

STRESS ANALYSIS OF A COMPOUND STRUCTURE UNDER STATIC LOADING

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A method for stress analysis of a special compound structure is stated. The structure, consisting of two reinforced cylindrical shells and a framework, is under longitudinal and lateral forces. The shells are analyzed using the finite-difference equations of the mixed method and the framework by the deformation method.

The stress analysis of systems consisting of different reinforced shells of revolution under local and other loads and complex fixation and boundary conditions is important theoretically and practically.

Designing discretely reinforced shells is the subject of many studies [1, 2, 4, 5, 7–10, etc.]. However, it is sometimes difficult to implement the techniques developed; therefore, approximate methods are widely applied.

The objective of the present paper is a numerical linear analysis of the stress–strain state of joined different shells under static loading.

The stress state of a multitiered structure consisting of a conic and two cylindrical shells (Fig. 1a) is analyzed. The conic shell has large cutouts and, therefore, can be regarded as a three-dimensional framework. The sections of its posts and crossbars include the adjoining parts of the casing. The posts are located symmetrically along the perimeter and, for a quarter of the perimeter, they coincide with the generatrices coming through nodes 1, 3, and 5 (the last node's post is more rigid than the others). The nodes may be thought of as rigid due to their structure. Some cells are connected by end rings (the conic shell may also have an intermediate ring — a two-tier frame) with cushion layers, which make it possible to transmit a vertical load as uniformly distributed.

A horizontal load is applied to the end of the upper shell and directed between the reinforced posts (1st type) or turned through an angle of 90° (2nd type). According to the superposition principle, the vertical and horizontal loads acting jointly are analyzed separately.

How fasteners are arranged on the upper and lower ends is shown in Fig. 1b, c, respectively. Bolts are at nodes 3 and 5 and pins are at nodes 1 and 6 (the lower joint) and at nodes 1, 4, and 6 (the upper joint). Owing to symmetry, only a quarter of the perimeter is shown. The sections of the end rings between nodes are assumed rectilinear.

When the action of the horizontal load is analyzed, the friction forces where parts of the structure are joined are neglected but the work of bolted and pin joints is taken into account, the former taking up normal and shearing forces and the latter only shearing forces. How these forces are distributed is unknown.

To solve the problem, a coordinate system is selected so that the structure is statically determinate. A quarter of the structure bounded by the planes of symmetry and antisymmetry is considered. The shells are analyzed using two differential equations of the mixed method and the framework by the deformation method. The forces of interaction between separate parts of the system are determined by the method of forces.

As mentioned above, the stress–strain state of the casing is determined using the technical theory of shells, whose equations are [4]

$$(E t)^{-1} \nabla^2 \nabla^2 \varphi - R w''_{\alpha\alpha} = 0, \quad D \nabla^2 \nabla^2 w + R \varphi''_{\alpha\alpha} - q R^4 = 0, \quad (1)$$

and the ribs are analyzed by the Kirchhoff–Clebsch theory of rods.

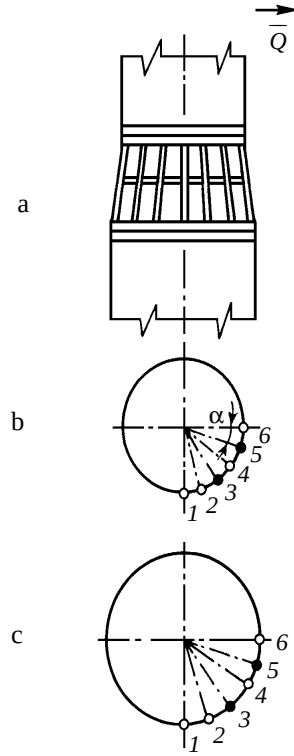


Fig. 1

Written in finite differences, these equations allow us to treat both smooth shells and shells discretely reinforced with stringers (s) or frames (f), an orthogonal rib mesh, or ribs of variable section, including zero-rigidity sections (breaks of reinforcements). The ribs may be of different rigidity and arranged either regularly or irregularly. The loads may be uniformly distributed, stepped, or stepwise-continuous.

