

THEORY OF CRACK GROWTH IN ROLLING CONTACT

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Roll pairs are widely used in mechanisms, e.g., in ball and roller bearings, roll mills, gear trains, wheelrail systems, and so on. Determining the working life in such a system is important.

In most cases, the working life of a roll pair is governed by the crack initiation and growth in the contact zone [1-3], and approaches from failure mechanics are used to obtain solutions. Experiments have indicated the damage in contact rolling fatigue, and in certain cases it has proved possible to determine the contact-fatigue working life of roll surfaces [4-8]. Importance is attached to determining the state of stress and strain around edge and subsurface stationary cracks. Most theoretical treatments concern two-dimensional cases, i.e., deriving stress intensity factors SIF at crack vertices in half-planes with given loading at the boundary (localized force, herzian contact loading) and with various boundary conditions at the crack edges, which simulate the contact interaction, the effects of lubricants, and so on [9-15]. One can use these data and cracking-resistance characteristics in deterministic models [10] and probabilistic ones [16, 17] to determine the working life. Keer and Bryant [10] used the experimentally determined crack growth path in calculations. It is a difficult theoretical task to calculate the crack path in a compound state of loading such as arises in rolling.

Here we propose a two-dimensional model for the working life in which a key element is an algorithm for defining the crack growth path in one of the bodies in the contacting pair. We use a local failure criterion derived from the mechanical properties of the material and the state of stress and strain, together with the equation for the crack growth rate and the characteristics of the cyclic cracking resistance. We use a concept for estimating the working life from the viewpoint of failure mechanics [18, 19] with realization [20] for cyclic loading, together with a stepwise method of predicting the crack growth path [21, 22] by the use of singular integral equations [21, 23]. The rolling body is represented as a half-plane, while the contact interaction in which rolling cycle is represented by transverse displacement along the boundary for normal and tangential forces distributed over the contact area in accordance with a certain law (elliptical or parabolic distribution, localized force, etc.). Two independent steps are used in deriving the path, which allow one to incorporate the stress redistribution related to the advance of the crack vertex and to the displacement of the contact area. It is assumed that we are given the crack growth path in a tribosystem in a fashion that enables one to determine the dimensions of the contacting-body surface fragmentation and to establish the mechanisms and causes of the damage, including pitting and flaking. The calculations are performed for the case where the contact forces are represented by a localized force, while the local failure criterion is the σ_θ one, and the crack growth rate is defined by Paris's equations. The growth paths are examined for edge cracks and an estimate is made of the working life for rolling surfaces composed of 75KhGST steel. Some trends in rolling-surface pitting are established.

Model for Calculating Rolling-Surface Life. For simplicity we assume that there is a surface crack in one of the bodies of the rolling pair of cylindrical form. The width of the contact area and the crack length are small by comparison with the surface radius of curvature, so instead of the real body we use a two-dimensional formulation for a half-plane containing an edge crack. We simulate the contact effect on the body containing the crack from the other body (counterbody) as repeated translational displacement along the boundary of the half-plane, with the contact forces distributed over the area (line) of contact in accordance with a certain law:

$$N(x) + i T(x) = -(1 + if) p(x) = -p_0(1 + if) s(x, x_0), \quad |x - x_0| \leq c. \quad (1)$$

Then the normal forces $N(x)$ and the tangential forces $T(x)$ are related by Coulomb's law via the coefficient of friction f , where x_0 is the abscissa for the center of the contact area (Fig. 1), $2c$ is the width of that area, $s(x, x_0)$ is the normalized contact

pressure at the boundary of the half-plane, and p_0 is the force parameter (maximum pressure in the contact area). We express the frictional forces that arise when the bodies move by means of the tangential forces.

We use linear failure mechanics techniques [3, 17] in the analysis. We consider

$$K = F(K_I(\lambda), K_{II}(\lambda), \theta^*(\lambda)), \quad (2)$$

derived from the local failure mechanism and the parameter K controlling it in the compound state of strain [24]. Here K_I and K_{II} are the SIF at the crack vertex, while $\lambda = x_0/l_1$, where l_1 is the length of the initial crack. In static loading, when $F(K_I, K_{II}, \theta^*)$ attains a critical value for a particular material, the crack begins to grow in the direction (Fig. 1) indicated by the angle

$$\theta^* = f(K_I(\lambda), K_{II}(\lambda)), \quad (3)$$

which provides extremal $F(K_I, K_{II}, \theta)$ for the given $K_I(\lambda)$ and $K_{II}(\lambda)$.

In rolling, the loading is cyclic. We assume that the local failure is quasistatic and use (2) and (3) to examine the crack growth in this case.

