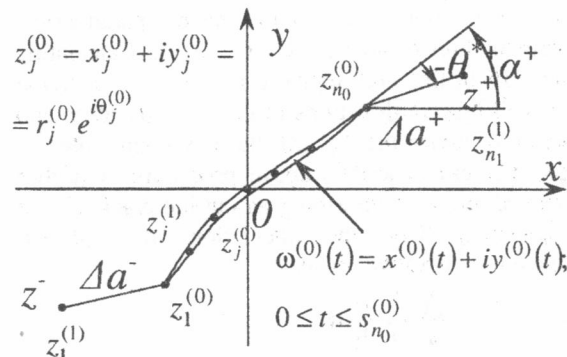


Fig. 3 Crack contour modeling.

On a specific gang of points we create cubic spline interpolation $x^{(0)}(t)$, $y^{(0)}(t)$ to approximate initial outline of a crack L_0 in form:

$$\omega^{(0)}(t) = x^{(0)}(t) + i y^{(0)}(t); \quad 0 \leq t \leq s_{n_0}^{(0)}.$$

We shall enter new variable for a parameterisation crack outline equations:

$$t(\zeta) = \frac{1+\zeta}{2} s_{n_0}^{(0)}; \quad |\zeta| \leq 1. \quad (2)$$

Thus the following conditions are fulfilled $t(-1) = 0$, $t(1) = s_{n_0}^{(0)}$. We have constructed functions crack in form:

$$L^{(0)} = \omega^{(0)}(\zeta) = x^{(0)}(\zeta) + i y^{(0)}(\zeta); \quad -1 \leq \zeta \leq 1.$$

The new coordinates of crack tips after crack growth can be calculated under the formulas:

$$z^{\pm} = \omega^{(0)}(\pm 1) + \Delta a^{\pm} e^{i(\theta^{\pm} + \alpha^{\pm})}, \quad (3)$$

where

$$\alpha^{\pm} = \begin{cases} \arg \varpi^{\pm}; & \text{if } \pm \operatorname{Re} \varpi^{\pm} > 0; \\ \pi + \arg \varpi^{\pm}; & \text{if } \pm \operatorname{Re} \varpi^{\pm} < 0; \\ \frac{\pi}{2}; & \text{if } \operatorname{Re} \varpi^{\pm} = 0; \pm \operatorname{Im} \varpi^{\pm} > 0; \\ -\frac{\pi}{2}; & \text{if } \operatorname{Re} \varpi^{\pm} = 0; \pm \operatorname{Im} \varpi^{\pm} < 0. \end{cases}$$

$$\varpi^{\pm} = \frac{d\omega^{(0)}(\pm 1)}{d\zeta}$$

After crack growth we have next arrays of points to outline new crack contour: $z_j^{(1)}$ $j=1, \dots, n_1$; $n_1 = n_0 + 2$; $z_1^{(1)} = z^-$; $z_{n_1}^{(1)} = z^+$; $z_j^{(1)} = z_{j-1}^{(0)}$; $j=2, \dots, n_1 - 1$ (Fig. 3).

Thus the functions crack $L^{(1)} = \omega^{(1)}(\zeta) = x^{(1)}(\zeta) + i y^{(1)}(\zeta)$; $-1 \leq \zeta \leq 1$ can be constructed based on the data arrays $z_j^{(1)}$ and with the help of eq. (1,2) by replacing subscript 0 to 1.

Repeating step-by-step circumscribed procedure on each step we shall have parametrical representation of an outline of a crack. Calculating then SIF in tips and applying any fracture criterion we shall receive directions of crack propagation and therefore, under the eq. (3) the new coordinates of tips. The circumscribed above procedure is further repeated and circumscribing an outline crack after it propagating. Thus the calculation crack growth trajectory is iterative process.

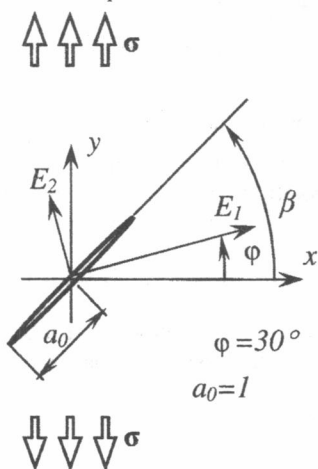


Fig. 4 Orthotropic cracked plate subjected uniaxial tensile on infinity.