Thus, for an intermediate point 0 in a square mesh ($R \Delta \alpha \times R \Delta \alpha$), we have [4]

$$\begin{aligned}
 (fF)_0 - \sum_{i=1}^4 (fF)_i + \sum_{i=5}^{12} (fF)_i - \sum_{i=1}^2 w_i + 2w_0 &= 0, \\
 (rw)_0 - \sum_{i=1}^4 (rw)_i + \sum_{i=5}^8 (rw)_i + 2 \sum_{i=9}^{12} (w)_i + \eta \left(\sum_{i=1}^2 F_i - 2F_0 \right) - \frac{q_0 (R \Delta \alpha)^4}{D} &= 0.
 \end{aligned} \tag{2}$$

Designating the node number by a subscript and the rigidity characteristics of a stringer or a frame by a superscript, we obtain the coefficients of Eqs. (2) in the form

$$\begin{aligned}
 f_0 &= I(B_0^{sf} + B_0^{fs}) - T_1^{sf-fs} - T_2^{sf-fs} - T_3^{fs-sf} - T_4^{fs-sf}, \\
 f_1 &= \tilde{I}(G_0^{sf-fs} + O_1^{sf-fs}), \quad f_5 = 1 - T_1^{sf-fs}, \quad f_9 = \hat{I}(U_3^{fs-sf} + U_1^{sf-fs}), \\
 f_2 &= \tilde{I}(G_0^{sf-fs} + O_2^{sf-fs}), \quad f_6 = 1 - T_2^{sf-fs}, \quad f_{10} = \hat{I}(U_4^{fs-sf} + U_1^{sf-fs}), \\
 f_3 &= \tilde{I}(G_0^{fs-sf} + O_3^{fs-sf}), \quad f_7 = 1 - T_3^{fs-sf}, \quad f_{11} = \tilde{I}(U_3^{fs-sf} + U_2^{sf-fs}), \\
 f_4 &= \tilde{I}(G_0^{fs-sf} + O_4^{fs-sf}), \quad f_8 = 1 - T_4^{fs-sf}, \quad f_{12} = \hat{I}(U_4^{fs-sf} + U_2^{sf-fs}).
 \end{aligned} \tag{3}$$

Here

$$\begin{aligned}
G^{\text{sf}-\text{fs}} &= B^{\text{sf}} - \nu B^{\text{fs}}, & O^{\text{sf}-\text{fs}} &= L^{\text{sf}} - \nu M^{\text{fs}}, & T^{\text{sf}-\text{fs}} &= L^{\text{sf}} + \nu^2 M^{\text{fs}}, \\
G^{\text{fs}-\text{sf}} &= B^{\text{fs}} - \nu B^{\text{sf}}, & O^{\text{fs}-\text{sf}} &= L^{\text{fs}} - \nu M^{\text{sf}}, & T^{\text{fs}-\text{sf}} &= L^{\text{fs}} + \nu^2 M^{\text{sf}}, \\
U^{\text{sf}-\text{fs}} &= L^{\text{sf}} + M^{\text{fs}}, & I &= 20 - 4(1 - \nu)^2, \\
\tilde{I} &= 8 - 2(1 - \nu), & U^{\text{fs}-\text{sf}} &= L^{\text{fs}} + M^{\text{sf}}, \\
\hat{I} &= 2 + \nu, & B^{\text{sf}} &= \beta a^s \mu^f, & L^{\text{sf}} &= \beta b^s \mu^f, & M^{\text{sf}} &= \beta c^s \mu^f, & a^s &= 1 + \nu \mu^s, \\
b^s &= 1 - \nu^2 \mu^s, & c^s &= 1 - \mu^s, & c^2 &= 1 - \mu^s, \\
F &= \varphi [(E t)_{\text{sh}} R \Delta \alpha^2]^{-1}, & \beta &= (1 - \nu^2) \mu^s \mu^f, \\
\mu^s &= (ES)^s [(ES)^s + (E t_{\text{sh}}) R \Delta \alpha]^{-1}.
\end{aligned} \tag{4}$$