In each cycle, the contact area is displaced along the boundary of the half-plane (with λ varying), and $F(K_I, K_{II}, \theta^*)$ varies, taking maximal and minimal values with respect to λ :

$$K_{\max} = F(K_I(\lambda'), K_{II}(\lambda'), \theta^*(\lambda')), \quad (4)$$

$$K_{\min} = F(K_I(\lambda''), K_{II}(\lambda''), \theta^*(\lambda'')).$$

The following is the range in $F(K_I, K_{II}, \theta^*)$ in a rolling cycle:

$$\Delta K = K_{\max} - K_{\min}. \quad (5)$$

We also derive the value of $F(K_I, K_{II}, \theta^*)$ maximum in modulus:

$$K_{\max}^* = \max \{ |K_{\max}|, |K_{\min}| \} = |F(K_I(\lambda^*), K_{II}(\lambda^*), \theta^*(\lambda^*))|, \quad (6)$$

in which

$$\lambda^* = \begin{cases} \lambda', & \text{if } |K_{\max}| > |K_{\min}|, \\ \lambda'', & \text{if } |K_{\max}| < |K_{\min}|. \end{cases} \quad (7)$$

We assume that the crack grows in each cycle only for $\lambda = \lambda^*$ in the direction defined by (3) when the following condition is obeyed:

$$\Delta K \geq \Delta K_{th}. \quad (8)$$

Here ΔK_{th} is the threshold range for $F(K_I, K_{II}, \theta^*)$, which is a characteristic of the cyclic cracking resistance for the given conditions (external medium, temperature).

We derive the path in stages. In the first stage, we consider the contact problem in elasticity theory for a half-plane with an initial edge crack (curvilinear in the general case) along the line L_1 with the (1) force acting on the boundary with abscissa for the center of the contact area x_0 and with boundary conditions on L_1 that incorporate the scope for contact between the edges of the crack. The solution gives the SIF. Further, from (3), (4), and (6) we determine that value of the parameter $\lambda = \lambda_1^*$ (i.e., the location of the contact area) and the crack development angle θ_1^* such that $F(K_I, K_{II}, \theta^*)$ attain $K_{\max}^*$. From (5) we also derive the range ΔK_1 . We assume that (8) is obeyed and that each time (in N_1 cycles) the crack acquires a small increment Δl_1 and develops in the direction denoted by θ_1^* , i.e., $\lambda_1^*, \theta_1^*, \Delta K_1$ do not vary from cycle to cycle. The newly formed path length Δl_1 is approximated by a polynomial of third degree. At each successive stage, the path is described by repeating the procedure (each time for the new crack edge). This gives us the crack growth path as a smooth curve.

When one estimates the working life under cyclic loading, one distinguishes [24] two periods, i.e., the total working life

$$N = N_0 + N_c = N_0 + \int_{l_1}^{l_c} \frac{dl}{v(\Delta K(l))}, \quad (9)$$

in which N_0 is the number of cycles needed to form an initial macrocrack[†] of length l_1 , while N_c is the number of cycles during the growth stage in which it grows along its path from the initial length l_1 to the critical length l_c , on completion of which the sample is no longer viable; the function

$$v = v(\Delta K(l)) \quad (10)$$

describes analytically the dependence of the growth rate on the range in K , which controls the growth mechanism. We call N_c the viability or the residual working life.

We seek to establish the range ΔK for function $F(K_I, K_{II}, \theta^*)$ at each stage along the crack growth path by the use of the analytic formula (10), which describes the fatigue failure diagram FFD for the material in $(v, \Delta K)$ coordinates, for which we derive the crack growth rate and thus N_c . Paris's formula required subsequently is

$$v = C(\Delta K)^n, \quad (11)$$

which is often used and which applies for the middle part of the FFD[‡] (C and n are characteristics of the cyclic cracking resistance).

There may be a change in the failure mechanism during the crack growth that is related to the state of stress and strain and the mechanical properties of the material. Then one should introduce the corresponding equation for the other mechanism into the algorithm for calculating the path together with the condition for transfer from one mechanism to the other. One can also consider cracks, for example, which develop with transition to shear mechanism on two areas. In that case, the task becomes more complicated, since one has to consider the additional failure contour (crack) and the stepwise growth of each part, including the growth rates. The stepwise path discussion mechanism is adopted also for internal cracks. Then the increases in length at each tip are also related to the growth rates.

Crack Growth Path Algorithm. The contact loading is simulated as a localized (equivalent) force and it is assumed that the crack growth involves the normal detachment mechanism.**

The criterion for generalized normal detachment (σ_θ criterion) indicates that the crack grows in a direction for which the stresses σ_θ are such that there is a maximum value for the stress intensity factor $K_{I\theta}$. Then (2) takes the form

$$K = K_{I\theta} = \cos^3(\theta^*/2) [K_I - 3 \operatorname{tg}(\theta^*/2) K_{II}], \quad (12)$$

and the angle θ^* is defined [23] by

$$\theta^* = 2 \operatorname{arctg} [(K_I - \sqrt{K_I^2 + 8K_{II}^2}) / (4K_{II})]. \quad (13)$$