As a sample, propagation of the inclined crack in anisotropic plate from composite material (T300-5208; $E_1=181$ GPa, $E_2=10,3$ GPa, $\nu_{12}=0,28$, $G_{12}=7,17$ GPa) subjected uniaxial tensile on infinity was considered (Fig. 4). It has been shown that the initially inclined crack tries to straighten and propagate in a direction of main modulus of elasticity.

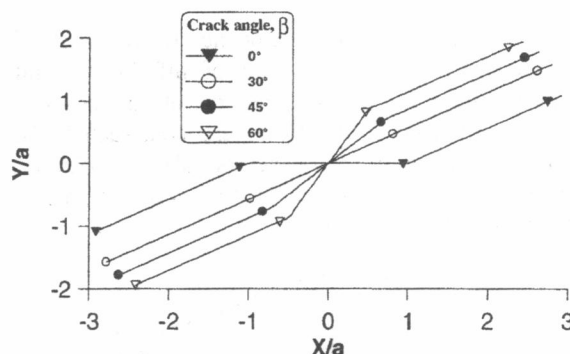


Fig. 5 Crack growth trajectory for any angles of initial crack.

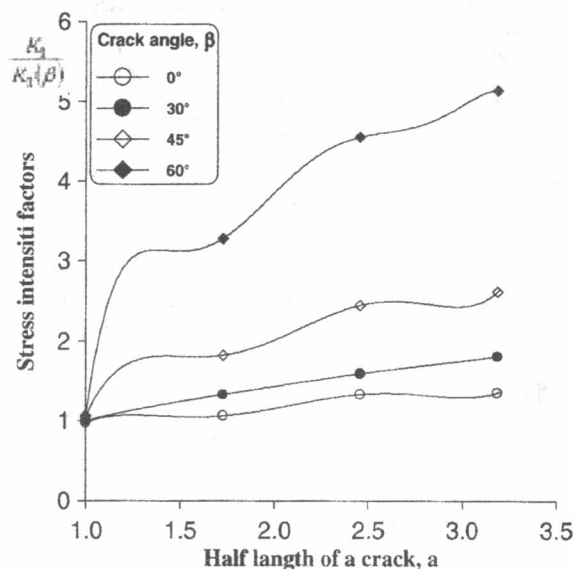


Fig. 6 Variation of stress intensity factor of mode I along trajectory.

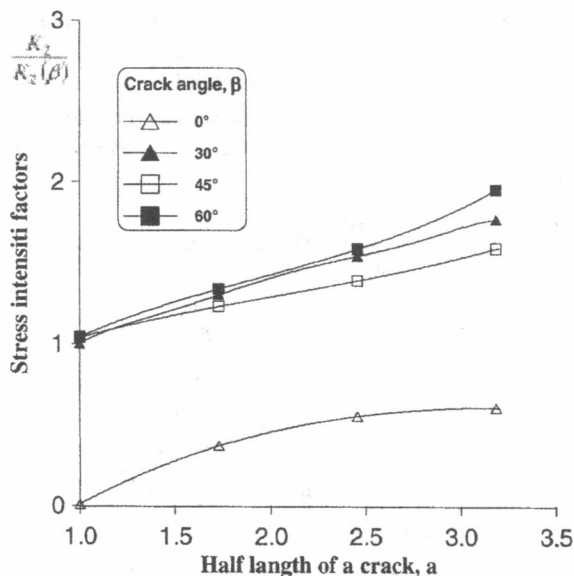


Fig. 7 Variation of stress intensity factor of mode II along trajectory.

As evident from the received results (fig. 5), the crack practically at once starts to be distributed along the main (greatest) direction of elasticity as against isotropic material where the crack under same loading conditions tries "to be straightened" and be

propagated perpendicularly lines of action of the external loading [7].

