

PLANE COLLISION PROBLEM FOR TWO IDENTICAL ELASTIC PARABOLIC BODIES—DIRECT CENTRAL IMPACT

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A direct central collision of two identical infinite cylindrical bodies is studied. A nonstationary plane elastic problem is solved. The variable boundary of the contact area is determined. A mixed boundary problem is formulated. Its solution is represented by Fourier series. An infinite system of Volterra equations of the second kind for the unknown expansion coefficients is derived by satisfying boundary conditions. The basic characteristics of the collision process are determined numerically depending on the curvature of the frontal surface of the bodies.

Keywords: collision of two parabolic bodies, direct central impact, plane problem, Fourier series, Volterra equations

In this paper, a technique is developed for determining the collision characteristics for blunted elastic bodies. A nonstationary elastic problem is formulated. It is solved by the approach used earlier to study the penetration of a body into a compressible liquid [5] and the impact of a rigid body on the surface of an elastic half-space [7, 8]. In the paper [6], the solution is found to the collision problem for two identical elastic cylindrical bodies whose material does not resist shear. Here we lift this restriction. We study the direct central collision of two infinite cylindrical bodies, assuming that the cylinders interact along their common generatrix at the initial stage of the collision. Since the collision characteristics vary in a similar fashion in all cross sections, the motion of the bodies is studied in one of them, i.e., the impact is treated within the framework of a plane elastic problem. The contact area varies during the collision. Mixed boundary conditions are formulated on the assumption that the time interval of interest is not greater than the collision duration (in this case, the perturbations arising during the collision reflect from the rear and side surfaces and then have no time to come back to the contact area and, hence, do not affect the interaction process) and the depth of mutual penetration and the frontal curvature of the cylinders are small. The bodies have finite masses, and their velocity varies during the collision. In this connection, the boundary conditions should be supplemented with the equations of motion of the bodies' centers of mass (their initial velocity is specified). To solve the initial-boundary-value problem formulated, the desired quantities are expanded into Fourier series and the mixed boundary conditions are satisfied. This results in a resolving system of integral Volterra equations of the second kind for the desired expansion coefficients, which can be solved numerically simultaneously with the equation of motion of the body as a whole. Both the resolving system and the equation of motion are discretized. The solution of the problem posed yields the normal stress on the surface of the bodies, the forces of interaction, the varying contact boundary, and the displacement and velocities of the bodies during the collision depending on the elastic characteristics of the material, the shape and mass of the bodies, and the collision velocity. Nonstationary mixed problems with an unknown boundary were studied in [2, 4, 13, 14] and recent experiments on collision of elastic bodies are addressed in [12].

1. Problem Formulation. Suppose that two infinite elastic cylindrical bodies move toward each other. Consider their direct central collision that occurs at some instant of time ($t = 0$). The cylinders are assumed identical in shape and material. Their relative velocity is directed along the common axis of symmetry of their cross sections, $V_0(0) = V_0^{(2)}(0) - V_0^{(1)}(0)$ at $t = 0$. At the initial instant of the collision, the bodies interact along their common generatrix. Since all the cross sections of the system are

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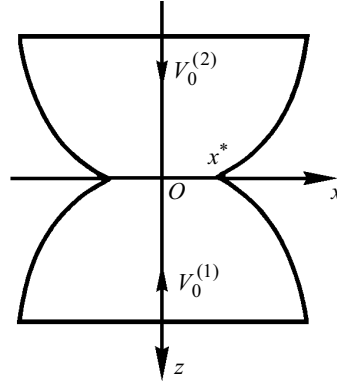


Fig. 1

equal, we will restrict the consideration to the motion along one of them. Let us introduce a Cartesian coordinate system Oxz with the origin at the initial contact point and the Oz -axis directed along the common axis of symmetry of the cross section (Fig. 1). The contact area varies during the collision, its width being determined by the boundary value of the coordinate x^* .