We also assume that in each rolling cycle (and thus in each stage in defining the path), there is a range in λ in which the crack edges are not in contact. This enables one to derive the path and estimate $K_{I\theta, \max}$ by using the solution to the first basic problem in elastic theory for a half-plane containing a crack when a localized force acts at the boundary of the half-plane. We also assume that $K_{I\theta, \min} = 0$. Then (6) becomes

$$K_{I\theta, \max}^* = K_{I\theta, \max}, \quad (14)$$

and $\Delta K_{I\theta} = K_{I\theta, \max}$. Correspondingly, crack growth condition (8) becomes

$$K_{I\theta, \max} \geq K_{Ith}. \quad (15)$$

Let the elastic isotropic half-plane occupy the lower part of the plane of the complex variable $z = x + iy$ ($y \leq 0$) and be weakened by a smooth edge curvilinear crack. Its initial edge L_1 is of length l_1 and is referred to the local $x'O'y'$ coordinate system, and it has its origin at point a on the edge of the half-plane and end at point b_1 (Fig. 1). The $x'O'y'$ system is related

[†]We do not examine the crack initiation stage in this paper.

[‡]Although Paris's formula is derived on the basis of describing the FFD in terms of the mechanism for normal detachment in a symmetrical loading pattern, many researchers have used it in the case of failure as a consequence of normal detachment in a compound state of strain [10, 20] or under transverse shear [10] by the use of the corresponding characteristics for the cyclic cracking resistance.

**In the general case, a fatigue crack can grow on rolling on account of normal detachment and because of transverse shear. Some researchers have shown by experiment that normal detachment predominates, particularly at high loading levels [25, 26].

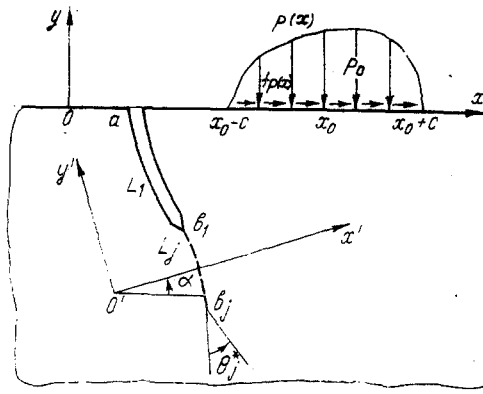


Fig. 1. General geometrical and force scheme.

to the basic coordinate system xOy by $z = z'e^{i\alpha} + z_0'$, in which the complex variable is $z' = x' + iy'$; z_0' is the affix of point O' in the xOy system, and α is the angle between the abscissas in the main and local coordinate systems.

The model and assumptions lead us to predict the crack growth in the next stage j by solving the first basic problem in elasticity theory for a half-plane weakened by a curvilinear edge crack having an edge L_j when a localized force acts on the boundary of the half-plane at the point $x = x_0$, i.e., with the following boundary conditions*:

$$N(x) + iT(x) = -P(1 + if)\delta(x - x_0), \quad y = 0; \quad (16)$$

$$N^\pm(\tau) + iT^\pm(\tau) = 0, \quad \tau \in L. \quad (17)$$

In (17), the superscripts $+$ and $-$ denote the limiting values of the stresses at the edges of a crack on approach from left and from right to L .

The complex loading potentials are written as sums:

$$\Phi(z) = \Phi_0(z) + \Phi_1(z); \quad \Psi(z) = \Psi_0(z) + \Psi_1(z). \quad (18)$$

Here the potentials $\Phi_0(z)$ and $\Psi_0(z)$ characterize the loaded state of the continuous half-plane with boundary loaded in accordance with boundary condition (16) and are defined [27] by

$$\Phi_0(z) = \frac{P}{2\pi} \frac{i-f}{z-x_0}; \quad \Psi_0(z) = \frac{P}{2\pi} \left[\frac{i+f}{z-x_0} + \frac{i-f}{(z-x_0)^2} x_0 \right], \quad (19)$$

while the functions $\Phi_1(z)$ and $\Psi_1(z)$ describe the loaded state related to the displacement $g(t)$ at L [21]:

$$\begin{aligned} \Phi_1(z) &= \frac{1}{2\pi} \int_L \left\{ \left[\frac{1}{T-z} - \frac{1}{\bar{T}-z} \right] e^{i\alpha} g'(t) dt + \frac{T-\bar{T}}{(\bar{T}-z)^2} e^{-i\alpha} \overline{g'(t)} d\bar{t} \right\}; \\ \Psi_1(z) &= \frac{1}{2\pi} \int_L \left\{ \left[\frac{1}{(\bar{T}-z)^2} - \frac{1}{(T-z)^2} \right] \bar{T} e^{i\alpha} g'(t) dt + \right. \\ &\quad \left. + \left[\frac{1}{T-z} - \frac{1}{\bar{T}-z} + \frac{(\bar{T}-T)(\bar{T}+z)}{(\bar{T}-z)^3} \right] e^{-i\alpha} \overline{g'(t)} d\bar{t} \right\}, \end{aligned} \quad (20)$$

in which $T = te^{i\alpha} + z_0'$.