The quantities $B^{\text{fs}}, L^{\text{fs}}, M^{\text{fs}}, a^f, b^f, c^f$, and μ^f are determined from (4) by appropriately permuting the indices, E, t, R, ν , and D are the elastic modulus, the thickness, the radius, Poisson's ratio, and the cylindrical rigidity of the shell, ES is the tension-compression rigidity of the ribs, and q is the radial load on the shell.

Equation (2) for the node 0 includes four sections of the stringer $(EJ)_{\text{I}} - (EJ)_{\text{IV}}$, which differ by flexural rigidity and have a length equal to the meshwidth, and as many sections of the frame $(EJ)_{\text{V}} - (EJ)_{\text{VIII}}$. The coefficients of this equation, where the Roman numerals (subscripts) designate the rigidities of the corresponding sections of the ribs, have the form

$$\begin{aligned}
r_0 &= 20 + \left\{ \alpha [(\vartheta_{\text{II}} + \vartheta_{\text{III}})(3 - 2\chi_{\text{II,III}}^2) + \vartheta_{\text{II}}\chi_{\text{I,II}} - \vartheta_{\text{III}}\chi_{\text{III,IV}}] \right\}^s \\
&+ \left\{ \alpha [(\vartheta_{\text{VI}} + \vartheta_{\text{VII}})(3 - 2\chi_{\text{VI,VII}}^2) + \vartheta_{\text{VI}}\chi_{\text{V,VI}} - \vartheta_{\text{VII}}\chi_{\text{VII,VIII}}] \right\}^f, \\
r_1 &= 8 + \left\{ \alpha \vartheta_{\text{III}} [4 + 2(\chi_{\text{II,III}} - \chi_{\text{III,IV}})] \right\}^s, & r_2 &= 8 + \left\{ \alpha \vartheta_{\text{II}} [4 + 2(\chi_{\text{I,II}} - \chi_{\text{II,III}})] \right\}^s, \\
r_5 &= 1 + \left(\alpha \frac{2\vartheta_{\text{III}}\vartheta_{\text{IV}}}{\vartheta_{\text{III}} + \vartheta_{\text{IV}}} \right)^s, & r_6 &= 1 + \left(\alpha \frac{2\vartheta_{\text{I}}\vartheta_{\text{II}}}{\vartheta_{\text{I}} + \vartheta_{\text{II}}} \right)^s, & \vartheta_{\text{I-IV}}^s &= \frac{(EJ)_{\text{I-IV}}^s}{(EJ)_{\text{arb}}}, \\
\chi_{\text{I,II}}^s &= \left(\frac{\vartheta_{\text{I}} - \vartheta_{\text{II}}}{\vartheta_{\text{I}} + \vartheta_{\text{II}}} \right)^s, & \chi_{\text{II,III}}^s &= \left(\frac{\vartheta_{\text{II}} - \vartheta_{\text{III}}}{\vartheta_{\text{II}} + \vartheta_{\text{III}}} \right)^s, & \chi_{\text{III,IV}}^s &= \left(\frac{\vartheta_{\text{III}} - \vartheta_{\text{IV}}}{\vartheta_{\text{III}} + \vartheta_{\text{IV}}} \right)^s, & \alpha_s &= \frac{(EJ)_{\text{arb}}^s}{R \Delta \alpha D},
\end{aligned} \tag{5}$$

$(EJ)_{\text{arb}}$ is the arbitrary rigidity of ribs, the quantities $r_3, r_4, r_7, r_8, \vartheta^f, \chi^f$, and α^f are determined from (5) by replacing stringer sections I-IV by frame sections V-VIII.