Let us introduce dimensionless coordinates and variables:

$$\bar{x} = \frac{x}{R}, \quad \bar{z} = \frac{z}{R}, \quad \bar{u}_j = \frac{u_j}{R}, \quad \bar{t} = \frac{c_0 t}{R}, \quad \bar{M} = \frac{M}{\rho R^2}, \quad \bar{\sigma}_{jk} = \frac{\sigma_{jk}}{K},$$

$$c_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}, \quad c_s = \sqrt{\frac{\mu}{\rho}}, \quad c_0 = \sqrt{\frac{K}{\rho}},$$

$$\alpha = \frac{c_p}{c_0}, \quad \beta = \frac{c_s}{c_0}, \quad b = \frac{\beta}{\alpha}, \quad j, k = x, z,$$

where K is the bulk modulus, μ is the shear modulus, R is some linear dimension (if the cross section is bounded by a parabolic curve, then this linear dimension may be the distance between the focus and the vertex of the parabola), M is the mass of the bodies, ρ is density, u_j are the components of the displacement vector, σ_{jk} are the components of the stress tensor, c_p and c_s are the velocities of longitudinal and transverse waves, respectively, and c_0 is the sonic velocity in the material on the assumption that its shear modulus is equal to zero. Hereafter, we will use only dimensionless coordinates and variables, omitting the overbar everywhere.

The deformation of the bodies is described by two scalar wave potentials $\varphi^{(i)}$ and $\psi^{(i)}$ ($i = 1, 2$), which satisfy the following equations if the stress-strain state is plane [9]:

$$\frac{\partial^2 \varphi^{(i)}}{\partial x^2} + \frac{\partial^2 \varphi^{(i)}}{\partial z^2} = \frac{1}{\alpha^2} \frac{\partial^2 \varphi^{(i)}}{\partial t^2}, \quad \frac{\partial^2 \psi^{(i)}}{\partial x^2} + \frac{\partial^2 \psi^{(i)}}{\partial z^2} = \frac{1}{\beta^2} \frac{\partial^2 \psi^{(i)}}{\partial t^2}. \quad (1.1)$$

The stresses and displacements are expressed in terms of the potentials $\varphi^{(i)}$ and $\psi^{(i)}$ as follows:

$$\begin{aligned} u_x^{(i)} &= \frac{\partial \varphi^{(i)}}{\partial x} + \frac{\partial \psi^{(i)}}{\partial z}, \quad u_z^{(i)} = \frac{\partial \varphi^{(i)}}{\partial z} - \frac{\partial \psi^{(i)}}{\partial x}, \\ \sigma_{zz}^{(i)} &= (1 - 2b^2) \frac{\partial^2 \varphi^{(i)}}{\partial t^2} + 2\beta^2 \left(\frac{\partial^2 \varphi^{(i)}}{\partial z^2} - \frac{\partial^2 \psi^{(i)}}{\partial x \partial z} \right), \\ \sigma_{zx}^{(i)} &= \beta^2 \left(2 \frac{\partial^2 \varphi^{(i)}}{\partial x \partial z} + \frac{\partial^2 \psi^{(i)}}{\partial z^2} - \frac{\partial^2 \psi^{(i)}}{\partial x^2} \right), \quad i = 1, 2. \end{aligned} \quad (1.2)$$

Assume that the depths of mutual penetration of the bodies during the collision are insignificant and the curvature of the plane curve bounding the cross section of the bodies is small. We will restrict the consideration to a time interval such that waves reflected by the rear surface of the bodies have no time to come back to the contact area and, hence, do not affect the interaction process. These assumptions enable us to linearize the boundary conditions and to formulate them on the axis $z = 0$. Then, in the absence of friction between the contacting surfaces, the boundary conditions express the equality of the displacements and normal stresses in the contact area (the contact stresses must be compressive), the absence of normal stresses on the free surface, and the absence of tangential stresses everywhere:

$$u_z^{(1)} \Big|_{z=0} + s^{(1)} = u_z^{(2)} \Big|_{z=0} + s^{(2)}, \quad |x| \in (0..|x^*|), \quad s^{(i)}(0) = 0, \quad (1.3)$$

$$\sigma_{zz}^{(1)} \Big|_{z=0} = \sigma_{zz}^{(2)} \Big|_{z=0}, \quad |x| \in (0..|x^*|), \quad \sigma_{zz}^{(i)} \Big|_{z=0} > 0, \quad |x| \in (0..|x^*|), \quad (1.4)$$

$$\sigma_{zz}^{(i)} \Big|_{z=0} = 0, \quad |x| \notin (0..|x^*|), \quad \sigma_{zx}^{(i)} \Big|_{z=0} = 0, \quad |x| \geq 0, \quad (1.5)$$

where $s^{(i)}$ ($i = 1, 2$) is the displacement of the center of mass of the i th body. For the sake of convenience, let us rearrange (1.3) in terms of deformation rates for the contacting surfaces:

$$V_0^{(1)} + \frac{\partial u_z^{(1)}}{\partial t} \Big|_{z=0} = V_0^{(2)} + \frac{\partial u_z^{(2)}}{\partial t} \Big|_{z=0}, \quad |x| \in (0..|x^*|), \quad (1.6)$$

where $V_0^{(i)}$ ($i = 1, 2$) is the velocity of the center of mass of the i th body.