We satisfy (17) with the (18) potentials to get a singular integral equation in terms of the displacement $g'(\tau)$, which in normalized form is [27, 28].

$$\int_{-1}^1 [R(\xi, \eta) \varphi(\xi) + S(\xi, \eta) \overline{\varphi(\xi)}] d\xi = \pi P(\eta, \lambda), \quad |\eta| < 1, \quad (21)$$

*Subsequently, the subscripts indicating the serial number of the stage are usually omitted. For example, in the first stage we have the initial crack and $L = L_1$.

in which

$$\begin{aligned}
 R(\xi, \eta) = & \operatorname{Re} \left[\frac{\omega'(\eta)}{\omega(\xi) - \omega(\eta)} + \frac{\bar{\omega}'(\eta)}{\bar{\omega}(\xi, \eta)} \right] + \frac{1}{2} \frac{\omega(\xi, \xi)}{\omega^2(\xi, \eta)} [\bar{\omega}'(\eta) + \\
 & + \omega'(\eta) e^{2i\alpha} + 2e^{2i\alpha} \bar{\omega}'(\eta) \frac{\omega(\xi) - \omega(\eta)}{\omega(\xi, \eta)}]; \quad S(\xi, \eta) = -\frac{1}{2} \frac{\partial}{\partial \eta} \frac{\omega(\xi) - \omega(\eta)}{\omega(\xi) - \bar{\omega}(\eta)} + \\
 & + \frac{\omega'(\eta)}{2} \left[\omega^{-1}(\xi, \eta) + \frac{\omega(\xi) - \omega(\eta)}{\omega^2(\xi, \eta)} \frac{\bar{\omega}'(\eta)}{\omega'(\eta)} + e^{-2i\alpha} \frac{\bar{\omega}(\xi, \xi)}{\bar{\omega}^2(\xi, \eta)} \right]; \\
 \omega(\xi, \eta) = & \bar{\omega}(\eta) - \omega(\xi) e^{2i\alpha} - 2ie^{i\alpha} \operatorname{Im} z'_0; \quad \varphi(\xi) = g'(t) \omega'(\xi); \\
 P(\eta, \lambda) = & \frac{P}{\pi} \left\{ \operatorname{Re} \left[\frac{(f-i)e^{-i\alpha}}{a(\eta, \lambda)} \right] \omega'(\eta) - \frac{\operatorname{Re} [a(\eta, \lambda)(f+i)e^{i\alpha}]}{a^2(\eta, \lambda)} \bar{\omega}'(\eta) \right\}; \\
 a(\eta, \lambda) = & \omega(\eta) + (z'_0 - \lambda I_1) e^{-i\alpha}.
 \end{aligned} \tag{22}$$

From

$$t = \omega(\xi) = x'(\xi) + iy'(x'(\xi)), \quad t \in L, \quad |\xi| \leq 1 \tag{23}$$

we specify the parametric equation for the edge of the crack L_j , which is the unknown in predicting the path increments in the $j-1$ previous stages. Here $x'(\xi) = \operatorname{Re}[(b-a)\xi + b + a]/2$, where b is the affix for the leading crack tip, and $y'(x')$ is the function that describes L_j in the $x'O'y'$ local coordinate system, which is located in such a way as to provide uniqueness in $y'(x')$ in all stages of path prediction.

Integral equation (21) was solved numerically by mechanical quadrature (see [21, 23]). $\varphi(\eta)$ is written as $\varphi(\eta) = u(\eta)/\sqrt{1-\eta^2}$, in which $u(\eta)$ is a function continuous in the interval $[-1, 1]$ that satisfies the additional condition

$$u(-1) = 0, \tag{24}$$

which provides a finite solution at $\eta = -1$ at the boundary of the half-plane. We apply gaussian quadrature formulas to (21) and represent $u(\eta)$ in (24) by means of an interpolation Lagrange polynomial at the nodes ξ_k to get a system of M linear algebraic equations for the M unknowns $u(\xi_k)$ ($k = 1, \dots, M$):

$$\begin{aligned}
 \sum_{k=1}^M [R(\xi_k, \eta_m) u(\xi_k) + S(\xi_k, \eta_m) \bar{u}(\xi_k)] &= MP(\eta_m, \lambda) \\
 (m=1, M-1); \\
 \sum_{k=1}^M \left[(-1)^k \operatorname{tg} \left(\frac{2k-1}{4M} \pi \right) u(\xi_k) \right] &= 0,
 \end{aligned} \tag{25}$$

in which $\xi_k = \cos[(2k-1)\pi/(2M)]$ and $\eta_m = \cos[\pi m/M]$ are the zeros of Chebyshev polynomials correspondingly of the first kind $T_M(\xi)$ and the second kind $U_{M-1}(\eta)$.

