

## STRESSES IN A HALF SPACE IN THE PROCESS OF CONTACT INTERACTION WITH A NOTCHED SURFACE

O. V. Maksymuk, V. V. Shyrokov, and A. O. Cherneha

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We propose a mathematical model and the solution of a periodic contact problem of interaction of a notched surface with an elastic half space, study the mutual influence of microasperities in the contact surfaces, and determine the isolines of the maximum level of tangential stresses near the contact zone. This enables us to predict the regions of fracture (wear) for actual tribocouples.

Since the physicochemical transformations and wear in the process of friction occur under the action of contact stresses, the stressed state realized in the contact zone is an important factor affecting these processes. In the case of friction of solid bodies, surface irregularities promote the formation of variable (as functions of time) stresses in some microvolumes adjacent to the regions of actual contact. In the surface layers, these stresses can exceed the critical levels corresponding to the onset of accumulation of damage to the material, initiation and growth of microcracks, and separation of worn particles. The comprehensive investigation of the distribution of stresses and strains in the surface layers is based on the use of various experimental and analytic methods [1–3]. However, in this case, the required experimental investigations encounter serious difficulties because all mechanical factors acting in the process of contact are localized near uneven (in most cases) surfaces and, hence, the applicability of the well-known methods is doubtful. Therefore, it is preferable to use numerical methods [4]. In [5–7], the mutual influence of different microasperities in the process of contact of regular rough surfaces was studied for the analysis of these methods and as a principal factor for the prediction of the serviceability of tribocouples.

### Statement of the Problem

In what follows, we study the stresses formed in an elastic half plane (two-dimensional problem) in contact with an uneven surface with periodic distribution of microasperities (Fig. 1).

In most cases, the model of microirregularities in the form of a periodic system of notches fairly adequately describes the topography of the surfaces obtained, e.g., after grinding, milling, planing, etc. [8, 9]. In addition, the model enables one to change the sharpness of the asperities (angle  $\gamma$ ), the distance between them  $l$ , and the width of their base  $d$  in order to get a sufficiently good approximation of the mean values of the parameters of actual roughness. In some cases, regular notched surfaces are specially created according to the operating or engineering requirements (working surfaces of clamping tools, the cases of high-speed conventional engines, decorative parts, etc.).

The unknown contact pressure is found as a solution of the following integral equation with Hilbert kernel [10]:

$$\int_{-l/2}^{l/2} p(t) \cot \frac{\pi(t-x)}{l} dt = q(x), \quad (1)$$

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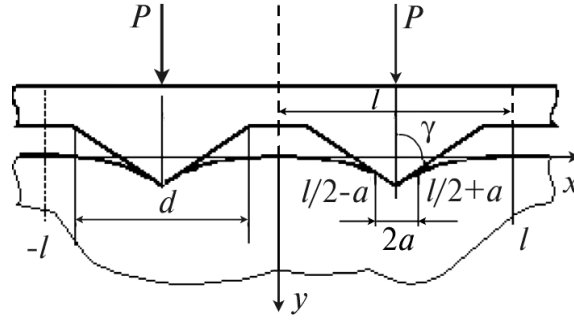


**Fig. 1.** Schematic diagram of contact interaction.

where  $q(x) = lE \cot \gamma$  for  $l/2 - a \leq x \leq l/2$ ,  $q(x) = -lE \cot \gamma$  for  $l/2 \leq x \leq l/2 + a$ ,

$$\frac{1}{E_*} = \frac{1 - \nu_p^2}{E_p} + \frac{1 - \nu^2}{E},$$

and  $\nu_p$  and  $E_p$  are, respectively, Poisson's ratio and Young's modulus of the rough surface.  
Under the conditions

$$p\left(\frac{l}{2} \pm a\right) = 0 \quad \text{and} \quad P = \int_{l/2-a}^{l/2+a} p(x) dx,$$

the solution of Eq. (1) has the form

$$p(x) = \frac{E_* \cot \gamma}{\pi} \ln \left| \frac{\sqrt{\sin^2(\pi a/l) - \cos^2(\pi x/l) + \sin(\pi a/l) \sin(\pi x/l)}}{\cos(\pi a/l) \cos(\pi x/l)} \right|. \quad (2)$$