By considering the interaction process on a small time interval, we can reduce the collision problem for two finite elastic bodies to the collision problem for two infinite elastic half-strips of finite width l . This width is selected so that waves reflected by the side surfaces have no time to come back to the contact area and, hence, do not affect the interaction process. Therefore, the conditions on the side surfaces of the half-strip may be dealt with as we see fit. In particular, for the variables in the general solution to be separated conveniently, we assume that the following conditions hold:

$$u_z^{(i)} \Big|_{|x|=l} = 0, \quad \sigma_{zx}^{(i)} \Big|_{|x|=l} = 0. \quad (1.7)$$

The boundary conditions at infinity are

$$\begin{aligned} \varphi^{(1)} &\rightarrow 0, & \psi^{(1)} &\rightarrow 0, & z &\rightarrow \infty, \\ \varphi^{(2)} &\rightarrow 0, & \psi^{(2)} &\rightarrow 0, & z &\rightarrow -\infty. \end{aligned} \quad (1.8)$$

The initial conditions (no perturbations in the bodies) are

$$\begin{aligned} \varphi^{(i)} \Big|_{t=0} = \frac{\partial \varphi^{(i)}}{\partial t} \Big|_{t=0} &= 0, \\ \psi^{(i)} \Big|_{t=0} = \frac{\partial \psi^{(i)}}{\partial t} \Big|_{t=0} &= 0, \quad i = 1, 2. \end{aligned} \quad (1.9)$$

The above boundary-value problem should be supplemented with the equation of motion of the body as a whole. To this end, we will use the equation of motion of the center of mass of each body:

$$\mu \frac{dV_c^1}{dt} = F(t), \quad \mu \frac{dV_c^2}{dt} = -F(t), \quad (1.10)$$

where

$$F(t) = 2 \int_0^{x^*} \sigma_{zz}(t, x, 0) dx,$$

$$V_c^{(1)}(0) = V_0^{(1)}, \quad V_c^{(2)}(0) = V_0^{(2)}. \quad (1.11)$$

In the general case, the boundary of the contact area, x^* , is determined from the condition

$$w(t) - f(x) - u_z(t, x, 0) = \begin{cases} 0, & |x| \leq x^*(t), \\ < 0, & |x| > x^*(t), \end{cases} \quad (1.12)$$

where $w(t)$ is the displacement of the center of mass of one of the bodies and $f(x)$ is the curve bounding the cross section of the bodies.

In a simpler formulation, the point of change in the boundary conditions, x^* , is determined as the abscissa of the intersection point of the undeformed contours if the depth of mutual penetration is z^* .

Relations (1.1)–(1.12) constitute a mixed initial–boundary–value problem with an unknown moving boundary.

2. Solution Technique. Let us apply the Laplace transform in t with a parameter s to the system of equations (1.1) and use conditions (1.9). We obtain the following equations in the Laplace transform space:

$$\frac{\partial^2 \varphi^{(i)L}}{\partial x^2} + \frac{\partial^2 \varphi^{(i)L}}{\partial z^2} = \frac{s^2}{\alpha^2} \varphi^{(i)L}, \quad \frac{\partial^2 \psi^{(i)L}}{\partial x^2} + \frac{\partial^2 \psi^{(i)L}}{\partial z^2} = \frac{s^2}{\beta^2} \psi^{(i)L}, \quad i = 1, 2. \quad (2.1)$$

The superscript L denotes a Laplace-transformed function.

Owing to the finiteness of the perturbed area, we can expand the solution of the wave equations into Fourier series on the interval $[-1, 1]$ containing the perturbed area.