The solution is used with the formula [21]

$$K_I - iK_{II} = \frac{1}{M} \frac{V\pi|\omega'(1)|}{\omega'(1)} \sum_{k=1}^M \left[(-1)^k \operatorname{ctg} \left(\frac{2k-1}{4M} \pi \right) u(\xi_k) \right] \tag{26}$$

to derive the SIF, while $K_{I\theta}$ and θ^* are derived from (12) and (13).

We can now derive the value $\lambda = \lambda^*$ for which $K_{I\theta}$ attains the maximum $K_{I\theta, \max}$. We derive λ^* in steps. In the first stage of path prediction, we examine the entire range in λ for which the crack is open ($K_I(\lambda) > 0$), and in the subsequent steps, the ranges around the previous values of λ^* , i.e.,

$$\lambda_j^* = \lambda_{j-1}^* \pm mH_1, \quad j = 2, 3, \dots, \quad m = 0, 1, 2, \dots, \tag{27}$$

where $H_1 > 0$ is a fixed increment in λ^* and m is the number of steps. At each step, one solves (21), but in the intervals within a stage, the procedure for finding $K_{I\theta, \max}$ simplifies because only the right-hand side of (21) is dependent on λ .

When λ_j^* is known, (13) gives θ_j^* , which is the basis for predicting segment $j+1$ of the path. The abscissa for the crack tip is determined from stage to stage by the following (Fig. 2) [29]:

$$x'_{j+1} = \operatorname{Re} b_1 + \sum_{k=1}^j h_k, \tag{28}$$

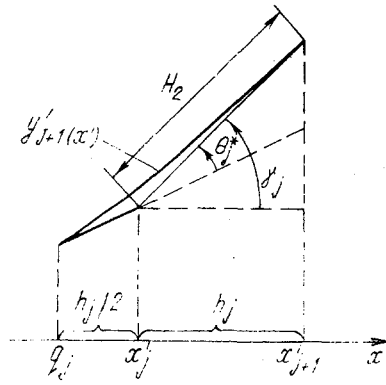


Fig. 2. Scheme for forecasting the next $(j + 1)$ -st section in the crack path increment.

in which $h_k = H_2 \cos \gamma_k$ is the increment in the abscissa at each stage in predicting the path, $H_2 > 0$ is a fixed step in the path increment, which lies in the direction of crack propagation, while γ_k is the angle between the local $O'x'$ -axis and the direction of path growth; clearly, $x'_j = \text{Re } b$. In the first stage, one considers the initial crack, while the next segments in the path increment are determined by laying them out in the region of the angular points as approximated by polynomials of third degree (see [21, 22]):

$$y'_{j+1}(x') = a_{j+1}(x' - q_j)^3 + b_{j+1}(x' - q_j)^2 + c_{j+1}(x' - q_j) + d_{j+1} \quad (29)$$

$$(q_j \leq x' \leq x'_{j+1}, \quad q_j = x'_j - h_j/2).$$

The parameters of these polynomials are determined by conditions for smoothness of link-up between adjacent parts of the path increment and by the direction of the next increment in the crack.

Then the equation for the entire path is

$$y'(x') = \begin{cases} y'_1(x'), & \text{Re } a \leq x' \leq q_1 = \text{Re } b_1 - h_1/2; \\ y'_k(x'), & q_{k-1} \leq x' \leq q_k, \quad k = \overline{2, j}; \\ y'_{j+1}(x'), & q_j \leq x' \leq x'_{j+1}, \end{cases} \quad (30)$$

in which $y'_1(x')$ describes the edge of the initial crack in the interval $[\text{Re } a, \text{Re } b_1]$.

We represent the expression for N_c in (11) as

$$N_c \approx \sum_{k=1}^{j_c} \frac{\Delta l_k}{v_k(K_{1\theta, \max})} = H_2 \sum_{k=1}^{j_c} v_k^{-1}(K_{1\theta, \max}), \quad (31)$$

which is used in stepped path prediction. Here j_c is the total number of steps in the path growth necessary for the extension from the initial length to the critical one.

Numerical Analysis (path and SIF). To forecast the propagation path for an edge crack initially rectilinear and of length l_1 at an angle β to the boundary of a half-plane ($z_0' = 0$, $a = 0$, $b_1 = l_1 e^{-i\beta}$, $\beta = -\alpha$, see Fig. 4a) on the assumption that the failure occurs via the normal detachment mechanism. The contact loads during rolling are simulated as a localized force. The parametric equation for the edge of the initial crack is

$$t = \omega(\xi) = l_1(\xi + 1)/2, \quad |\xi| \leq 1. \quad (32)$$

The coefficient of friction f is 0.25 or 0.30, which correspond for example to the friction between a wheel and rail in dry weather [30].

Figure 3 shows how the loaded-state characteristics around the crack tip vary during the initial rolling cycles (in the first stage of path prediction) [27].