The conjugation equations for the end ring and the shell are

$$\begin{aligned}
q_\alpha(\beta) &- \frac{1}{R^2} \frac{\partial^2 \varphi}{\partial \beta^2} + \frac{(E t)_{\text{sh}}}{2R} (A^1 \beta + A^2 \beta) + \frac{\partial m_z(\beta)}{R \partial \beta} \\
&= \frac{(EJ_z)_k}{R^4} \frac{\partial^4 u}{\partial \beta^4} - \frac{(GJ)_k}{R^4} \frac{\partial^2 u}{\partial \beta^2} - \frac{(EJ_z + GJ)_k}{R^4} \frac{\partial^3 w}{\partial \alpha \partial \beta^2},
\end{aligned}$$

$$\begin{aligned}
m_{\beta}(\beta) - \frac{D}{R^2} \left(\frac{\partial^2 w}{\partial \alpha^2} + \nu \frac{\partial^2 w}{\partial \beta^2} \right) - \left[\frac{(EJ_{\beta})^s}{R^2} \frac{\partial^2 w}{\partial \alpha^2} - (eN_1)^s \frac{1}{R} \frac{\partial}{\partial \beta} \right] \\
= \frac{(EJ_{\alpha})_k}{R^3} \frac{\partial w}{\partial \alpha} - \frac{(GJ)_k}{R^3} \frac{\partial^3 w}{\partial \alpha \partial \beta^2} - \frac{(EJ_z + GJ)_k}{R^3} \frac{\partial^2 u}{\partial \beta^2}, \\
\frac{(EJ)_{sh}}{(ES)_k} \left(\frac{\partial \phi^0}{\partial n} - \frac{\partial \phi}{\partial n} \right) = \frac{\partial^2 \phi}{\partial \alpha^2} - \nu \frac{\partial^2 \phi}{\partial \beta^2}, \\
q_z(\beta) - Q_1^* + \frac{Q^s}{R} \frac{\partial}{\partial \beta} + \frac{\partial m_{\alpha}(\beta)}{R \partial \beta} = \frac{(EJ_z)_k}{R^4} \frac{\partial^4 w}{\partial \beta^4} + \frac{1}{R^2} \left(\frac{\partial \phi^0}{\partial n} - \frac{\partial \phi}{\partial n} \right). \quad (6)
\end{aligned}$$

Here q and m are the external distributed loads on the ring, EJ , GJ , and ES are the rigidities of the ring, Q_1 and Q^s are the lateral forces in the shell and in the ring, e and N_1 are the eccentricity and the longitudinal force in the stringer, u and w are the longitudinal and radial displacements, $\frac{\partial \phi}{R \partial n}$ and $\frac{\partial \phi^0}{R \partial n}$ are the derivatives along the normal to the shell contour without and with the ring (in the latter case, it is determined by the frame analogy from the external load), and $A^{1\beta}$ and $A^{2\beta}$ are the values of these quantities on the left and on the right of the stringer [7].

The contact finite-difference equations for the shell and the end ring [3] have the form:

(a) in the ring plane,

$$\begin{aligned}
\tau A_{m,n}^{\alpha} - (1 - \nu) F_{m,n} + F_{m-1,n} - 0.5\nu (F_{m,n+1} + F_{m,n-1}) - \Pi A_{m,n}^{0\alpha} = 0, \\
\eta (F_{m-1,n} - F_{m,n} + 0.5 A_{m,n}^{0\alpha}) + o w_{m,n} - 8 w_{m-1,n} + w_{m-2,n} - r (w_{m,n+1} + w_{m,n-1}) \\
+ s (w_{m,n+2} + w_{m,n-2}) + 2 (w_{m-1,n+1} + w_{m-1,n-1}) - (1 + \nu) K_{m,n} \\
+ 0.5 \nu (K_{m,n+1} + K_{m,n-1}) - p_{m,n} (R \Delta \alpha)^2 (D)^{-1} = 0, \quad (7)
\end{aligned}$$