Recall that the two colliding cylindrical bodies are identical in shape and material. Taking this fact into account, using conditions (1.7) and (1.8), and applying the Fourier method to separate variables, we obtain the following solution of (2.1):

$$\begin{aligned} \varphi^{(1)L} &= \sum_{n=0}^{\infty} A_n^{(1)}(s) \cdot e^{-zP} \cos \bar{n}x, & \psi^{(1)L} &= \sum_{n=0}^{\infty} B_n^{(1)}(s) \cdot e^{-zS} \sin \bar{n}x, \\ \varphi^{(2)L} &= \sum_{n=0}^{\infty} A_n^{(2)}(s) \cdot e^{zP} \cos \bar{n}x, & \psi^{(2)L} &= \sum_{n=0}^{\infty} B_n^{(2)}(s) \cdot e^{zS} \sin \bar{n}x, \\ P &= \sqrt{\frac{s^2}{\alpha^2} + \bar{n}^2}, & S &= \sqrt{\frac{s^2}{\beta^2} + \bar{n}^2}, & \bar{n} &= \frac{n\pi}{l}, \end{aligned} \quad (2.2)$$

where $A_n^{(i)}$ and $B_n^{(i)}$ are coefficients to be determined from the boundary conditions (1.5).

Let us expand the normal stress on the surface of the bodies and the rate of its deformation into Fourier series ($z=0$):

$$V^{(i)}(t, x, 0) = \sum_{n=0}^{\infty} V_n^{(i)}(t) \cos \bar{n}x, \quad (2.3)$$

$$\sigma_{zz}^{(i)}(t, x, 0) = \sum_{n=0}^{\infty} \sigma_{zzn}^{(i)}(t) \cos \bar{n}x, \quad (2.4)$$

$$\sigma_{zx}^{(i)}(t, x, 0) = \sum_{n=0}^{\infty} \sigma_{zxn}^{(i)}(t) \cos \bar{n}x. \quad (2.5)$$

Prior to solving the mixed problem, let us solve an auxiliary, nonmixed problem for system (1.1) and the boundary conditions on the interval $[-1, 1]$:

$$V^{(i)}(t, x) = \frac{\partial u_z^{(i)}}{\partial t} \Big|_{z=0}, \quad (2.6)$$

$$\sigma_{zx}^{(i)}(t, x, 0) \Big|_{z=0} = 0. \quad (2.7)$$

Applying the Laplace transform to Eqs. (2.5) and (2.7), we determine how the coefficients $A_n^{(i)}$ and $B_n^{(i)}$ are related:

$$A_n^{(i)} = (-1)^i \frac{s^2 + 2\beta^2 \bar{n}^2}{2\beta^2 P \bar{n}} B_n^{(i)}, \quad i = 1, 2, \quad n = \overline{0, \infty}.$$

Using this relation and the Laplace-transformed equations (1.2), (2.3), (2.4), and (2.6), we obtain, in the Laplace transform space, the following relation among the coefficients of the n th harmonics in the Fourier expansion of the normal stress and the deformation rate:

$$\begin{aligned} \sigma_{zz}^{(i)L}(s, x, 0) &= \sum_{n=0}^{\infty} \sigma_{zzn}^{(i)L}(s) \cos \bar{n}x, \quad V^{(i)L}(s, x, 0) = \sum_{n=0}^{\infty} V_n^{(i)L}(s) \cos \bar{n}x, \\ \sigma_{zzn}^{(i)L} &= (-1)^i \alpha V_n^{(i)L} \left(1 - \frac{\sqrt{s^2 + \alpha^2 \bar{n}^2} - s}{\sqrt{s^2 + \alpha^2 \bar{n}^2}} + 4\beta^2 \bar{n}^2 \left(\frac{1}{\sqrt{s^2 + \alpha^2 \bar{n}^2}} - \frac{\beta}{\alpha} \frac{1}{\sqrt{s^2 + \beta^2 \bar{n}^2}} \right) \left(\frac{1}{s} + \frac{\beta^2 \bar{n}^2}{s^3} \right) \right), \quad i = 1, 2. \end{aligned}$$

Applying the inverse Laplace transform and using the convolution theorem [3], we get

$$\sigma_{zzn}^{(i)} = (-1)^i \alpha \left(V_n^{(i)}(t) + \int_0^t V_n^{(i)}(\tau) F_n(t - \tau) d\tau \right), \quad i = 1, 2, \quad (2.8)$$

$$F_n(t) = -\bar{n} \alpha J_1(\bar{n} \alpha t) + 4\beta^2 \bar{n}^2 \int_0^t \left(J_0(\bar{n} \alpha \tau) - \frac{\beta}{\alpha} J_0(\bar{n} \beta \tau) \right) \left(1 + \beta^2 \bar{n}^2 \frac{(t - \tau)^2}{2} \right) d\tau, \quad (2.9)$$

where $J_0(t)$ and $J_1(t)$ are Bessel functions of the first kind and of the zero and first orders, respectively [1].