The numerical data gave the ranges in λ in which the initial crack is completely open ($K_I(\lambda) > 0$). With the direction used for the tangential forces fP (Fig. 4a), the relevant ranges in λ lie to the right of the crack (λ additional). For other λ , there is complete or partial contact between the crack edges. The analysis is based on Fig. 3a, which shows the SIF $K_I(\lambda > 0)$. For identical angles of the crack to the boundary, the range in λ for which $K_I(\lambda) > 0$ is widest for obtuse β , which is

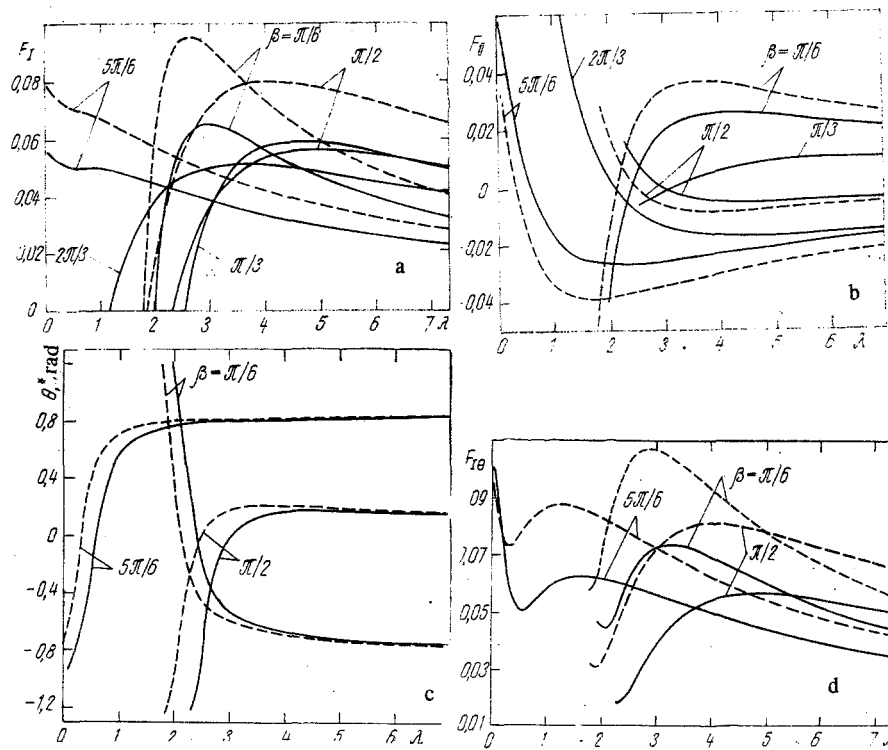


Fig. 3. Behavior of the normalized SIF $F_I = \sqrt{\pi l_1} K_I/P$ (a) and $F_{II} = \sqrt{\pi l_1} K_{II}/P$ (b) and of the angle of initial crack growth (c) and the normalized SIF $F_{I0} = \sqrt{\pi l_1} K_{I0}/P$ (d) for an initially rectilinear edge crack of length l_1 oriented at an angle β to the boundary of the half-plane, the independent variable being the distance of the point of application for the localized force at the boundary from the crack $x_0 = l_1 \lambda$. The solid lines are for $f = 0.25$ and the dashed ones for $f = 0.30$.

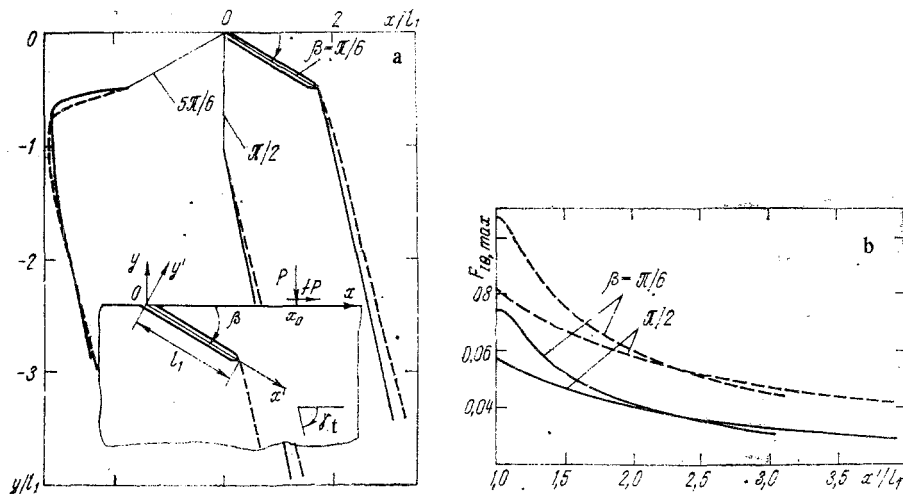


Fig. 4. Growth path of an initially rectilinear edge crack of length l_1 oriented at angle β to the boundary of a half-plane as a consequence of the normal detachment mechanism on cyclic displacement of a localized force, which simulates rolling contact interaction (a); normalized SIF $F_{I0,max} = \sqrt{\pi l_1} K_{I0,max}/P$ along path (b). The solid lines are for a coefficient of friction at the contact $f = 0.25$ and the dashed lines for $f = 0.30$.