(b) from the ring plane,

$$\begin{aligned}
(F_{m,n+1} - 2F_{m,n} + F_{m,n-1}) - 2\gamma (3 + \omega \Delta \alpha^2) (\Delta \alpha)^{-1} u_{m,n} \\
+ \gamma (4 + \omega \Delta \alpha^2) (\Delta \alpha)^{-1} (u_{m,n+1} + u_{m,n-1}) - \gamma (\Delta \alpha)^{-1} (u_{m,n+2} + u_{m,n-2}) \\
+ 0.5 \gamma (1 + \omega) (K_{m,n+1} - 2K_{m,n} + K_{m,n-1}) + q_{m,n} K(Et)_{sh}^{-1} = 0, \\
- \left\{ 0.5 (\eta)^{-1} + 0.25 \gamma [1 + 2\omega (\Delta \alpha^2)^{-1}] \right\} K_{m,n} + 0.25 \gamma \omega (\Delta \alpha^2)^{-1} (K_{m,n+1} + K_{m,n-1}) \\
+ 0.5 \gamma (1 + \omega) (\Delta \alpha)^{-1} (u_{m,n+1} - 2u_{m,n} + u_{m,n-1} + (\eta)^{-1} [(1 + \nu) w_{m,n} - 0.5\nu (w_{m,n+1} + w_{m,n-1}) - w_{m-1,n} = 0. \quad (8)
\end{aligned}$$

Here $\eta = [R^2 \Delta \alpha^4 (Et)_{sh}] \cdot D^{-1}$, $A_{cont} = \frac{2}{Et \Delta \alpha} \left(\frac{\partial \phi}{\partial n} \right)_{cont}$, $K_{cont} = 2 \Delta \alpha \left(\frac{\partial w}{\partial n} \right)$, A^0 is determined by the frame analogy from the given planimetric load, q and p are the loads on the ring, and

$$\tau = (2\mu)^{-1}, \quad \Pi = (1 - \mu) \tau, \quad o = 10 + 6\alpha, \quad r = 4(1 + \alpha),$$

$$s = 0.5 + \alpha, \quad \alpha = (EJ_\alpha) (DR \Delta \alpha)^{-1}, \quad \gamma = (EJ_z) [E t (R \Delta \alpha)^3]^{-1},$$

$$\omega = (GJ_{\alpha}) (EJ_z)^{-1}, \quad \mu = 2ES (2ES + EtR \Delta \alpha)^{-1}.$$

The equation for the horizontal displacement Δ of the end of the upper shell, where the bending load $\bar{Q}$ is applied, has the form [8]

$$\begin{aligned} o_\Delta \Delta + \eta \left(\sum_{n=2}^{k-1} \cos \beta_n F_{1,n} + 0.5 \cos \beta_k F_{1,k} \right) + \sum_{n=2}^{k-1} \psi_n w_{1,n} + 0.5 \psi_k w_{1,k} \\ + s_s \left(\sum_{n=2}^{k-1} \cos \beta_n w_{2,n} + 0.5 \cos \beta_k w_{2,k} \right) - 0.5 (k-1) (R^2 \Delta \alpha^3 / \pi D) = 0. \end{aligned}$$

Here

$$\begin{aligned} o_\Delta = 0.5 (k-1) o_d + s_d [\cos^2 0.5 \pi (k-1)^{-1} - \cos^2 0.5 \pi (k-2) (k-1)^{-1}] \\ - 2 r_d \sum_{n=2}^{k-1} \cos \beta_n \cos \beta_{n+1} + 2 s_d \sum_{n=2}^{k-2} \cos \beta_n \cos \beta_{n+2}, \\ \psi_n = -\bar{r}_s \cos \beta_n + (2 - \nu) (\cos \beta_{n+1} + \cos \beta_{n-1}), \\ o_d = 10 + 3 \alpha_s - 2 e_s^2 (s_s)^{-1}, \quad s_d = 0.5 (1 - \nu^2) (s_s)^{-1}, \quad r_d = 4 - 2 e_s (s_s)^{-1}, \\ \bar{r}_s = 6 - 2\nu + 2 \alpha_s, \quad \alpha_s = (EJ_\beta)_s (R \Delta \alpha D)^{-1}, \quad e_s = 1 + \nu + \alpha_s, \quad s_s = 1 + \alpha_s, \end{aligned} \quad (9)$$