Let us now solve the mixed problem. Using relations (2.8) and (2.9) and the boundary conditions (1.4) and (1.6), we arrive at the following systems of equations:

$$\begin{cases} -2V^{(1)} - \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(1)}(\tau) F_n(t - \tau) d\tau + \int_0^t V_n^{(2)}(\tau) F_n(t - \tau) d\tau \right) \cos \bar{n}x + V_0(t) = 0, & |x| \leq x^*, \\ V^{(1)} + \sum_{n=0}^{\infty} \int_0^t V_n^{(1)}(\tau) F_n(t - \tau) d\tau \cos \bar{n}x = 0, & x^* \leq |x| \leq l, \end{cases} \quad (2.10)$$

$$\begin{cases} 2V^{(2)} + \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(2)}(\tau) F_n(t - \tau) d\tau + \int_0^t V_n^{(1)}(\tau) F_n(t - \tau) d\tau \right) \cos \bar{n}x + V_0(t) = 0, & |x| \leq x^*, \\ V^{(2)} + \sum_{n=0}^{\infty} \int_0^t V_n^{(2)}(\tau) F_n(t - \tau) d\tau \cos \bar{n}x = 0, & x^* \leq |x| \leq l, \end{cases} \quad (2.11)$$

where $V_0 = V_0^{(2)} - V_0^{(1)}$.

Using the unit Heaviside function $H(x)$, we can combine the equations in systems (2.10) and (2.11):

$$\begin{aligned} & -2V^{(1)} - \sum_{n=0}^{\infty} \int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau \cos \bar{n}x \\ & = \left(-V_0(t) + \sum_{n=0}^{\infty} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau \cos \bar{n}x \right) H(x^* - x) + \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau \right) \cos \bar{n}x H(x - x^*), \end{aligned} \quad (2.12)$$

$$\begin{aligned} & 2V^{(2)} + \sum_{n=0}^{\infty} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau \cos \bar{n}x \\ & = \left(-V_0(t) - \sum_{n=0}^{\infty} \int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau \cos \bar{n}x \right) H(x^* - x) - \sum_{n=0}^{\infty} \left(\int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau \right) \cos \bar{n}x H(x - x^*). \end{aligned} \quad (2.13)$$

Expanding the right-hand sides of the latter formulas into Fourier series and equating the coefficients of like harmonics (using the property of orthogonality of trigonometric functions), from relations (2.12) and (2.13) we get two infinite systems of Volterra equations of the second kind for the unknown coefficients $V_n^{(1)}$ and $V_n^{(2)}$:

$$\begin{aligned} & V_n^{(1)}(t) + \frac{1}{2} \int_0^t V_n^{(1)}(\tau) F_n(t-\tau) d\tau + \frac{1}{2} \sum_{m=0}^{\infty} \alpha_{mn} \int_0^t V_m^{(2)}(\tau) F_m(t-\tau) d\tau \\ & - \frac{1}{2} \sum_{m=0}^{\infty} \alpha'_{mn} \int_0^t V_m^{(1)}(\tau) F_m(t-\tau) d\tau = \frac{1}{2} V_{0n}(t), \end{aligned} \quad (2.14)$$

$$\begin{aligned} & V_n^{(2)}(t) + \frac{1}{2} \int_0^t V_n^{(2)}(\tau) F_n(t-\tau) d\tau + \frac{1}{2} \sum_{m=0}^{\infty} \alpha_{mn} \int_0^t V_m^{(1)}(\tau) F_m(t-\tau) d\tau \\ & - \frac{1}{2} \sum_{m=0}^{\infty} \alpha'_{mn} \int_0^t V_m^{(2)}(\tau) F_m(t-\tau) d\tau = -\frac{1}{2} V_{0n}(t). \end{aligned} \quad (2.15)$$

The coefficients of systems (2.14) and (2.15) are determined as follows:

$$\begin{aligned} & V_{00} = x^* \frac{V_0(t)}{l}, \quad V_{0n} = \frac{2V_0(t)}{l} \frac{\sin \bar{n}x^*}{\bar{n}}, \\ & \alpha_{00} = \frac{x^*}{l}, \quad \alpha'_{00} = \frac{1}{l}(-l + x^*), \quad \alpha_{m0} = \frac{1}{l} \frac{\sin \bar{m}x^*}{\bar{m}}, \\ & \alpha'_{m0} = \frac{1}{l} \left(\frac{\sin \bar{m}l}{\bar{m}} - \frac{\sin \bar{m}x^*}{\bar{m}} \right), \quad m \neq 0, \quad \alpha_{mn} = \frac{1}{l} \left(\frac{\sin(\bar{m}-\bar{n})x^*}{\bar{m}-\bar{n}} + \frac{\sin(\bar{m}+\bar{n})x^*}{\bar{m}+\bar{n}} \right), \\ & \alpha'_{mn} = \frac{1}{l} \left(\frac{\sin(\bar{m}-\bar{n})l}{\bar{m}-\bar{n}} + \frac{\sin(\bar{m}+\bar{n})l}{\bar{m}+\bar{n}} - \frac{\sin(\bar{m}-\bar{n})x^*}{\bar{m}-\bar{n}} - \frac{\sin(\bar{m}+\bar{n})x^*}{\bar{m}+\bar{n}} \right), \quad m \neq n, \\ & \alpha_{mm} = \frac{1}{l} \left(x^* + \frac{\sin 2\bar{m}x^*}{2\bar{m}} \right), \quad \alpha'_{mm} = \frac{1}{l} \left(l - x^* + \frac{\sin 2\bar{m}l}{2\bar{m}} - \frac{\sin 2\bar{m}x^*}{2\bar{m}} \right), \quad m \neq 0. \end{aligned} \quad (2.16)$$

From the structure of the infinite systems (2.14) and (2.15) (they differ only by the sign of the right-hand side), it follows that it is sufficient to find the solution of one of them because $V_n^{(2)}(t) = -V_n^{(1)}(t)$. System (2.14) should be supplemented with the equation of motion of the center of mass (1.10) where the force $F(t)$ is defined by

$$F(t) = 2 \sum_{n=0}^{\infty} \sigma_{zzn}(t) \sin \bar{n} x^*(t). \quad (2.17)$$

Thus, we have reduced the mixed boundary-value problem to the infinite system of Volterra equations of the second kind (2.14) (or (2.15)), which is solved simultaneously with Eq. (1.10). After determining the coefficients of $V_n(t)$ from (2.14), we find the stress $\sigma_{zz}(t)$ on the surface of the body from relations (2.4), (2.8), and (2.9).

3. Supersonic Stage. Consider the earliest, so-called “supersonic” stage of collision. At this stage, a point x^* of the contact boundary moves with a velocity greater than that of waves and the perturbations in the bodies do not reach the frontal surface. Therefore, the conditions on this surface may be dealt with as we see fit. Let a velocity normal to the frontal surface of the body be specified on it (equal to zero outside the contact area). Then the boundary conditions on the interval $[-1, 1]$ for $z = 0$ are the following:

$$\left. \frac{\partial u_z^{(1)}}{\partial t} \right|_{z=0} = V_0 + \left. \frac{\partial u_z^{(2)}}{\partial t} \right|_{z=0}, \quad \sigma_{zz}^{(1)} = \sigma_{zz}^{(2)}, \quad \sigma_{xz}^{(i)} = 0, \quad i = 1, 2, \quad (3.1)$$

where $V_0 = H(r^* - |h|) V_0(t) = \sum_{n=0}^{\infty} V_{0n}(t) \cos \bar{n} x$, $V_0(t) = V_0^{(2)}(t) - V_0^{(1)}(t)$. The normal stresses are compressive and the coefficients V_{0n} are determined from formulas (2.16).

Satisfying the boundary conditions (3.1) in the Laplace-transform space, we determine the coefficients $A_n^{(i)}$ and $B_n^{(i)}$:

$$A_n^{(i)} = -\frac{s^2 + 2\beta^2 \bar{n}^2}{2s^3 P} V_{0n}^L, \quad B_n^{(i)} = (-1)^i \frac{\bar{n} \beta^2}{s^3} V_{0n}^L, \quad i = 1, 2, \quad n = \overline{0, \infty}. \quad (3.2)$$

Using expansion (2.4) and general solution (2.2), we substitute expressions (3.2) into the Laplace-transformed equations (1.2). As a result, we obtain the following relation between the Fourier coefficients in the expansion of the normal stress and the coefficients V_{0n} determined earlier:

$$\sigma_{zzn}^L = -\frac{\alpha}{2} V_{0n}^L \left(1 - \frac{\sqrt{s^2 + \alpha^2 \bar{n}^2} - s}{\sqrt{s^2 + \alpha^2 \bar{n}^2}} + 4\beta^2 \bar{n}^2 \left(\frac{1}{\sqrt{s^2 + \alpha^2 \bar{n}^2}} - \frac{\beta}{\alpha} \frac{1}{\sqrt{s^2 + \beta^2 \bar{n}^2}} \right) \left(\frac{1}{s} + \frac{\beta^2 \bar{n}^2}{s^3} \right) \right). \quad (3.3)$$

Applying the inverse Laplace transform and the convolution theorem, we get

$$\sigma_{zzn} = -\frac{\alpha}{2} \left(V_{0n}(t) + \int_0^t V_{0n}(\tau) F_n(t - \tau) d\tau \right), \quad (3.4)$$

where the function $F_n(t)$ is defined by (2.9).