In this case, the contact region is specified by the formula

$$P^* = \frac{\pi P \tan \gamma}{dE} = \frac{l}{d} \ln \left| \tan \left( \frac{\pi}{4} + \frac{\pi a}{2l} \right) \right|. \quad (3)$$

To find stresses in the elastic half plane, we use the expression for the complex potential of a two-dimensional periodic problem [3]

$$\Phi(z) = \frac{1}{2il} \int_L (p(t) + ig(t)) \cot \frac{\pi(t-z)}{l} dt \quad (4)$$

and the expressions for stresses

$$\sigma_{xx} + \sigma_{yy} = 4 \operatorname{Re} \Phi(z) \quad \text{and} \quad \sigma_{yy} - i\tau_{xy} = \Phi(z) - \Phi(\bar{z}) - (z - \bar{z}) \overline{\Phi'(\bar{z})}, \quad (5)$$

where  $p(x)$  and  $g(z)$  are, respectively, normal and tangential loads in the region  $L$  for a period  $l$ . By using the formulas for the principal stresses  $\sigma_1 + \sigma_2 = \sigma_{xx} + \sigma_{yy}$  and  $\sigma_1 - \sigma_2 = \tau_{\max}$ , we get

$$\begin{aligned}\tau_{\max} &= \left| \overline{\Phi(z)} + \Phi(\bar{z}) - (z - \bar{z})\overline{\Phi'(\bar{z})} \right|, \quad \sigma_1 = 2 \operatorname{Re} \Phi(z) + \left| \overline{\Phi(z)} + \Phi(\bar{z}) - (z - \bar{z})\overline{\Phi'(\bar{z})} \right|, \\ \sigma_2 &= 2 \operatorname{Re} \Phi(z) - \left| \overline{\Phi(z)} + \Phi(\bar{z}) - (z - \bar{z})\overline{\Phi'(\bar{z})} \right|.\end{aligned}\tag{6}$$

Relations (4)–(6) specify the required stresses.

### Solution of the Problem and Its Analysis

In numerical calculations, we use the following relations for the evaluation of stresses via the real variables  $x$  and  $y$  [10]:

$$\begin{aligned}\tau_{xy} &= -\frac{1}{2l} \left( \frac{\pi y}{2l} \right) \int_{-a}^a \frac{\sin(2\pi(x-s)/l) \sinh(2\pi y/l)}{(\sin^2(\pi(x-s)/l) + \sinh^2(\pi y/l))^2} p(s) ds \\ &\quad - \frac{1}{2l} \int_{-a}^a \frac{\Theta_+(x)}{(\sin^2(\pi(x-s)/l) + \sinh^2(\pi y/l))^2} g(s) ds, \\ \sigma_{xx} &= -\frac{1}{2l} \int_{-a}^a \frac{\Theta_+(x-s)}{(\sin^2(\pi(x-s)/l) + \sinh^2(\pi y/l))^2} p(s) ds \\ &\quad - \frac{1}{l} \left( 1 - \frac{2\pi y}{l} \sinh \frac{2\pi y}{l} \right) \int_{-a}^a \frac{\sin(2\pi(x-s)/l)}{(\sin^2(\pi(x-s)/l) + \sinh^2(\pi y/l))^2} g(s) ds, \\ \sigma_{yy} &= -\frac{1}{2l} \int_{-a}^a \frac{\Theta_-(x-s)}{(\sin^2(\pi(x-s)/l) + \sinh^2(\pi y/l))^2} p(s) ds \\ &\quad - \frac{1}{2l} \sinh \frac{2\pi y}{l} \int_{-a}^a \frac{\sin(2\pi(x-s)/l)}{(\sin^2(\pi(x-s)/l) + \sinh^2(\pi y/l))^2} g(s) ds,\end{aligned}\tag{7}$$

where

$$\begin{aligned}\Theta_{\pm}(x-s) &= \cos^2 \frac{\pi(x-s)}{l} \sinh^2 \frac{\pi y}{l} \eta_{\mp}(y) + \sin^2 \frac{\pi(x-s)}{l} \cosh^2 \frac{\pi y}{l} \eta_{\pm}(y), \\ \eta_{\pm} &= \sin \frac{\pi y}{l} \pm \frac{\pi y}{l}.\end{aligned}$$