TABLE 1

β , rad	λ_1^*	θ_1^* , rad	$F_{I\theta}$, max
$\pi/6$	3,3	-0,580	0,0739
	2,9	-0,562	0,1071
$\pi/2$	4,9	0,177	0,0569
	4,1	0,210	0,0815
$5\pi/6$	0,1	-0,904	0,94
	0,1	-0,551	0,88

Note. The data above the lines are $f = 0.25$, and below them for $f = 0.30$.

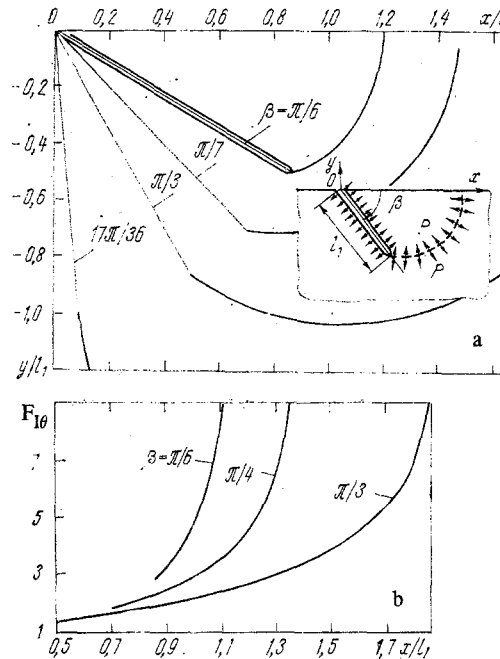


Fig. 5. Growth path in the normal detachment mechanism for an initially rectilinear edge crack of length l_1 at an angle β to the boundary of a half-plane with a normal pulsating pressure on the edges (a); normalized SIF $F_{I\theta} = K_{I\theta}/(p\sqrt{\pi l_1})$ along path (b).

conditioned by the direction of action for the tangential forces fP . As the angle of the crack to the boundary increases, the relative ranges in λ on the whole narrow. As the tangential force fP increases (coefficient f) for a fixed β , the range in λ for which the crack is open will extend.

The SIF (for open cracks) varies rapidly when the localized force moves near the crack (Fig. 3a and b), which is reflected in the behavior of the dependent quantities θ^* and $K_{I\theta}$ (Fig. 3c and d). Here the SIF attain their maximum values in absolute terms. Increased friction in the contact leads in the main to increase in the stress intensity around the tip (Fig. 3a, b and d). As the contact point recedes from the mouth of the crack, the SIF, including $K_{I\theta}$, fall gradually to zero, while θ^* varies little and for each orientation of the crack tends to the corresponding asymptotic value.

These numerical values for K_I and K_{II} agree well with the analytical results for a crack perpendicular to the boundary of a half-plane [31].

From Fig. 3, one can establish the instant of propagation for the initial crack (when $K_{I\theta}$ for the open crack attains the maximum value during the loading cycle) and also the initial growth direction (θ_1^* and corresponding λ_1^*). Table 1 gives the numerical values of these for various β and f .

The crack growth path in Fig. 4a shows that the direction of the subsequent crack growth is only slightly dependent on the initial orientation, and after a certain initial period, the growth into the material along a straight line at any acute angle

TABLE 2

β , rad	B	H_1	H_2	λ_1^*	λ_e^*	Δl , mm	N_e , cycles
Localized force at boundary of half-plane							
$f=0,25$							
$\pi/6$	10	$0,1l_1$	$0,05l_1$	3,3	18,0	3,0	$2,0 \times 10^4$
$\pi/2$	10	$0,1l_1$	$0,05l_1$	4,9	19,5	3,0	$2,8 \times 10^5$
$5\pi/6$	10	$0,1l_1$	$0,01l_1$	0,1	12,8	3,0	$1,4 \times 10^5$
$f=0,30$							
$\pi/6$	10	$0,1l_1$	$0,05l_1$	2,9	15,2	3,0	$5,6 \times 10^4$
$\pi/2$	10	$0,1l_1$	$0,05l_1$	4,1	16,4	3,0	$8,0 \times 10^4$
$5\pi/6$	10	$0,1l_1$	$0,01l_1$	0,1	10,6	3,0	$4,7 \times 10^4$
Pressure uniformly distributed along crack edges							
$\pi/6$	0,4	—	$0,01l_1$	—	—	0,40	$5,6 \times 10^4$
$\pi/4$	0,6	—	$0,01l_1$	—	—	0,76	$1,4 \times 10^5$
$\pi/3$	0,8	—	$0,1l_1$	—	—	1,43	$2,4 \times 10^5$

γ_t to the boundary is in the direction of action of the tangential contact force. The greater the friction at the contact, the more strongly the crack is deflected towards the boundary: for $f = 0.25$, $\gamma_t \approx 78^\circ$, while $\gamma_t \approx 76^\circ$ for $f = 0.30$.