Thus, in the supersonic case, the solution is much simpler and is reduced to an infinite sequence of linear integral equations (3.4) for the coefficients $V_{0n}(t)$ defined by (2.16). It is well to bear in mind that relation (3.4) must be used simultaneously with Eq. (1.10), where the force of reaction $F(t)$ is defined by (2.17).

4. Solution Algorithm and Results. The integrals in (2.8), (2.14), and (3.4) were evaluated by the Gregory formula [10], with retaining differences up to the fourth order. Equation (1.11) with a truncated right-hand side was solved by the Adams method [3]. Differences up to the fourth order were retained in both Gregory and Adams formulas. Infinite systems of integral equations were subjected to reduction. The order of reduction was selected from the considerations of practical convergence. In particular, reliable numerical results were obtained with 150 equations in the infinite system (2.14) and $\Delta t = 0.02$ as a step of integration. The Gibbs phenomena were smoothed by using Gibbs σ factors [10].

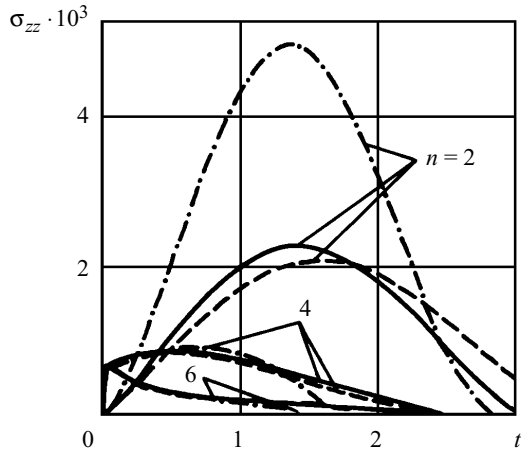


Fig. 2

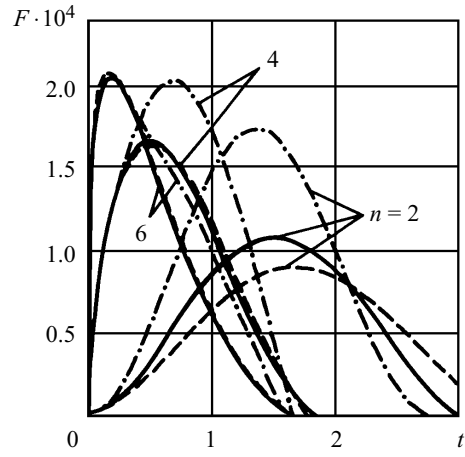


Fig. 3

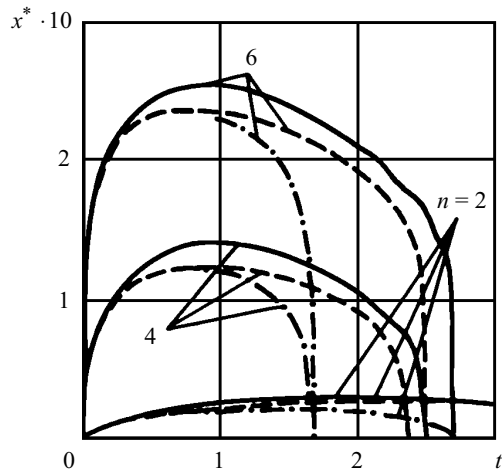


Fig. 4

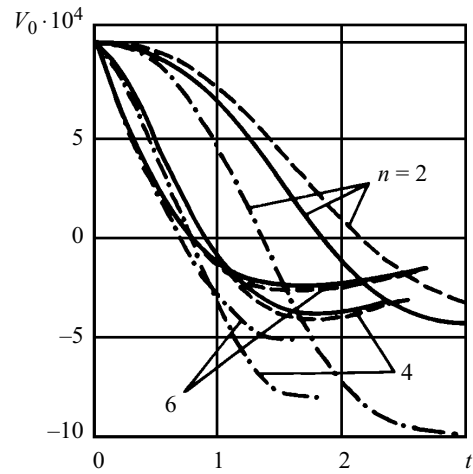


Fig. 5

The task was to determine the normal stress on the surface of the bodies, the forces of interaction, the variable contact area, and the relative velocity depending on the shapes of the colliding bodies.