Relations (7) specify the stresses acting in the half plane and relations (6) give the principal stresses used to determine the strength of materials in the contact couple.

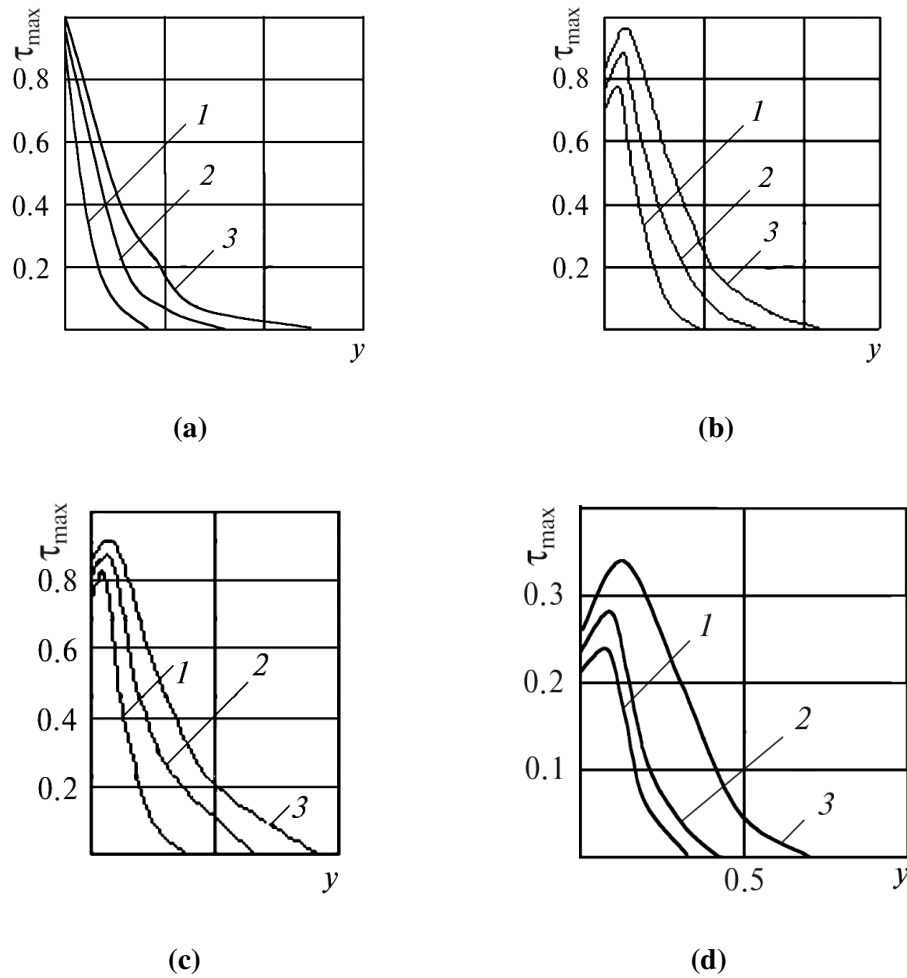


**Fig. 2.** Distributions of  $\tau_{\max}$  in the contact zone for various values of  $l/d$ : (1) 0.5, (2) 1, (3) 2: (a)  $x = l/2$ , (b)  $x = l/2 + a/4l$ , (c)  $x = l/2 + a/2l$ , (d)  $x = l/2 + a/l$ .