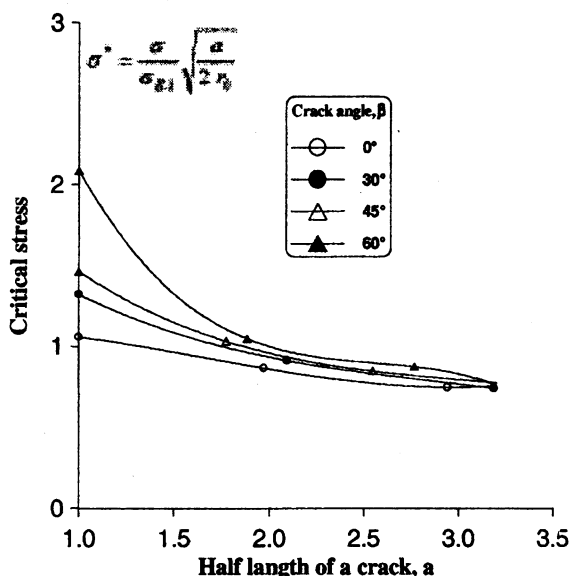


Fig. 8 Variation of critical stress along trajectory.

Thus K_1 and K_2 increases along a crack growth trajectory and have to tend to value of stress intensity factor for inclined crack with angle $\beta = \varphi = 30^\circ$ (fig. 6,7). Critical loading along the trajectory decreases (fig. 8).

As show calculations, crack propagation in isotropic and an anisotropic material essentially differ from each other that it is necessary to take into account at designing structures and their further operation.

The developed technique allows to predict mixed mode crack growth trajectories in thin-walled sheet elements of structures from the metal and composite materials operating in plane stress state conditions.

The algorithm submitted in work is easily distributed to a case of several cracks, and also on a case of cyclic loading.

References

- [3] Maksimenko V.N., Cendrovsky A.V., The definition of stress of intensity factors for cracks of the complicated form in anisotropic slices, *J. Applied mechanics and engineering physics*. 1986, n6, p. 124-128.
- [4] Maksimenko V.N., Limiting equilibrium of an anisotropic slice, loosed by an elliptic cut and a system of cracks of the complicated form, *The scientist slip CAGI*. 1987, v.18, n3, p. 105-117.
- [5] Maksimenko V.N., *Concentration of stress in the elements of air designs*, Novosibirsk: Publishing House of NSTU, 1989.
- [6] Buszek M.B., Heracovich C.T., Direction of crack growth in fibrous composites, *Mech. Comp. Mater.* 1983, v. 58, p. 75-82.
- [7] Sih G.C., Strain-energy-density factor applied to mixed mode crack problems, *Int. J. of Fract.* 1974, v. 10, n3.
- [8] Zhang S.Q., Jang B.Z., Valaire B.T., Suhling J.S., A new criterion for composite material mixed mode fracture analysis, *Eng. Fract. Mech.* 1989, v.34, n 3, p. 749-769.
- [9] Saveliev A.V., Mixed mode crack growth trajectory prediction in the plate from metal and composite material, *Collected of proceeding, of NSTU*, 2002. n2(28). p. 71-78.
- [10] Saveliev A.V., Maksimenko V.N., About crack propagation in anisotropic plate, *Pros. 1st Russian Korean International Symposium on Applied Mechanics "RUSKOAM-2001"*, September 21-24, 2001. Novosibirsk.
- [11] Saveliev A.V., Maksimenko V.N., Crack growth trajectory determination in plane elements of structure, *Pros. 6th All-Russian scientific conf.*, November 29 – December 1, 2003. Novokuzneck, p. 58-61.

POSSIBLE APPROACHES TO DESCRIPTION OF UNELASTIC MATERIALS BEHAVIOR OF DIFFERENT MODULES

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Abstract

In this work presentation approaches to description of behavior of isotropic materials sensitive to the stress type are given below.

Many materials have different characteristics at various stresses. This phenomenon was called the different Module State. Physical characteristics of different Modules materials are given in the work [1]. The experimental data of influence of the stress and level on the mechanical characteristics of materials and their approximations are given in [2]. The existing approaches to description of behavior of isotropic materials sensitive to the stress type are given below.

G.S. Shapiro [3] assumed that in physical Correlations constants take different values dependent on the average stress sign σ_{cp} . The share Modules G and volume expansion K have different forms dependent on the σ_{cp} sign:

$$\sigma_{cp} = \begin{cases} K^+ \theta, & \sigma_{cp} > 0; \\ K^- \theta, & \sigma_{cp} < 0, \end{cases} \quad (1)$$