The collision problem was numerically solved for two identical cylindrical bodies with parabolic cross sections $z = x^n$ ($n = 2, 4, 6$). As n increases, so does the bluntness of the frontal surface of the bodies. Three formulations were considered: the supersonic solution (3.4), (1.10), the mixed problem (2.14), (1.10), and the mixed problem with regard for the change in the contact area because of the deformation of the contacting bodies. In the first two formulations, the contact boundary is determined as the radial coordinate of the intersection point of the undeformed contours of the bodies when z^* is the depth of mutual penetration. In the third formulation, the contact boundary is determined from relation (1.12). In the figures above, the dash-dot line corresponds to the supersonic solution, the dashed line to the mixed problem, and the solid line to the case of accounting for the deformation of the surface of the interacting bodies at the contact boundary (the third formulation).

Figures 2–5 present the results obtained in all the three formulations for $V_0 = 10^{-3}$, $\mu = 0.45$ K, and $M = 0.25$.

Figure 2 shows how the normal stress at the frontal point vary in time for various values of n . As the degree of bluntness (the power of the parabolic curve) increases, the impact-induced stress decreases and reaches the maximum value earlier. For $n = 4$ and $n = 6$, the curves corresponding to the second and third formulations practically coincide. For $n = 2$, the curve of change in the stress at the supersonic stage differs considerably from the curves resulted from the solution of the problem in the other formulations.

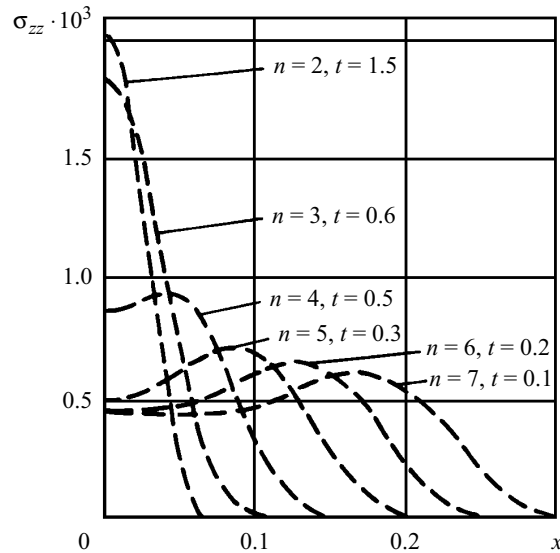


Fig. 6

Figure 3 shows how the force of interaction between the bodies varies with time. On the time interval considered, the force of reaction reaches the maximum value, which increases with n . For $n = 4$, the curves corresponding to the second and third formulations coincide. For $n = 6$, all the results coincide at the initial stage and then, after reaching the maximum value, the force of interaction decreases more slowly in the supersonic case.

Figure 4 demonstrates how the contact area varies during the collision. The width of the contact half-strip increases with the power of the parabolic curve that bounds the cross section of the cylinder. The resultant curves for the mixed and supersonic problems coincide only at the initial stage, and the collision duration is much greater in the mixed problem. The results for the second and third formulations are slightly different.

Figure 5 shows the velocity of the body as a whole versus time. As in the previous figures, the curves for the second and third formulations practically coincide when $n = 4, 6$. As the degree of bluntness (the parameter n) increases, the velocity decreases faster and the difference between the solutions in the third formulation and the other two formulations decreases.

An analysis of Figs. 2–5 reveals that the difference between the solutions in all the three formulations decreases with increase in n . However, the supersonic solution can be used only at the initial stage of interaction because since then the difference increases, becoming significant at the last stage. As regards the other two formulations, the curves of the second formulation may be used to approximate the curves of the third formulation.

Figure 6 compares the distributions of the normal stress over the surface of the colliding bodies for various values of n . The curves correspond to the time point the stresses reach their maximum. It is seen from the graphs that the greater the bluntness of the cross section, the lower the maximum stress, the earlier it occurs, and the farther it shifts toward the boundary of the contact area from its center. The stress distribution shown in Fig. 6 is in qualitative agreement with that (for various values of n) for two parabolic bodies under static compression [11].

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