Note that the quantities  $\sigma_1$  and  $\sigma_2$  can easily be found if we know  $\tau_{\max}$  [see relations (6)]. Therefore, in what follows, we study the behavior of the quantity  $\tau_{\max}$  whose properties are determining for materials with pronounced plastic properties.

In our problem, we have  $g(x) \equiv 0$ . Therefore,  $\sigma_{xx}$  and  $\sigma_{yy}$  are the principal stresses along the  $Oy$ -axis (for  $x = 0$ ). This fact seriously facilitates the procedure of evaluation of  $\tau_{\max}$ . This quantity can be found in the analytic form as follows:

$$\tau_{\max} = \frac{|\sigma_{yy} - \sigma_{xx}|}{2} = \frac{E \cot \gamma}{\pi} \left( \frac{\pi y}{l} \right) \frac{\sin(2\pi/l)}{\sinh(2\pi/l) \sqrt{\sin^2(\pi a/l) + \sinh^2(\pi y/l)}}. \quad (8)$$

In Fig. 2, we present the distributions of the maximum tangential stress for various values of the parameter  $l/d$ . The distributions obtained for various  $x$  and fixed values of  $l/d$  are qualitatively similar to the presented distributions.

In relation (7), the integrand possesses a singularity under the center of a microasperity and vanishes as the distance from the surface approaches zero.

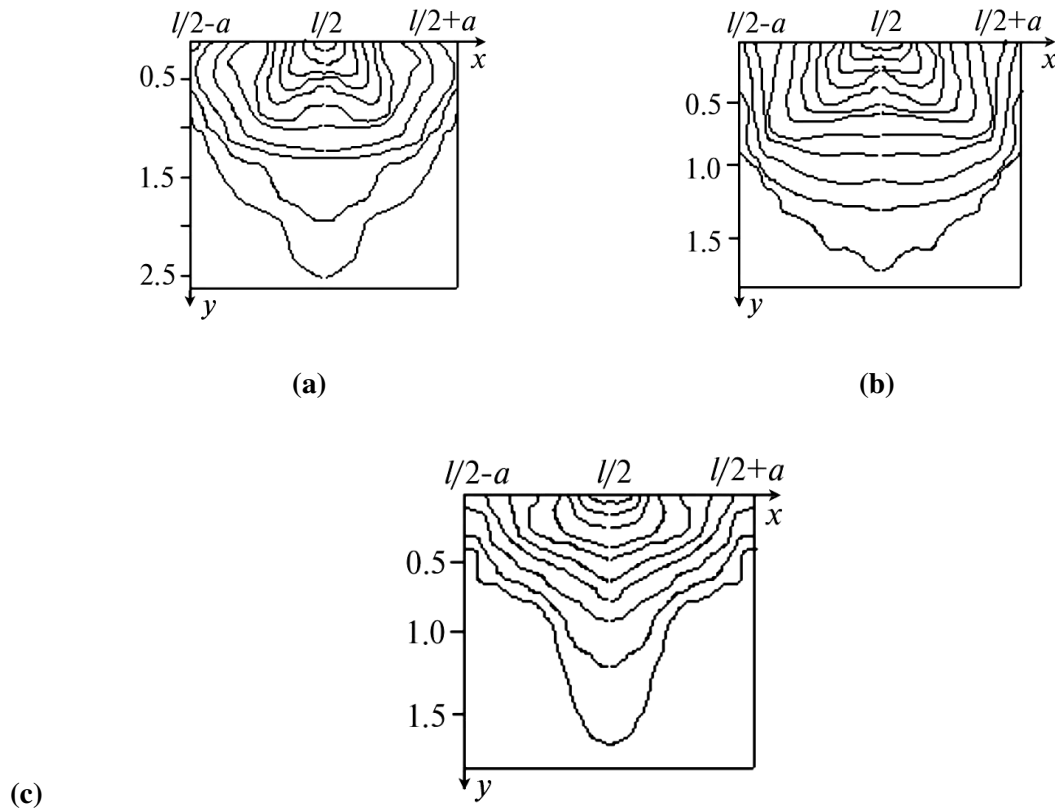


**Fig. 3.** Level lines of  $\tau_{\max}$  for various values of the parameter  $l/d$  inside the contact zone: (a)  $l/d = 0.5$ , (b) 1, (c) 2.