k is the number of nodes along a quarter of the perimeter of the shell, n is the current mesh number, β is the angular coordinate of a mesh node reckoned from the line along which the force $\bar{Q}$ acts, and the subscript “s” corresponds to the stringer.

To analyze the lower shell, it is assumed fixed as a rigid body so as not to displace horizontally at the lower end and vertically at the upper end. It is assumed that $\varepsilon_2 = w = M_1 = 0$ at the lower end section and the forces N_1 are distributed linearly (the function F is determined as in a beam structure due to a bending moment).

To analyze the three-dimensional framework, we will present the well-known formulas [6], which should be written in a spatial coordinate system,

$$\begin{aligned} M_{ab} = 2 i_{ab} (2 \varphi_a + \varphi_b - 3 \psi_{ab}), \quad M_{ba} = 2 i_{ab} (2 \varphi_b + \varphi_a - 3 \psi_{ab}), \\ Q_{ab} = Q_{ba} = -6 i_{ab} / l_{ab} (\varphi_a + \varphi_b - 2 \psi_{ab}), \\ K_{ab} = -K_{ba} = i_{ab}^k (\varphi_a - \varphi_b), \quad \psi_{ab} = \psi_{ba} = \frac{\Delta_{ab}}{l_{ab}}. \end{aligned} \quad (10)$$

The angular displacements φ^r , φ^t , and φ^v relative to the coordinate axes are independent quantities and the displacements Δ^r , Δ^t , and Δ^v are dependent (Fig. 2), they being expressed in terms of the displacements δ along crossbars as follows [3]:

$$\begin{aligned} \Delta_{i,k}^r = \frac{1}{2 \sin \alpha/2} [\delta_{i,(k,k+1)} - \delta_{i,(k-1)}], \quad \Delta_{i,k}^t = \frac{1}{2 \cos \alpha/2} [\delta_{i,(k,k+1)} + \delta_{i,(k,k-1)}], \\ \Delta_{i,k}^v = -\frac{\cot \beta}{2 \sin \alpha/2} [\delta_{i,(k,k+1)} - \delta_{i,(k-1)}]. \end{aligned} \quad (11)$$

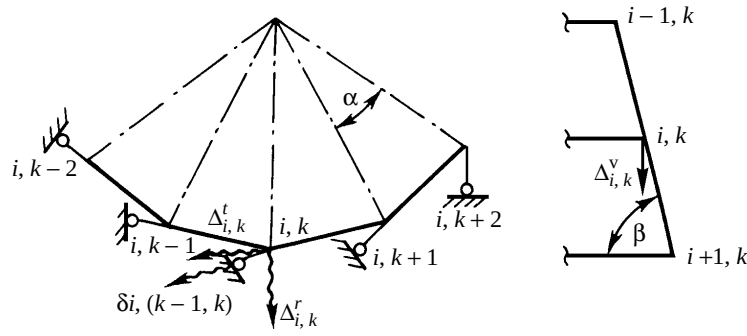


Fig. 2

Here the indices “ $k, k + 1$ ” and “ $k, k - 1$ ” are the second indices of the ends of the crossbars converging at the node i, k and $\Delta_{i,k}^v$ is the vertical displacement relative to the plane of the lower tier.