Clearly, there is an angle for the initial rectilinear crack $\beta = \gamma_t$ for which $\theta_1^* \approx 0$ and the crack will propagate along its continuation. Also, near $\beta = \gamma_t$, the behavior of the $K_{II}(\lambda)$ curve alters (Fig. 3b). If this angle is determined, one can predict the growth direction for an arbitrarily oriented crack that develops by the normal detachment mechanism.[†] Predicting the path from the initial growth direction, as has been done by some researchers, can lead to errors (the path for $\beta = 5\pi/6$ in Fig. 4a).

In this mode of loading, the crack propagates into the plate, and it should be emphasized that this is accompanied by falls in $K_{I\theta, \max}$ (Fig. 4b), which is a basic characteristic in the calculation of crack growth rates and rolling surface life, which ultimately may lead to the crack halting or a change in the growth mechanism.

We compare these paths with the growth paths for cracks whose edges are subject to cyclic uniformly-distributed pressure (Fig. 5a) [32]. That loading enables one to simulate the effects of oil at the edges of a surface crack involving contacts. When there is pressure on the edges of a crack oriented in any way (apart, clearly, from the symmetry case, when $\beta = \pi/2$), the growth occurs to the boundary. Pressure from the oil or other filling media penetrating to the middle of the crack is one of the reasons for pitting during rolling, which has been confirmed by experiment [4] (blackening does not occur for the unlubricated surfaces[‡]). The scale of the pitting is dependent on the length of the initial crack and the orientation: the shorter the initial crack and the greater the inclination to the boundary, the smaller the pitting recess. Also, as the crack tip approaches the boundary, there is rapid increase in $K_{I\theta}$ (Fig. 5b), which leads to spontaneous growth.

Rolling-Surface Life Estimation. One can estimate the working life for a rolling surface having a surface damaged by an initially rectilinear edge crack of length $l_1 = 1$ mm in 75KhGST steel having a platey pearlite structure, which is used to make railroad rails. The calculation is based on (31) with the numerical data for $K_{I\theta, \max}$ (Fig. 4b and 5b). The crack growth rate is determined from the Paris formula (9). The cyclic cracking resistance characteristics of the steel are [33]: $K_{1-2} = 5.2$, $\text{MPa}\sqrt{\text{m}}$, $k_{2-3} = 29.1 \text{ MPa}\sqrt{\text{m}}$, $C = 3.09 \times 10^{-12} \text{ MPa}^{-n} \cdot \text{m}^{1-n/2}$, $n = 3.48$.

We consider two modes of plate loading: cyclic translational displacement of a localized force along the boundary of a half-plane, and crack edges subject to a cyclic uniformly-distributed pressure (to simulate the effects of oil on rolling). In the first case, there was a contact force $P = BK_{2-3}\sqrt{\pi l_1}$ in each cycle, while in the second, the maximum pressure at the crack edges was $p_{\max} = BK_{1-2}/\sqrt{\pi l_1}$. In each case, the constant B was chosen to satisfy the following condition when the crack started and grew:

[†]We note that edge cracks grow by the transverse shear mechanism in the rolling pair at an angle of $20-30^\circ$ to the surface in the direction of the tangential forces. That trend has been demonstrated by experiment [5, 26] in which contact fatigue was examined with low coefficients of friction.

[‡]Pitting may arise at unlubricated surfaces when a surface crack propagates under the action of contact forces into the depth of the material and is deflected by other cracks or deviates toward the boundary. We do not consider here the peeling in a rolling surface usually associated with the growth of subsurface cracks.

$$K_{1-2} \leq K_{I\theta, \max} \leq K_{2-3}, \quad (33)$$

in which K_{1-2} and K_{2-3} are the boundaries of the Paris section in the FFD.

Table 2 shows the numerical results, together with λ^* at the start ($\lambda^* = \lambda_1^*$) and at the end ($\lambda^* = \lambda_e^*$) and N_e , the number of cycles during which the initial crack grows to a length l_e , together with $\Delta l = l_e - l_1$. In the first mode of loading, the initial crack could grow to 3 mm, while in the other one, the calculation was continued until $K_{I\theta, \max}$ exceeded K_{2-3} .

The working life decreased as the friction increased. As f increased from 0.25 to 0.30, the number of cycles needed for the crack to grow to 3 mm was reduced by an order of magnitude (Table 2). In the second mode of loading, we find that the larger the angle of the initial crack to the boundary, the greater the tendency toward the boundary at the tip, which in turn increased the surface viability as the crack increased to a permissible length. The working life with uniform pressure on the edges basically agrees as to order of magnitude with standard values [4, 5, 10], which in our view confirms Tallian's deduction [16] that the growth stage for an edge crack (the working life) in rolling contact constitutes the main part of the total working life.

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