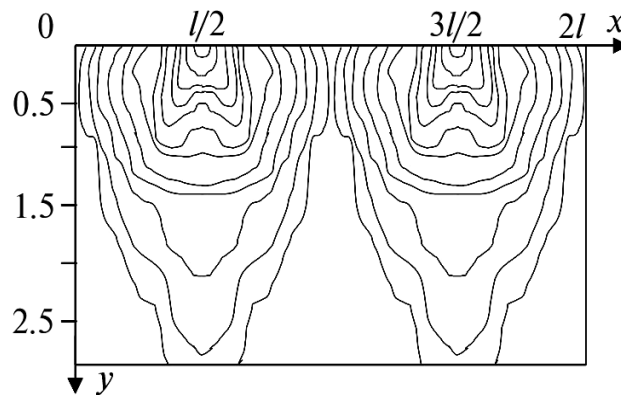


**Fig. 4.** Periodicity of level lines of the maximum tangential stresses.

To find  $\tau_{\max}$ , we use the methods of numerical integration performed by using a computer program written in Turbo Pascal. By using a uniform partition of the interval of integration with steps  $h = a/n$  and parabolic approximation in each step, we determine the value of the integral according to the formula

$$\int_{-a}^a f(x) dx = \frac{a}{3n} \left( f(-a) + f(a) + 2 \sum_{k=1}^{n-1} f(x_{2k}) + 4 \sum_{k=0}^{n-1} f(x_{2k+1}) \right).$$

The singularities of the integrand are found and separated by using small vicinities. In the course of numerical experiments, the size of the vicinities is directed to zero. To construct the plots, we use the Newton interpolating polynomial. The results of numerical calculations can be found in Figs. 3 and 4, where the quantities

$$\tau = \frac{\pi\tau_{\max}}{E \cot \gamma}$$

are given for  $P^* = 0.36$ .

The results presented in Fig. 2a are used for the verification of the numerical algorithm because the analytic relation (8) is true below the center of the microasperity. According to Fig. 2, the quantity  $\tau_{\max}$  attains its maximum value on the contact surface only under the center of the microasperity. At all other points  $x$ , the maximum value is attained at a certain distance from the surface and decreases as the distance from the center of the microasperity increases. The analysis of the data on the distribution of  $\tau_{\max}$  at the points

$$x = \frac{l}{2}, \quad x = \frac{l}{2} + \frac{a}{4l}, \quad x = \frac{l}{2} + \frac{a}{2l}, \quad x = \frac{l}{2} + \frac{3a}{4l}, \quad \text{and} \quad x = \frac{l}{2} + \frac{a}{l}$$

shows that the plots of the corresponding functions intersect. As follows from Figs. 3 and 4, the zones of high gradients of  $\tau_{\max}$  become more pronounced as the density of the distribution of microasperities increases. Despite some differences between the accepted scheme of contact interaction and the well-known schemes used in [2, 5, 11], the distributions of level lines of the maximum tangential stresses obtained by using these schemes are qualitatively similar (except the zone adjacent to the vertex of the asperity).

## CONCLUSIONS

The results obtained in the present work enable us to give a fairly complete description of the stressed state of the surface layers of triboelements with regular surfaces. To estimate the mutual influence of different microasperities, it is necessary to take into account not only the distances between them but also the entire complex of factors affecting the process of contact interaction, namely, the contact loads and physical, mechanical, and geometric characteristics of the contact couple. The results of determination of the maximum level of tangential stresses in the surface layers of the half plane and its location enable one to predict the regions of possible fracture (wear) caused by friction and, thus, substantiate the choice of materials for triboelements [12], the thickness of wear-resistant coatings [13, 14], and the parameters of surface hardening [15, 16].

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