Both a plane node (a crossbar has a break and a post does not have) and a spatial node of the framework may have an independent component of vertical displacement $\Delta_{i,k}^v$, depending on the boundary conditions. In view of this component, the total displacement of the spatial node of a multitiered structure with all posts inclined at a constant angle can be expressed as follows, based on formulas (10) from [3]:

$$\Delta_{i,k}^v = -\frac{\cot \beta}{2 \sin \alpha/2} [\delta_{i, (k, k+1)} - \delta_{i, (k, k-1)}] + \bar{\Delta}_{i,k}^v. \quad (12)$$

Thus, the unknown $\Delta_{i,k}^v$ is replaced by the group independent unknown $\bar{\Delta}_{i,k}^v$ at a node in the lower section.

The system of compatibility equations for the strains of joined parts of the structure is set up based on the coefficients δ calculated independently for the framework and the shells from the equations for the work the forces of the state i do during displacements of the state k . The final values of δ are determined as the sum of these components.

In the upper shell, displacements are fixed (assumed zero) where the reaction forces are applied. In this case, the components of the coefficients $\delta_{i,k}$ are determined by the displacements of the state k , i.e., by the circumferential displacement v and the radial displacement w , respectively, at points of application of the unknown i introduced instead of the tangent and radial forces of interaction. The components of $\delta_{i,3}$ are determined by the displacement u of the point 3 at the lower end (Fig. 1). The displacements u and w are determined directly from the solution of the system of equations. Considering that

$$\varepsilon_2 = R^{-1} (v_\beta + w) = (E t)_{sh}^{-1} (N_1 - v N_2) = (E t R^2)^{-1} (\varphi_\alpha \alpha - v \varphi_\beta \beta), \quad (13)$$

and, consequently,

$$v_\beta = (E t R)^{-1} (\varphi_\alpha \alpha - v \varphi_\beta \beta) - w, \quad (14)$$

for the displacement v , we obtain

$$v = \int_0^{\beta_1} v_\beta d\beta = (E t R)^{-1} \int_0^{\beta_1} [(\varphi_\alpha \alpha - v \varphi_\beta \beta) - w] d\beta, \quad (15)$$

where the integration is performed over the section of the contour between the point with the displacement $v = 0$ and the point in question.

Expression (14) written in finite differences has the form

$$(v_\beta)_{m,n} = A_{m,n} - 2F_{m,n} + 2F_{m-1,n} - v (F_{m,n+1} - 2F_{m,n} + F_{m,n-1}) - w.$$

Replacing the integration in (15) by summation over area elements, we find

$$v_{m,n} = \left\{ -F_{m,0} - 2 \sum_{j=1}^{n-1} F_{m,j} - F_{m,n} + F_{m-1,0} + 2 \sum_{j=1}^{n-1} F_{m-1,j} + F_{m-1,n} + 0.5 A_{m,0}^{\alpha} + \sum_{j=1}^{n-1} A_{m,j} + 0.5 [A_{m,n}^{\alpha} - \nu (A_{m,0}^{\beta} + A_{m,n}^{\beta})] - 0.5 w_{m,0} - \sum_{j=1}^{n-1} w_{m,j} - 0.5 w_{m,n} \right\} \Delta \alpha.$$

Solving the system of compatibility equations for strains, we find the forces of interaction between the three-dimensional framework and the shells, which correspond to the load transmitted to the framework. A reanalysis of the three-dimensional framework under the joint action of the found unknown forces and the forces of the load state yields the final values of the unknowns in the deformation method and, consequently, the forces in separate elements of the framework.

The above-described technique was used to analyze a structure with two cylindrical shells (regardless of the lower shell) with different rigidity parameters and a structure in which the bending load acts at an angle of 90° relative to the principal case. In addition, experimental investigations were carried out.

The results of the analysis of the system under the load acting at an angle of 90° depend on the arrangement of posts relative to the planes of symmetry and antisymmetry and on the arrangement of bolts and pins connecting the shells with the framework. The lower shell may be neglected in the analysis with this load since the results of calculations with and without regard for the compliances of this shell differ insignificantly.

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